Westfälische Wilhelms-Universität Münster

GEOPHYSIK WWU MÜNSTER

Magnetotelluric inversion with wavelet sparsity regularization

C. Nittinger[1], M. Becken[1]

[1]University of Münster, Institute of Geophysics, Correnstr. 24, 48149 Münster,

Contact: cnittinger@wwu.de

23.09.2015

EMTF 2015

Smoothing regularization in Magnetotellurics – an inevitable tool?

Occam's inversion: A practical algorithm for generating smooth models from electromagnetic sounding data

Steven C. Constable*, Robert L. Parker*, and Catherine G. Constable*

ABSTRACT

...stigate a new algorithm for computing regularized solutio... sional magnetotelluric inverse problem. The algorithm employs a... gradients (NLCG) scheme to minimize an objective function that p... als and second spatial derivatives of resistivity. We compare this al...

Occam's inversion to generate smooth, two-dimensional models from magnetotelluric data

C. deGroot-Hedlin* and S. Constable*

Abstract

...ree-dimensional magnetotelluric (MT) minimum structure inversion alg... t of the Occam approach. Computational costs associated with construct... el-space Occam approach to 3D MT inversion impractical. These diffic... matrix dimensions depend on the size of the data set, rather than the numb... space it becomes feasible to invert modest 3D MT data sets on a PC. To r... on is used for the iterative forward modeling code used to compute ... utational time by more than 70%, without affecting the inversion results. ...asonable fits can be obtained within a small number of iterations, with ...essary structure and find the model with minimum norm.
...4 Elsevier B.V. All rights reserved.

Magnetotelluric inversion for minimum structure

J. Torquil Smith* and John R. Booker*

ABSTRACT

Structure can be measured in terms of a norm of the derivative of a model with respect to a function of depth $f(z)$, where the model $m(z)$ is either the conductivity σ or $\log\sigma$. An iterative linearized algorithm can find models

Others result in s... quencies, a "red" fit... robust statistic is us... be made acceptably ...

Regularization is needed for a stable solution

Regularization

- Inverse problem:

$$\min\{\underbrace{\|G(m)-d^{obs}\|_2^2}_{data\,term} + \underbrace{\lambda\Phi(m)}_{regularization}\}$$

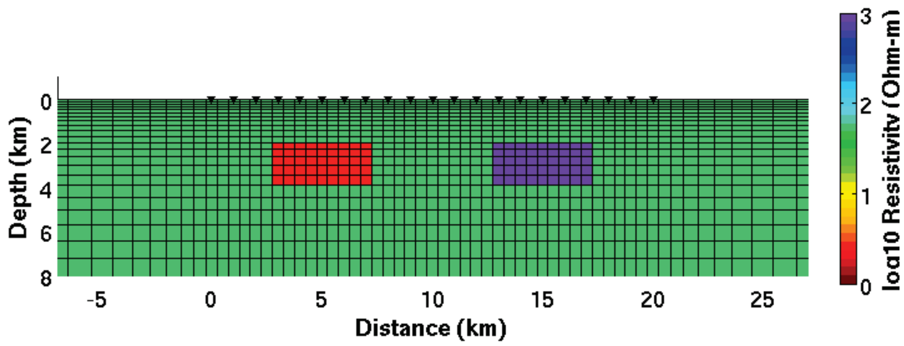
- Objective: smoothest possible representation of the model that fits the data
- non-linear and ill-posed problem

  → infinite number of solutions

- L2-Norm: $\Phi(m)=\sqrt{\sum|m|^2}$

  → smoothness

How to describe a resistivity model?



Usual inversion grid:

- Piecewise constant functions
- **Many** Blocks

Compressed model described by wavelets: [Daubechies, 1992]

$$m(x) = \sum_j^M c_j\Phi_j(x) = \underbrace{\sum_j a_j\Phi_j(x)}_{Approximation.\ Coeffs.} + \underbrace{\sum_j\sum_k d_k^j\Psi_{j,k}(x)}_{Detail\,Coeffs.}$$

j - translation
k - scale

- **few** coefficients

  → no loss of Information

Can we benefit from sparse inversion?

Sparse Inversion: Regularize for sparsity of wavelet coefficients

→ applications in various fields: medical imaging, image processing, signal analysis, seismic tomography

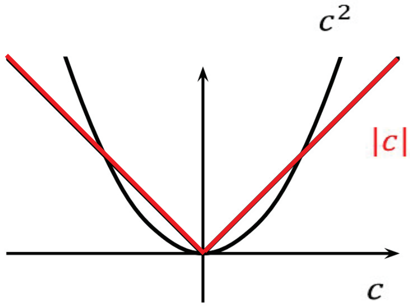
- No structural penalty
- Model structures dependent on chosen wavelet basis
  → can be sharp, smooth or both.

Regularization

- Inverse problem:

$$min\{ \underbrace{\| G(c) - d^{obs} \|_2^2}_{data\,term} + \underbrace{\lambda\Phi(c)}_{regularization} \}$$

- Objective: **sparsest** possible representation of the model that fits the data
- ~~L2-Norm: $\Phi(m) = \sqrt{\sum |m|^2}$ → smoothness~~
- L1-Norm: $\Phi(c) = \sum |c|$ → sparsity



Objective function in Wavelet domain

$$\Phi(c,m) = \| G(m) - d^{obs} \|_2^2 \quad + \quad \lambda \| c \|_1^1$$

- Wavelet Transformation: $m = Rc$
- Finite element forward code by Lee et al. (2009)

Block model domain		Wavelet domain
$G(m)$	→	$G(Rc)$
$J(m)$		$J(c) = J(m)R$

↓

$$\Phi(c) = \| W_e (G(c) - d^{obs}) \|_2^2 \quad + \quad \lambda \| c \|_1^1$$

Gauss Newton approach to solve linearized objective

- Linearized Iteration step:

