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Tectonic interaction between the Pamir and Tien Shan observed by GPS

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Abstract The complex tectonic interplay between the Central Asian Southwest Tien Shan and the north advancing Pamir as well as the role of the Pamir Frontal Thrust (PFT) separating these two orogens along the intervening Alai Valley is yet unclear. In this paper we present data of the newly installed Western Alai GPS profile (WAGP), capturing the deformation signal of both mountain ranges. The 20 km long WAGP records a maximum displacement rate of $9.3 \pm 0.8$ mm yr$^{-1}$. The lion’s share of displacement ($6.0 \pm 0.8$ mm yr$^{-1}$) is accommodated between the two stations located directly north and south of the PFT in 5 km distance. The WAGP data nicely complement the existing South Tien Shan and the Pamir GPS network data, which we present here in a combined reference frame and use it as input for horizontal block rotation/strain models. The model results show that both the Southwest Tien Shan and the Pamir behave as uniformly strained blocks and rotate counterclockwise (with respect to Eurasia) by $0.93 \pm 0.11^\circ$ Myr$^{-1}$ and $0.62 \pm 0.05^\circ$ Myr$^{-1}$, respectively. The Southwest Tien Shan undergoes NNE-SSW shortening of $-22.1 \pm 1.5 \times 10^{-9}$ year$^{-1}$ with an insignificant perpendicular extension. The Pamir is shortening with a rate of $-10.2 \pm 3.8 \times 10^{-9}$ year$^{-1}$ in a NNE-SSW direction, which is nearly 2.5 times less than its lateral extension rate. A band of increased deformation along the PFT is bounded to the north by the northern rim of the Alai Valley and extends up to 30–50 km south into the Pamir.

1. Introduction

Located in the northwestern part of the India-Asia collisional belt, the Pamir and Tien Shan are mountain belts formed by the same tectonic processes as the ongoing Indian-Eurasian collision (Figure 1). The Tien Shan is a mountain belt of $\sim 2000$ km length consisting of alternating ranges and valleys striking W-E to WSW-ENE. The belt width varies between $\sim 100$ km in the east and $\sim 350$ km in the west, where it is parted by the NW-SE oriented, right-lateral Talas-Fergana fault (TFR). West of the TFR the Fergana Valley splits the Southwest Tien Shan into the northern Chatkal-Kurama range system and the South Tien Shan with the series of E-W oriented, narrow ranges, and valleys (Figure 1). The Pamir is a high-mountain plateau elevated to 4000 m and more with a complex interior structure of an arcuate northward convex shape. Today, it acts as a relatively rigid indenter penetrating northward into the Eurasian plate and thus overriding the former Tajik-Tarim basin [Burtman and Molnar, 1993; Sobel et al., 2013].

The Pamir links to the western flank of the massive Tibetan Plateau with a mean altitude of 5000 m [Fielding et al., 1994]. The Pamir-Tien Shan region accommodates a higher deformation over a shorter distance compared to the Tibetan Plateau [Schmidt et al., 2011; van Hinsbergen et al., 2011] and is capable to produce magnitude 7 earthquakes in nearly decadal repeat times (ISC-EM catalog) [Storchak et al., 2013]. The last large seismic event in the vicinity was the 2008 magnitude 6.6 Nura earthquake with an epicenter just east of the Alai Valley [Sippl et al., 2014; Teshebaeva et al., 2014]. The seismic data indicating an inclined zone at the northern Pamir front and matching high strain rate at the surface suggest a subduction of the Tien Shan lithosphere beneath the Pamir [Burtman and Molnar, 1993; Strecker et al., 2003; Zubovich et al., 2010; Sobel et al., 2013] excluding the upper crust [Mechie et al., 2012; Sippl et al., 2013a, 2013b]. The Alai Valley, the last remnant of the formerly connected Tajik-Tarim basin, is squeezed between the South Tien Shan and the Pamir. It has an extent of $\sim 120$ km in lateral and $\sim 20$ km in longitudinal directions, an average elevation of 3000 m, and the Neogene sediment thickness reaches at least 3 km adjacent to the Trans Alai range (Figure 2a) [Armstrong and Strecker, 1999]. Its main tectonic features are the recently active Pamir Frontal Thrust (PFT) system (Figure 1), which runs parallel to the Pamir’s front and, a 10–15 km farther south but rooting in the same decollement, the Main Pamir Thrust (MPT) that was activated $\sim 25$ Ma ago [Sobol and Dumitru, 1997; Coutand et al., 2002; Sobel et al., 2013; Thompson et al., 2015]. Published displacement rates...
Figure 1. Topographic map of the Pamir and Southwest Tien Shan located at the northwestern end of the India-Asia collisional belt (inset). Brown lines here and on other figures are faults from Schurr et al. [2014]. The red line indicates the location of the Western Alai GPS profile (WAGP). The red arrow on the insert represents the northeastern advance of India (IGS station IISC) relative to the Eurasian plate.