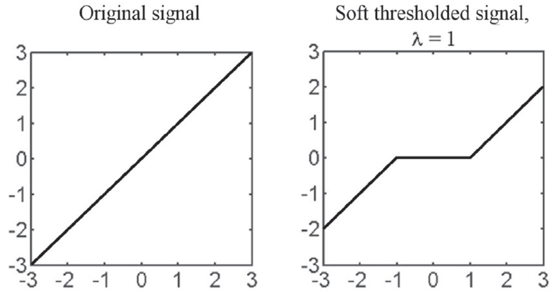
$$\| W_e (J(c^k) c^{k+1} - \hat{d}(c^k)) \|_2^2 \quad + \quad \lambda \| c^{k+1} \|_1^1 \quad \rightarrow \quad min.$$

$$\hat{d}(c^k) = d^{obs} - G(c^k) + J(c^k) c^k$$

- Solve with iterative soft thresholding (IST) by Daubechies et al. (2004)

$$c_{n+1}^{k+1} = Shrink_\lambda \left(c_n^{k+1} + (W_e J(c^k))^T (W_e (\hat{d}(c^k) - J(c^k) c_n^{k+1})) \right)$$

$$\mathrm{Shrink}_\lambda(c) = \begin{cases} c - \lambda & c \geq \lambda \\ 0 & |c| \leq \lambda \\ c + \lambda & c \leq -\lambda \end{cases}$$

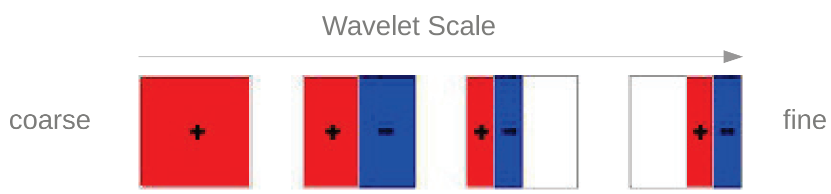


Inversion of magnetotelluric data in a sparse model domain

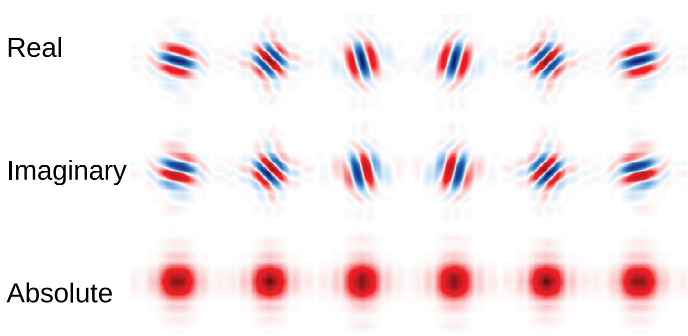
- Sparsity regularization
- Model Representation
- Synthetic examples

Wavelet functions

Simple Haar wavelet (only vertical):

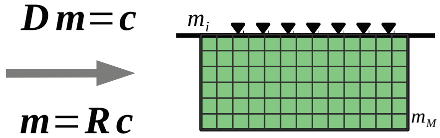


Complex dual tree wavelets [Kingsbury, 2001; Selesnik et al., 2005]:

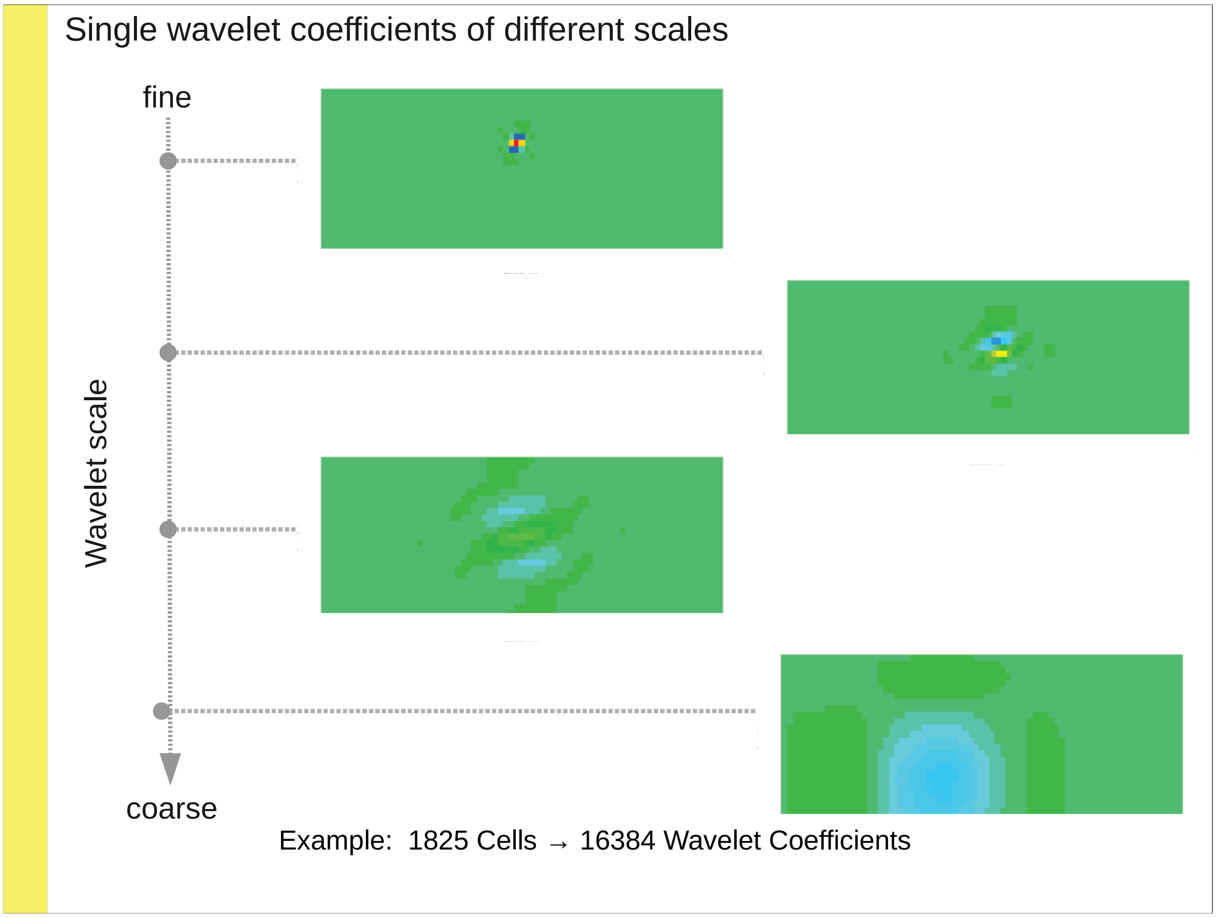


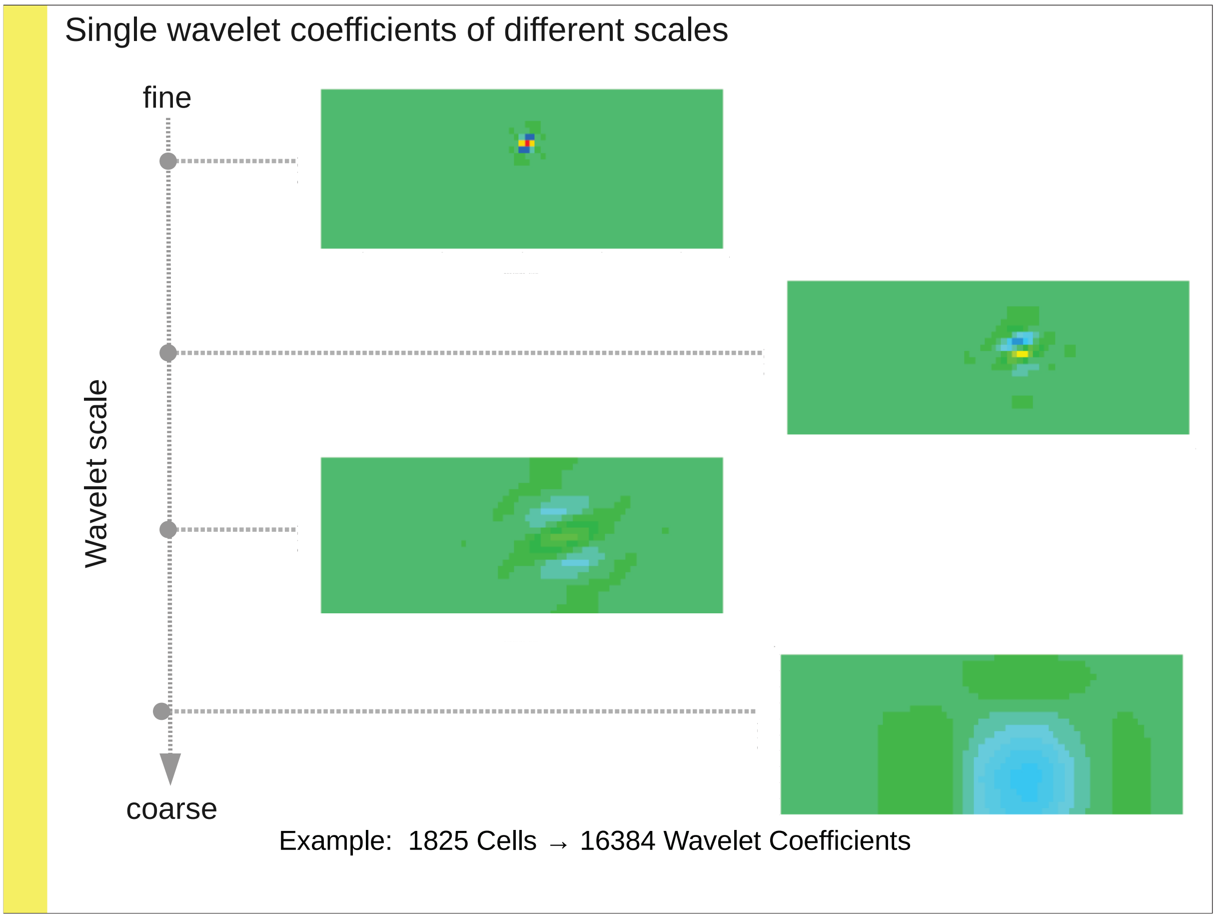
$$\mathbf{D}\,\mathbf{m}=\mathbf{c}$$

$$\mathbf{m}=\mathbf{R}\,\mathbf{c}$$

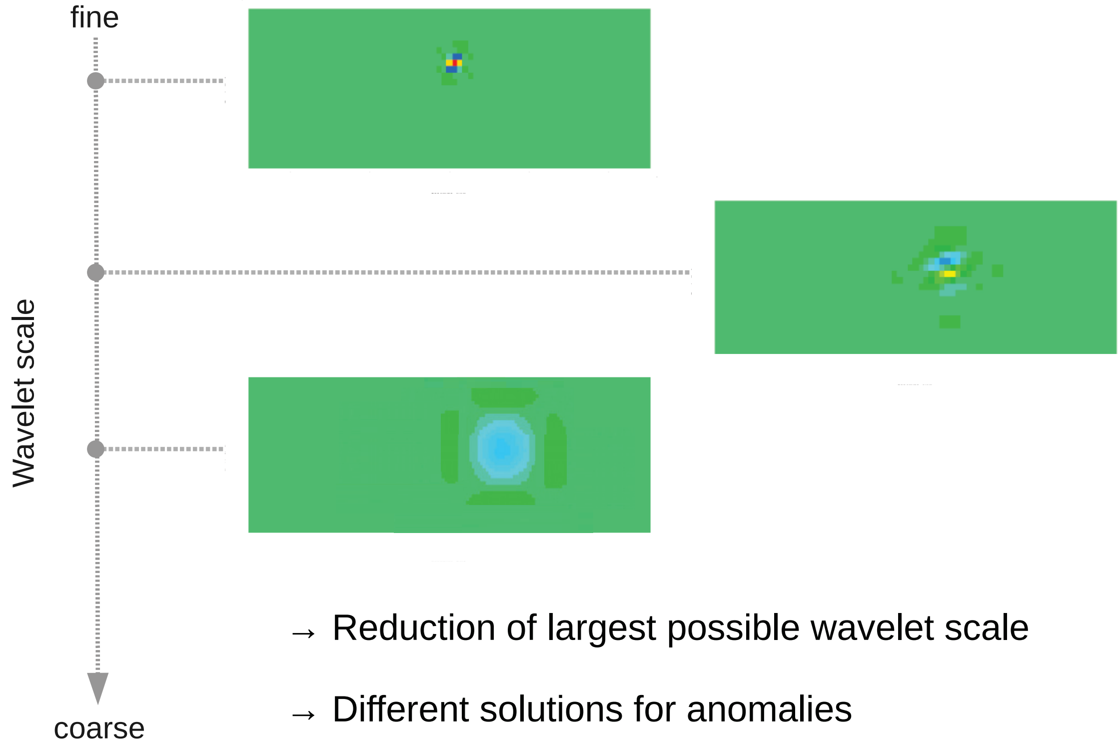


[Loris et al., 2007]