Figure 2. (a) View of the Alai Valley from its western toward its eastern end (3-D Google™ image). The blue pins mark the WAGP stations along the Altyndara River. The stations are named from north to south as ALA1, ALA2, ALA3, and ALA4. The white dashed lines illustrate the data transfer directions and the brown line marks the PFT surface fault trace. (b) Simplified sketch of the N-S cross section of Alai Valley illustrating the Pamir overthrusting the Tien Shan. Topography after Figure 12d of Coutand et al. [2002].
Table 1. Locations, Horizontal Rates and Uncertainties of WAGP Stations

<table>
<thead>
<tr>
<th>Station</th>
<th>Longitude</th>
<th>Latitude</th>
<th>East</th>
<th>North</th>
<th>East</th>
<th>North</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALA1</td>
<td>72°10.748′</td>
<td>39°33.179′</td>
<td>−0.7 ± 1.6</td>
<td>8.4 ± 1.6</td>
<td>−0.5 ± 0.6</td>
<td>1.1 ± 0.7</td>
</tr>
<tr>
<td>ALA2</td>
<td>72°09.953′</td>
<td>39°31.592′</td>
<td>−1.4 ± 1.5</td>
<td>9.5 ± 1.5</td>
<td>−2.1 ± 0.7</td>
<td>2.6 ± 0.8</td>
</tr>
<tr>
<td>ALA3</td>
<td>72°15.104′</td>
<td>39°26.596′</td>
<td>−2.8 ± 1.6</td>
<td>11.0 ± 1.6</td>
<td>−6.8 ± 0.8</td>
<td>6.4 ± 0.8</td>
</tr>
</tbody>
</table>

across these marginal features span a wide range. Coutand et al. [2002] reconstructed rates of 0.6–0.8 mm/yr from balanced cross sections. Burtman and Molnar [1993] estimated rates of ~3.5 mm/yr based on the seismic moment release calculation of earthquakes between 1963 and 1988, and Arrowsmith and Strecker [1999] found ~6 mm/yr rates during the Holocene by measuring the displaced river terraces. The highest observed rate of the N-S convergence is between 10 and 15 mm/yr as derived from Global Positioning System (GPS) measurements [Zubovich et al., 2010].

The tectonically highly interesting area of the Pamir and Tien Shan was subject of various GPS research projects [Abdrakhmatov et al., 1996; Reigber et al., 2001; Mohadjer et al., 2010; Zubovich et al., 2010; Ischuk et al., 2013]. First GPS observations in the Tien Shan were carried out in 1992, and since then more than 300 GPS points were installed and measured. Tajik GPS observations started in 2007 and covered the Pamir and Tajik depression, although the Northeast Pamir was measured earlier within the campaigns mentioned above. The obtained data showed that the internal deformation of the Pamir is minor and that E-W elongation exceeds the N-S shortening [Ischuk et al., 2013]. The highest deformation rates in the Pamir-Tien Shan region are observed across the Alai Valley with at least 10 mm yr$^{-1}$ to possibly 15 mm yr$^{-1}$ [Zubovich et al., 2010]. The authors suggested that this high velocity gradient is accommodated by the PFT. However, due to the sparse spatial data sampling it has so far been impossible to define the exact distribution of the deformation. This motivated us to study the high deformation zone in the Alai Valley in more detail in order to better understand the Pamir-Tien Shan interaction and also the relation between tectonic faults like the PFT and seismicity.