Single wavelet coefficients of different scales
fine
Wavelet scale
coarse
Example: 1825 Cells → 16384 Wavelet Coefficients


Single wavelet coefficients of different scales
fine
Wavelet scale
coarse
Example: 1825 Cells → 16384 Wavelet Coefficients

How to make use of different scales



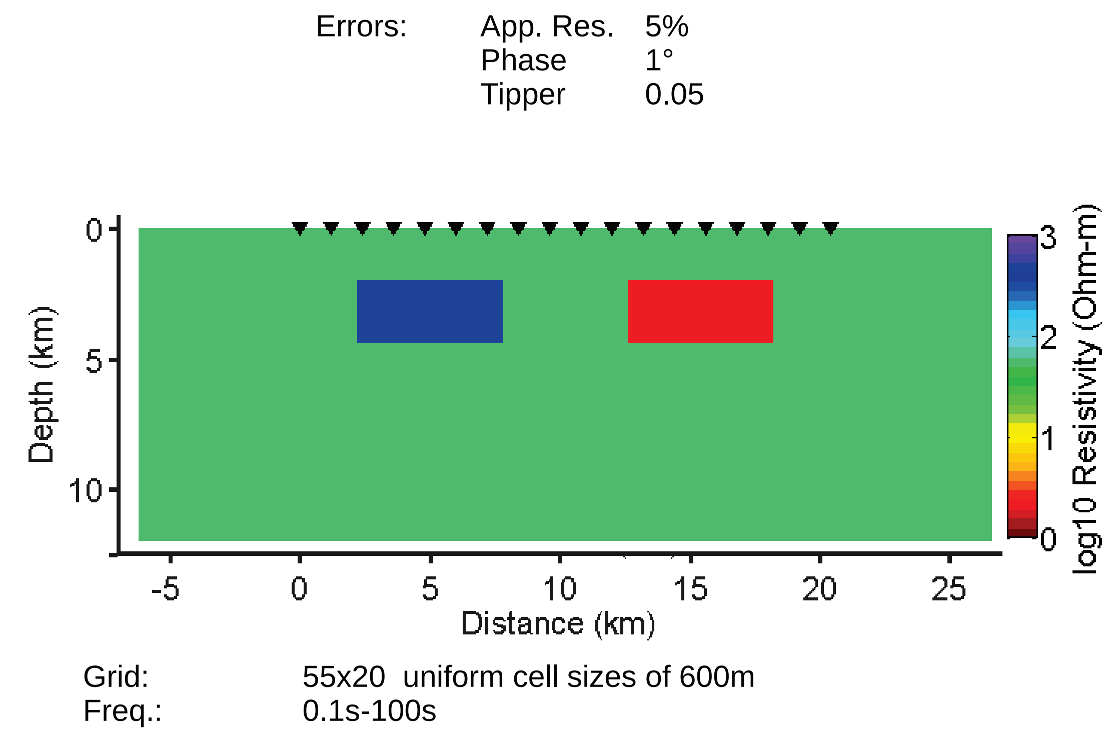
→ Reduction of largest possible wavelet scale

→ Different solutions for anomalies

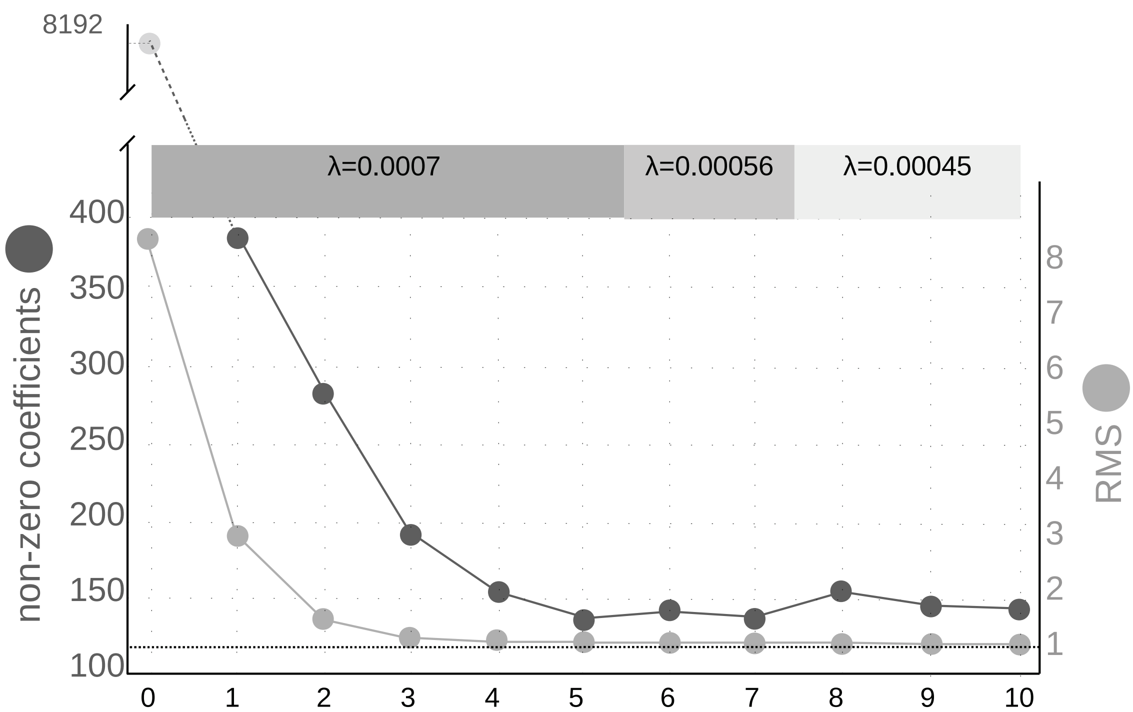
Inversion of magnetotelluric data in a sparse model domain