2. West Alai GPS Profile

The Alai Valley with the PFT fault system located at its southern rim is a suitable area to study active fault strands and their behavior using GPS data. Some of the highest shortening rates in central Asia are observed here. Displacement rates of more than 10 mm yr$^{-1}$ over 20 km and data uncertainties of less than 1–2 mm yr$^{-1}$ allow relatively quick and reliable rate estimates.

In fall 2013 we installed a profile of four continuously operating GPS stations in the western part of the Alai Valley (Figure 2a and Table 1) to better identify the recent deformation pattern between the Pamir and the Tien Shan. The profile layout was partly chosen by the location of a clearly identified surface trace of the PFT system [Arrowsmith and Strecker, 1999; Strecker et al., 2003] (Figure 3). The area is relatively easy accessible and inhabited, which allows regular station maintenance. Limiting factors for the installation were the spatial extent of the profile (~20 km), the topography (high-frequency data transfer), and restricted site access due to the nearby state border.

The stations ALA3 and ALA1 mark the southern and northern ends of the profile. They were installed on bedrock outcrops of the Pamir Trans Alai and Tien Shan Alai ranges, respectively, on a metallic tube of 8 cm diameter and ~100 cm length. The inner stations ALA1 and ALA2 were installed on sediment on a 350 cm long metallic tube driven and concreted 160–180 cm into the ground. All stations are equipped with Septentrio (Global Navigation Satellite Systems) GNSS receivers and NavXperience antennas. Power supply of each of the stations ALA1, ALA2, and ALA3 is ensured by a 120 Wp solar panel and a 30 Ah battery for backup. The power management is controlled by a solar regulator, which also provides fail-over prevention and reboots the system every other day. The sampling rate is 10 s. All three stations are connected to the main station, ALAI, by 2.4 GHz radio communication links. While ALA1 and ALA3 send the data directly to the main station, ALA2 is routing the data through ALA1, using it as a bridge (Figure 2b). Being the master station, ALAI is built as consistent as possible with the Remotely Operated Monitoring Station concept [Schöne et al., 2013].
It transfers the data of all four stations to our processing center via the satellite system VSAT. ALAI is also equipped with a low-bandwidth satellite system link (Iridium) that is used for information reception and station management during VSAT outages. The ALAI GPS data are sampled at 1 Hz in accordance to Schöne et al. [2013].

3. Data Processing and WAGP Velocity Results

Standard GPS processing with GAMIT/GLOBK is generally performed in three steps [Herring, 2004; Herring et al., 2009]: (1) Calculation of daily solutions including station coordinates and covariance matrices (H files), (2) combination of the daily solutions to one campaign solution for the time period of each field campaign, and (3) the computation of velocity vectors on the basis of the campaign solutions. For the continuous Western Alai GPS profile (WAGP) data we skipped step (2) and combined the WAGP daily solutions with the GPS campaign data of the Tien Shan-Pamir region acquired since 1995 [Zubovich et al., 2010; Ischuk et al., 2013] into a common Eurasian-fixed reference frame tied by the 15 IGS (International GNSS Service) stations VILL, MADR, IRKT, NRIL, NVSK, POL2, ARTU, GLSV, POTS, WTZR, ONSA, NYAL, CAGL, WSRT, and KOSG (http://igs.org/network). By doing so, we can compare the current WAGP displacement rates with the existing campaign GPS rates. The resulting data set covers the vast territory from the Tarim basin and the Tajik depression to the Kazakh platform including the Tien Shan and the Pamir. The resulting relative rates of the WAGP stations sufficiently exceed the signal-to-noise ratio and can be used for further analysis. In Table 1 they present by the east and north velocity components parallel and perpendicular to the Pamir front—one of the main structures here.

The endpoints of the GPS profile ALA3 and ALA1 are located in the Pamir and the Tien Shan, respectively, and measure the relative motion of the margins of these mountain belts. They show a total relative displacement of $9.3 \pm 0.8 \text{ mm yr}^{-1}$ with an azimuth of $-43^\circ$ (measured clockwise from north). This is slightly less but within the error of the $10-15 \text{ mm yr}^{-1}$ of shortening observed at the eastern end of the Alai Valley [Zubovich et al., 2010]. Reasons for this lower shortening rate might be the fact that the motion of ALA3 is affected by the partially or fully locked MPT, or that there exist other active fault strands farther south in the Altyndara Valley, outside the WAGP. The highest rate change is observed between stations ALA3 and ALA2 with $6.0 \pm 0.8 \text{ mm yr}^{-1}$ over a distance of only 5 km along an azimuth of $-39^\circ$. We assume that the observed difference is associated with an active fault branch of the PFT system located between both stations (Figures 3 and 4).