- Sparsity regularization
- Model Representation
- Synthetic examples

True model

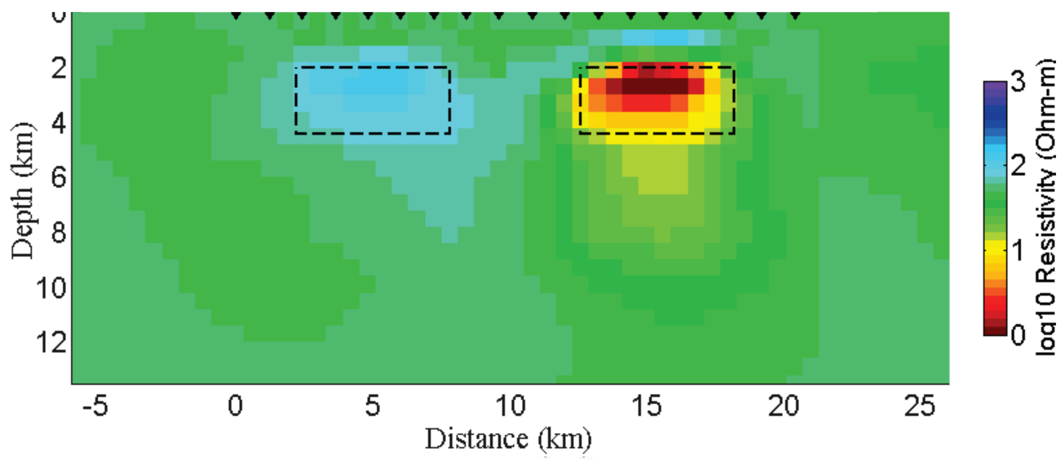


Sparsity of Iteration steps



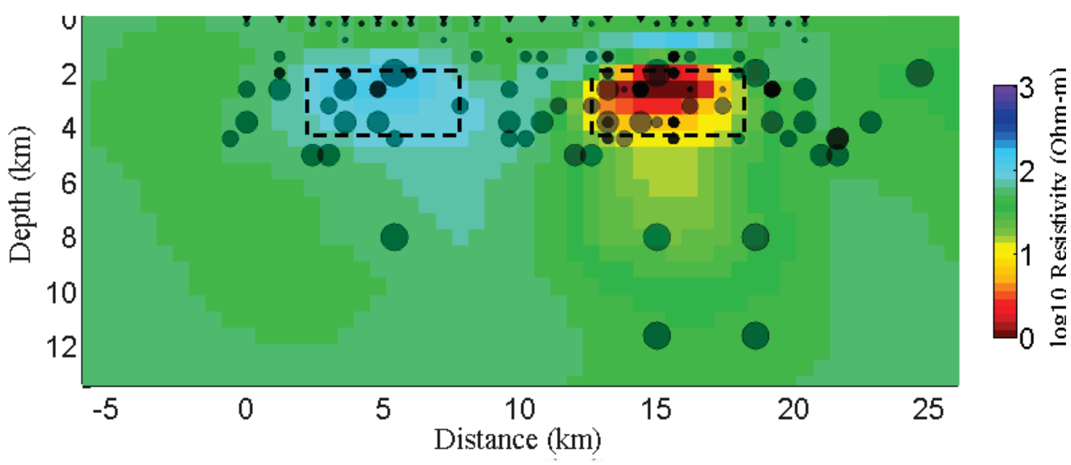
Inverse model

Coeffs: 143 RMS: 1.03

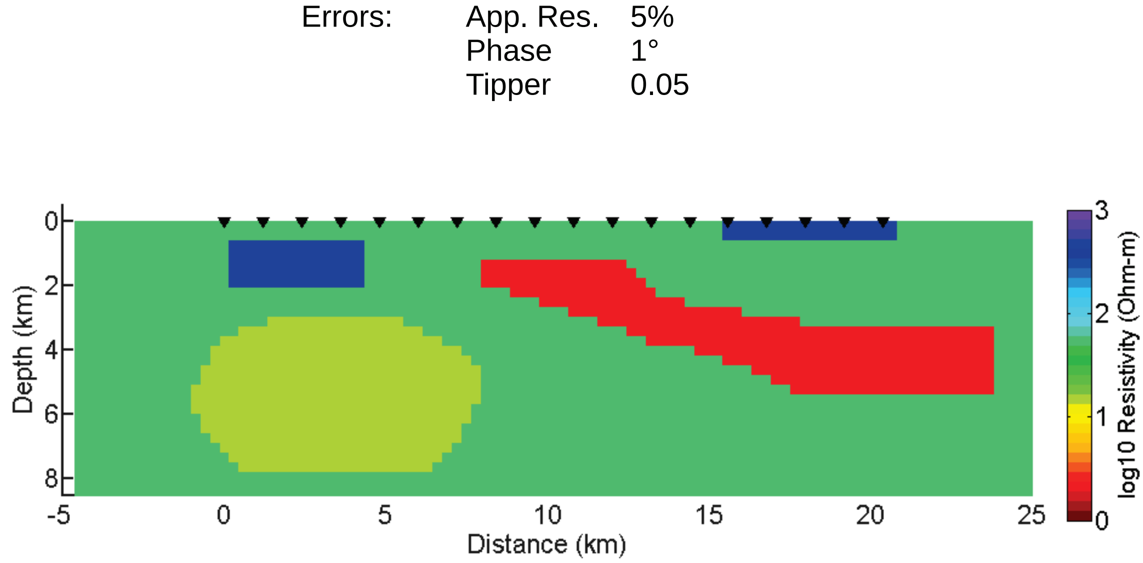


Inverse model

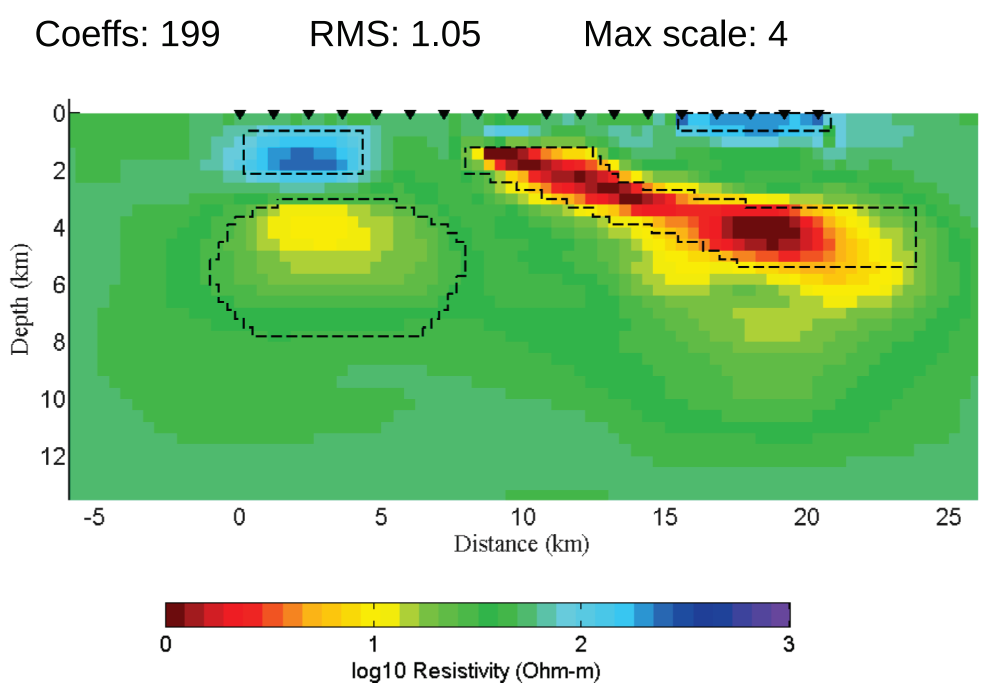
Coeffs: 143 RMS: 1.03



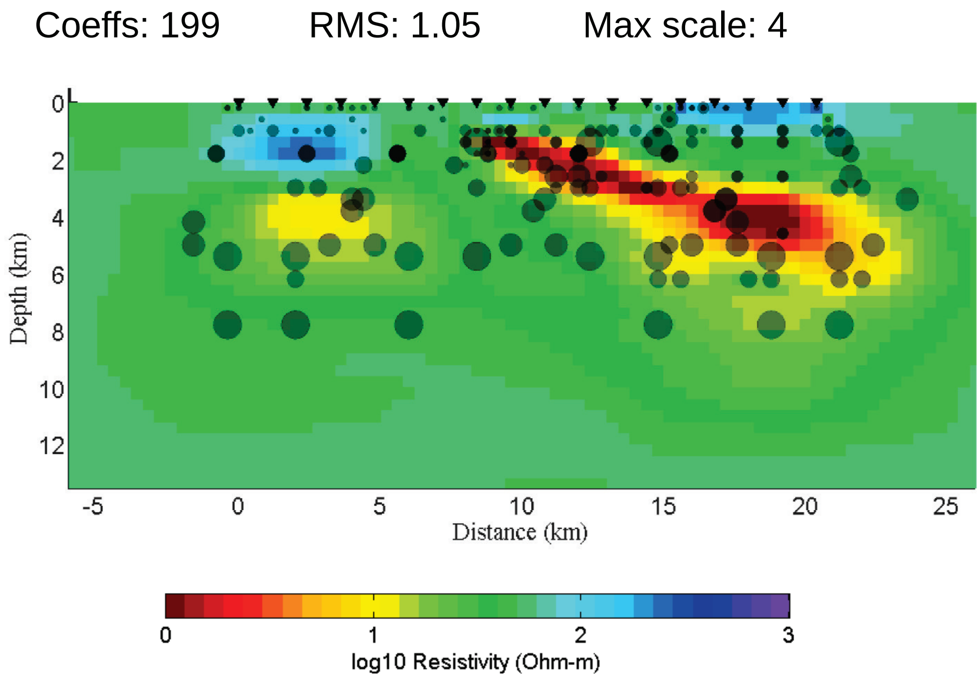
True model



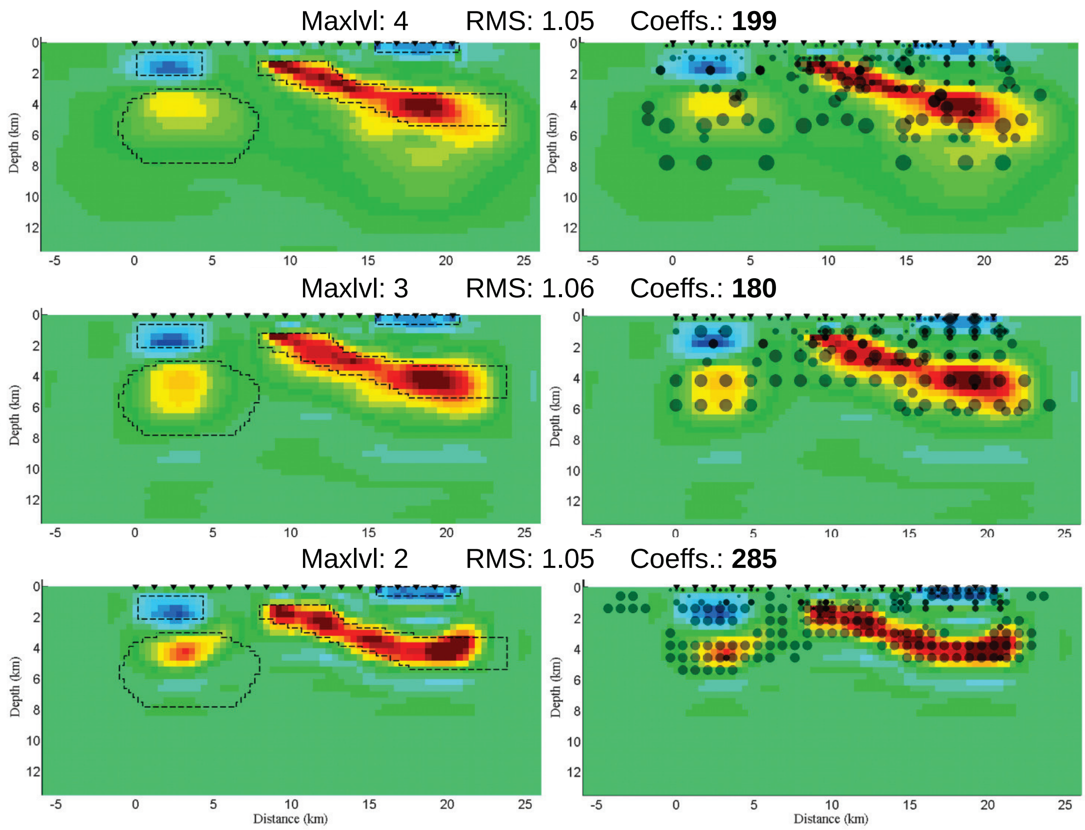
Inverse model



Inverse model

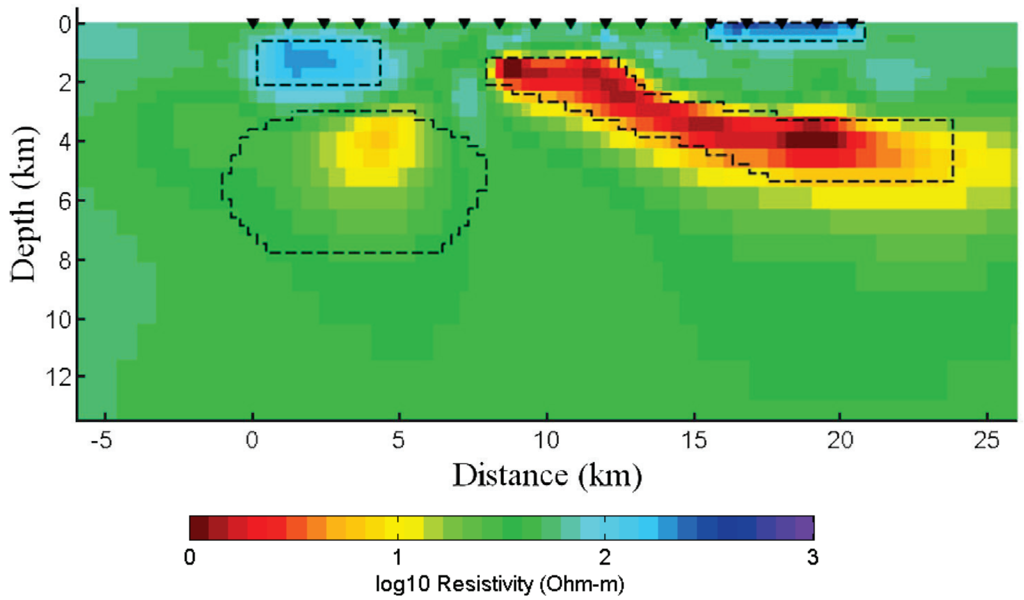


Reduction of maximal scale