Figure 3. PFT surface fault trace, highlighted by white arrows in Google™ aerial imagery and on personal field observations (a) with a view to the northwest.
Google™ aerial imagery nicely delineates the surface trace of this active fault with an azimuth of \(61°\) inside the ALA3-ALA2 sector (Figure 3). From this, we calculate a right-lateral fault-parallel rate component of \(5.6\pm0.8\) mm yr\(^{-1}\) and a fault-perpendicular shortening of \(2.2\pm0.8\) mm yr\(^{-1}\) between ALA3 and ALA2. Despite the relatively low shortening rate across the fault there is field evidence of at least 3 m of vertical offset on its southern side (insert on Figure 3). Undoubtedly, this fault was described by Arrowsmith and Strecker [1999] as clearly shown by a 4 m vertical offset (\(115°-120°\) strike, south side up). The fault was reactivated several times during Quaternary time, and Quaternary slip rate for this zone was defined at least 2.5 mm/yr and could be as high as 6 mm/yr.

The relative displacement rate between the stations ALA2 and ALAI is \(2.2\pm0.6\) mm yr\(^{-1}\) with an azimuth of \(42°\) over a distance of \(~12\) km. The data do not give evidence whether this deformation is due to another active fault strand north of ALA2 as published on several maps [e.g., Coutand et al., 2002; Strecker et al., 2003; Kalmetieva et al., 2009] or if this is a gradient due to a (partial) locking of the PFT. Finally, the relative displacement rates between the northernmost stations ALAI and ALA1 is \(1.2\pm0.7\) mm yr\(^{-1}\) to the direction of an azimuth of \(66°\) over a distance of slightly more than 3 km. Although for both ALA2-ALAI and ALAI-ALA1 segments the velocity differences are hardly distinguishable from the uncertainties, they do not conflict with the right-lateral strike-slip shift of the Pamir relative to the Tien Shan and the total trend of their convergence.

4. Interaction of the Pamir and Tien Shan

The last two large earthquakes occurred nearly two decades before the beginning of the GPS time series (the 1974, \(m_b=6.4\) Markansu earthquake) [Jackson et al., 1979] or at the outer extent of the study area, just east of the Alai Valley (the 2008, magnitude 6.6 Nura earthquake [Sippl et al., 2014; Teshebaeva et al., 2014]). We therefore assume that the GPS observations represent the interseismic phase of the current seismic cycle, where the faults are either locked or exhibit creeping. Using the full GPS data set, we model the Pamir and Southwest Tien Shan mountain belts as flat, uniformly strained bodies [Zubovich and Mukhamediev, 2010] (also see Appendix A for mathematical details), estimate the strain rate uniformity of these belts and define how the deformation on the WAGP relate to them. We thereby applied an iterative inversion procedure: First, we calculated the rotation and strain rate parameters separately for the Pamir and Southwest Tien Shan using two GPS data subsets. The resulting best fit parameters were used to forward model the horizontal velocity vectors. Then, we discarded the point with the largest residual between the modeled data and the observations. The procedure was repeated until the remaining data points showed residuals below a certain threshold. For the Pamir region we selected 10 GPS points with residuals below 1.3 mm yr\(^{-1}\) (black dots in Figure 5) and for the Southwest Tien Shan region 24 GPS points with residuals below 1 mm yr\(^{-1}\) (Figure 6). The stricter criterion for the Southwest Tien Shan data is justified by the larger amount of available GPS data and smaller data uncertainties. In addition, we propagated the GPS data uncertainties through the inversion (using the best fit model parameters) by adding a random Gaussian noise (normalized by the single data uncertainties) to the input data and repeated this procedure 1000 times, in order to obtain a statistical model parameter uncertainty estimation following [Metzger et al., 2011].
For the Pamir region, the best fit model nicely matches the input GPS observations in terms of azimuth and amplitudes (Figure 5). This leads to the conclusion that the interior of the Pamir is uniformly strained within data uncertainties. This observation does not support the findings based on microseismicity of Schurr et al. [2014] about the NNE-SSW trending Sarez-Karakul sinistral normal fault system (SKF on Figure 5). However, the recent 2015 magnitude 7.2 earthquake with the epicenter near Lake Sarez most probably reactivated exactly this fault (U.S. Geological Survey). One explanation might be that the deep seismic processes observed by Schurr et al. [2014] did not yet reach the Earth’s surface.

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The best fit strain rate parameters for the Pamir have a maximal strain rate $E_1 = 26.6 \pm 1.8 \times 10^{-9} \text{ year}^{-1}$ and a minimal strain rate $E_2 = 10.2 \pm 3.8 \times 10^{-9} \text{ year}^{-1}$ with an $E_2$ axis azimuth of $17.5 \pm 2.4^\circ$ in NNE-SSW direction (positive strain rates represent extension and negative rates are shortening). The extension rate is thus nearly 2.5 times larger than the orthogonal shortening rate, which can indicate gravitation-driven, lateral mass outflux of the Pamir as proposed by, e.g., Burtman [2013], Stübner et al. [2013], and Schurr et al. [2014]. The faster ESE-WNW lengthening with respect to the NNE-SSW shortening could be a sign of crustal thinning of the Pamir but that is apparently compensated by the subducting Tien Shan below the Pamir. The angular velocity of the Pamir rotation with respect to Eurasia is estimated with $0.93 \pm 0.11^\circ \text{ Myr}^{-1}$ counterclockwise around a pole at $61.2 \pm 1.7^\circ$ east and $33.1 \pm 0.6^\circ$ north.

The Pamir GPS data points, which were not used for the inversion but forward modeled (cyan dots, Figure 5), are located along the active northern perimeter of the Pamir, where strain is not uniform anymore. The closer the data points are to the Pamir’s margin, the higher the difference between the observed data and the model. This indicates that the northern frontal strip of the Pamir is deformed more rapidly than its internal parts. It might be possible that one or more active fault strands south of the PFT system (e.g., the Main Pamir Trust [Sobel et al., 2013]) absorb parts of the strain rate along the Pamir’s front. Another explanation could be that the Pamir’s front has a greater capability to be strained than its interior, due to the decreasing crustal thickness toward the north. In addition, the farther the Northern Pamir data points (except SHKA) are to the west, the smaller the east component of the observed velocities are compared to the model.

For the Southwest Tien Shan region the fit between model and observations is very good (Figure 6). We find a maximum strain rate $E_1 = -0.7 \pm 1.1 \times 10^{-9} \text{ year}^{-1}$ and a minimal strain rate $E_2 = -22.1 \pm 1.5 \times 10^{-9} \text{ year}^{-1}$ with an azimuth of $-19.8 \pm 2.4^\circ$. We thus observe a NNW shortening in the Tien Shan without any lengthening in ENE direction. The angular velocity of the Southwest Tien Shan with respect to Eurasia is estimated with $0.62 \pm 0.05^\circ \text{ Myr}^{-1}$ counterclockwise around a pole at $62.6 \pm 0.8^\circ$ east and $39.0 \pm 0.3^\circ$ north. This result agrees well with calculations of Zubovich and Mukhamediev [2010], where the field of horizontal velocity gradient was determined by a method of superimposed triangulations, as well as with the previously derived angular velocity of the Fergana Valley of $0.73 \pm 0.08^\circ \text{ Myr}^{-1}$ [Zubovich et al., 2010]. From this we conclude that the mountain parts of the Southwest Tien Shan covered by the GPS data presented here and the Fergana basin are in fact one uniform strain unit.

We also forward modeled the GPS points in the Alai Valley including WAGP stations and using the best fit model parameters both for the Pamir and for the Southwest Tien Shan (Figure 7). The Southwest Tien Shan model nicely predicts the data of the GPS points north of the valley, but it clearly overestimates the southern points located at the foot of the Trans Alai Range. The GPS velocities acquired close to the PFT system differ from the modeled velocities both in amplitude and azimuth. This could mean that the uniformly deformed, differently rotating moving Pamir and Tien Shan interact with each other along the PFT and...
thereby produce strike-slip and thrust displacements. A partial or complete locking of the faults enforces these displacements to be nonuniformly distributed around the PFT.