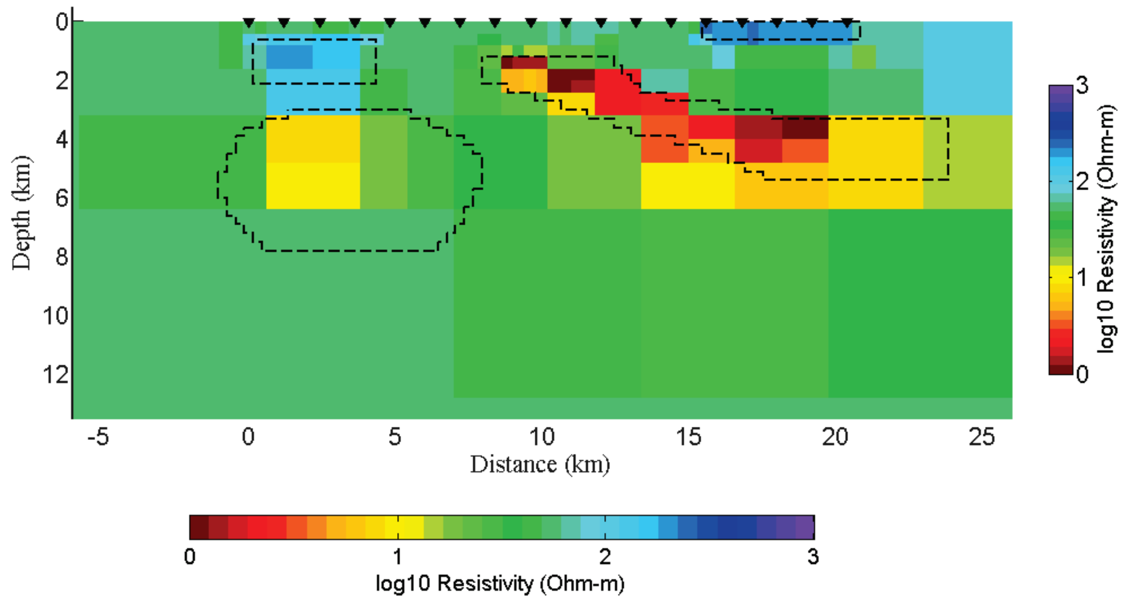
Smooth Inversion model is very similar

RMS: 1.02

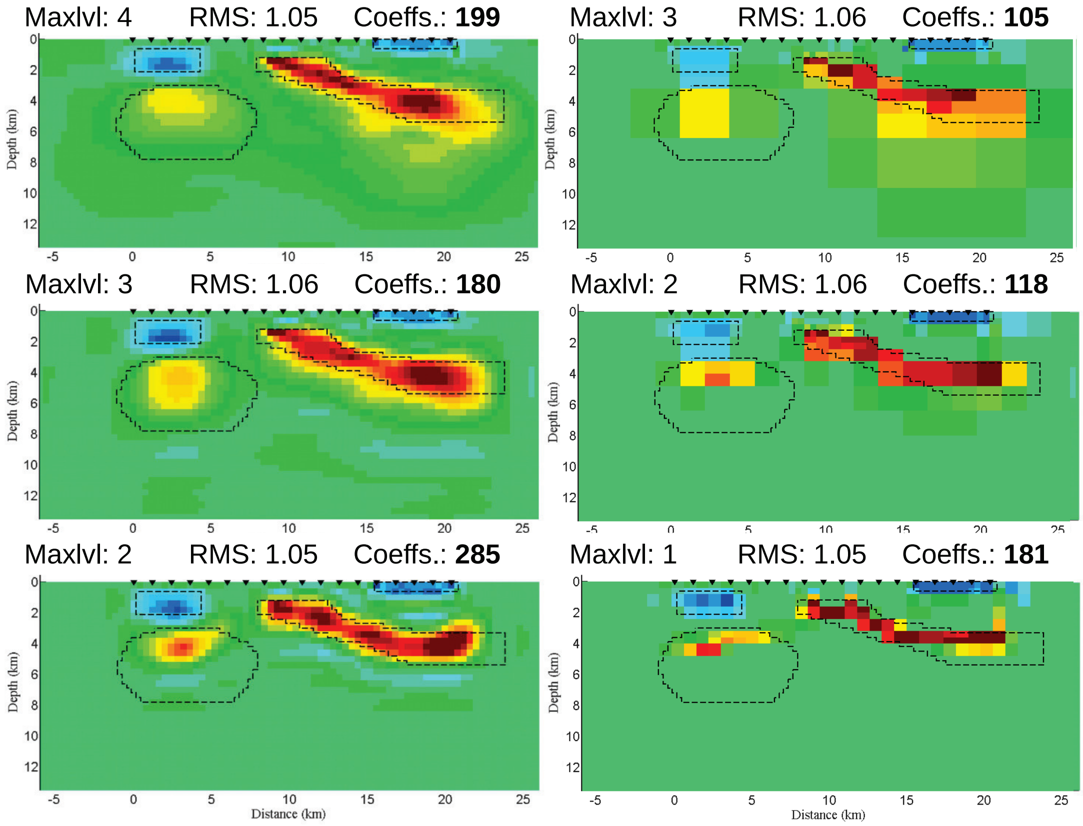


The Haar wavelet representation

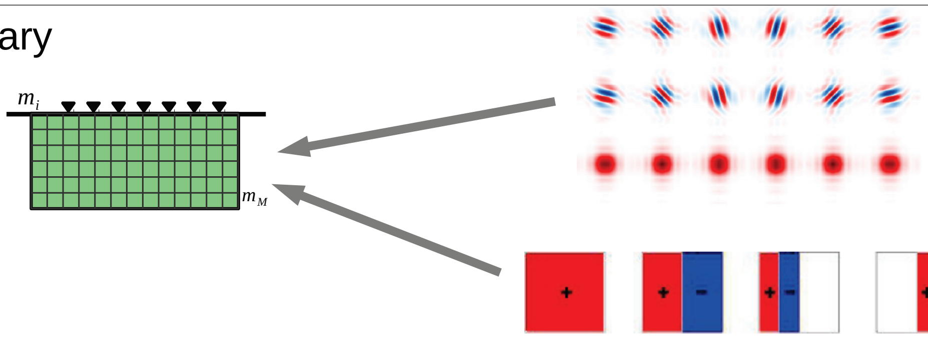
Coeffs: 96 RMS: 1.05 Max Scale: 4



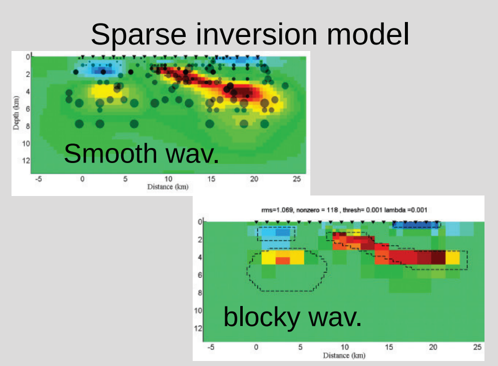
Inverse model



Summary



L1-Norm Regularization of *c*: $\Psi(c)=\lambda \| c \|_1^1$



Model Evaluation:

- Determine possible scales of anomalies
- Different representations allow comparison of different models
- Evaluation of non-unique model structures and artifacts