5. Conclusion

In this paper we present the first observations of the Western Alai GPS profile (WAGP, Figure 4 and Table 1), a profile of four continuous GPS stations installed in 2013 across the West of the Alai Valley, and the surface trace of the Pamir Frontal Thrust (PFT), where the highest deformation rates in the Pamir-Tien Shan region are revealed. These new GPS data help to better localize areas of high strain and identify the nature of slip of the local strand of the PFT system. In addition, the data nicely complement the existing GPS networks in the Pamir and the Southwest Tien Shan, which we present here in a combined reference frame.

Inversions of block rotation/strain models show that the inner part of both the Southwest Tien Shan (including the Ferghana Valley) and the Pamir behave as uniformly strained blocks rotating in horizontal plane with respect to Eurasia counterclockwise with the angular velocities of $0.62 \pm 0.05^\circ \text{Myr}^{-1}$ and $0.93 \pm 0.11^\circ \text{Myr}^{-1}$, respectively (Figures 5 and 6). This agrees well with previous results using different methods and data sets [Zubovich and Mukhamediev, 2010; Zubovich et al., 2010]. The Southwest Tien Shan exceeds a NNE-SSW shortening of $-22.1 \pm 1.5 \times 10^{-9} \text{year}^{-1}$ without any extension in perpendicular direction. The best fit model parameters for the Pamir region show that its shortening in a NNE-SSW direction of $-10.2 \pm 3.8 \times 10^{-9} \text{year}^{-1}$ is nearly 2.5 times lower than its lateral extension. This is further evidence for gravitational-driven lateral mass outflux of the Pamir as it was suggested before [Burtman, 2013; Stübner et al., 2013; Schurr et al., 2014]. A faster ESE-WNW lengthening than NNE-SSW shortening could lead to crustal thinning of the Pamir, but that is apparently compensated by subduction of the Tien Shan below the Pamir. The GPS data observed in and near the Alai Valley, which marks the Southwest Tien Shan and the Pamir border area, agree poorly with the model predictions. We conclude that along the PFT we find a zone of increased deformation, which is bounded by the northern rim of the Alai Valley on the north and in the south enters 30–50 km into the Pamir.

The 20 km long WAGP reveals a maximum displacement rate of $9.3 \pm 0.8 \text{mm yr}^{-1}$ between the outermost stations. Considering the fact that the southernmost station is located only 3 km from the fault surface trace, this agrees well with the $10–15 \text{mm yr}^{-1}$ measured earlier across the eastern end of Alai Valley [Zubovich et al., 2010]. The spatial resolution of the data points, however, does not give clear evidence if the total deformation on the PFT is accommodated by a single, locked fault or if the PFT is creeping freely and additional active strands without clear surface expressions exist. To answer this question and assess the locking degree of the PFT we extend the WAGP particularly toward the south and densify the station spacing with additional GPS points. From the deformation rates provided by the stations directly north and south of the PFT, however, we can separate the fault slip into a stronger right-lateral, fault-parallel component of at least $5.6 \pm 0.8 \text{mm yr}^{-1}$ and a less dominant fault-perpendicular shortening of at least $2.2 \pm 0.8 \text{mm yr}^{-1}$.

Appendix A: Inverting 2-D Linear Velocity Field

In the following, vectors and second-order tensors are designated by bold Latin small and uppercase letters. Symbols $\cdot$, $\times$, and $\otimes$ stand for the scalar, vector, and tensor (dyadic) products, correspondingly. Superscripts $^T$ and $^{-1}$ denote the transpose and inverse tensor.