WESTFÄLISCHE WILHELMS-UNIVERSITÄT MÜNSTER

GEOPHYSIK WWU MÜNSTER

References

Daubechies, I., (1992). Ten lectures on wavelets, Society for Industrial and Applied Mathematics (SIAM).

Daubechies, I., Defrise, M., & De Mol, C., (2004). An iterative thresholding algorithm for linear inverse problems with a sparsity constraint, Communications on Pure and Applied Mathematics, 57(11), 1413-1457.

Donoho, D. L. (2006). Compressed sensing. Information Theory, IEEE Transactions on, 52(4), 1289-1306.

Kingsbury, N. (2001). Complex wavelets for shift invariant analysis and filtering of signals. Applied and computational harmonic analysis, 10(3), 234-253.

Lee, S. K., Kim, H. J., Song, Y., & Lee, C.-K., (2009). Mt2dinvmatlab -a program in matlab and fortran for two-dimensional magnetotelluric inversion, Computers & Geosciences, 35(8),1722-1734.

Loris, I., Nolet, G., Daubechies, I., & Dahlen, F., (2007). Tomographic inversion using l1-norm regularization of wavelet coffecients, GJI, 170(1), 359-370.

Selesnick, I. W., Baraniuk, R. G., & Kingsbury, N. C., (2005). The dual-tree complex wavelettransform, Signal Processing Magazine, IEEE, 22(6), 123{151.