Let a linear velocity field $\mathbf{v}(\mathbf{x})$ be realized in a plane material domain $\Omega$, i.e.,

$$\mathbf{v}(\mathbf{x}) = \mathbf{x} \mathbf{G} + \mathbf{b}, \quad \mathbf{x} \in \Omega, \quad (A1)$$

where $\mathbf{v}$ is the horizontal velocity of a material point possessing the radius vector $\mathbf{x}$, and the second-order tensor $\mathbf{G}$ and the vector $\mathbf{b}$ are constants throughout $\Omega$. By differentiating $\mathbf{v}$ with respect to $\mathbf{x}$ it is apparent that $\mathbf{G}$ is the velocity gradient (i.e., $\mathbf{G} = \text{grad} \mathbf{v}$) which can be uniquely decomposed into symmetric and antisymmetric parts by

$$\mathbf{G} = \mathbf{E} + \mathbf{W} \quad (\mathbf{E} = \mathbf{E}^T, \quad \mathbf{W} = -\mathbf{W}^T). \quad (A2)$$
Here \( \mathbf{E} \) is the constant strain rate tensor of the homogeneously deformed domain \( \Omega \); \( \mathbf{W} \) is the spin associated with the angular velocity vector \( \mathbf{w} \) by equation
\[
\mathbf{x} \cdot \mathbf{W} = \mathbf{x} \times \mathbf{w}, \quad (\mathbf{w} = \omega \mathbf{n}),
\]
which is valid for arbitrary vector \( \mathbf{x} \). In equation (A3) \( \mathbf{n} \) is a unit vector orthogonal to the deformation plane and \( \omega \) is the angular velocity of the domain which is positive for counterclockwise rotation when looking from the end of \( \mathbf{n} \).

Information about the real horizontal velocity field \( \mathbf{v}^{\text{real}}(\mathbf{x}) \) in \( \Omega \) is represented by the velocity vectors \( \mathbf{v}^{(i)} \) specified at the set of \( K \) discrete points (GPS stations) \( \mathbf{x}^{(i)} \in \Omega, \ i = 1, \ldots, K \). The problem is to determine \( \mathbf{G} \) and \( \mathbf{b} \) in equation (A1) in such a way as to find the best fit between \( \mathbf{v}^{\text{real}}(\mathbf{x}) \) and the model field \( \mathbf{v}(\mathbf{x}) \). For this purpose one can minimize the residual functional \( J \),
\[
J = \sum_{i=1}^{K} \left( \mathbf{v}^{(i)} - \mathbf{x}^{(i)} \cdot \mathbf{G} - \mathbf{b} \right) \cdot \left( \mathbf{v}^{(i)} - \mathbf{x}^{(i)} \cdot \mathbf{G} - \mathbf{b} \right) \to \text{min}
\]
for which the following equations should be solved
\[
\frac{\partial J}{\partial \mathbf{G}} = 0, \quad \frac{\partial J}{\partial \mathbf{b}} = 0.
\]
The solution of equation (A5) can be expressed as
\[
\mathbf{G} = \mathbf{C}^{-1} \cdot \mathbf{A}, \quad \mathbf{b} = \mathbf{v}^{0} - \mathbf{x}^{0} \cdot \mathbf{C}^{-1} \cdot \mathbf{A},
\]
where
\[
\mathbf{x}^{0} = \frac{1}{K} \sum_{i=1}^{K} \mathbf{x}^{(i)}, \quad \mathbf{v}^{0} = \frac{1}{K} \sum_{i=1}^{K} \mathbf{v}^{(i)}
\]
\[
\mathbf{A} = \sum_{i=1}^{K} \left( \mathbf{x}^{(i)} \otimes \mathbf{v}^{(i)} \right) - K \mathbf{x}^{0} \otimes \mathbf{v}^{0}, \quad \mathbf{C} = \sum_{i=1}^{K} \left( \mathbf{x}^{(i)} \otimes \mathbf{x}^{(i)} \right) - K \mathbf{x}^{0} \otimes \mathbf{x}^{0}.
\]
The desired result for the model field \( \mathbf{v}(\mathbf{x}) \),
\[
\mathbf{v}(\mathbf{x}) = \mathbf{v}^{0} + \left( \mathbf{x} - \mathbf{x}^{0} \right) \cdot \mathbf{C}^{-1} \cdot \mathbf{A},
\]
exists if \( \det \mathbf{C} \neq 0 \), i.e., the GPS stations are not located along a straight line.

The seeking strain rate \( \mathbf{E} \) and the angular velocity \( \omega \) are determined from the first relation in equation (A6) by using equations (A2) and (A3). The \( \mathbf{E} \)'s eigenvalues \( E_1 \) and \( E_2 \) (\( E_1 \geq E_2 \)) and orientation of principal axes are found from \( \mathbf{E} \) by conventional methods.

**References**


