High Accuracy Long Distance Measurement with Frequency Combs

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1. Introduction
The ability to determine absolute distance to an object is one of the most basic measurements of remote sensing. High precision ranging finds important application in future tight formation flying satellite missions, where rapid High Accuracy Absolute Long Distance Measurements (HAALDM) are critical for maintaining the relative pointing and position of the individual satellites, or movable parts therein. Frequency comb lasers [1] have the potential to revolutionize long distance absolute measurements in space by allowing a sub-micrometer accuracy of distances up to, and possibly beyond 10000 km. Comb lasers are pulsed (ultra-fast) mode-locked lasers with a precisely controlled repetition rate and phase of the pulses. Stabilizing the output of a femtosecond laser provides a spectrum of well-defined frequencies, originally used as a ruler in frequency space and to measure differences between optical frequencies. The periodic pulse train of a femtosecond laser generates a »comb« of equally spaced modes that can be stabilized by a phase-locked loop to a precision radio-frequency reference oscillator (e.g. an atomic clock) to achieve the same timing stability in the optical domain. This optical frequency synthesizer can be used to measure or synthesize not only almost any optical frequency but also to provide a multitude of well-stabilized frequencies for multi-wavelength interferometry and to link the time-of-flight domain of long-distance measurement with the interferometric regime of sub-wavelength accuracy. The basic concept is to use this incredibly regular pulse structure to measure a distance in units of the pulse separation length. Because units of length and time are fixed to each other by the definition of the vacuum speed of light, every timing measurement can be immediately translated into length with the same precision. Because the phase of every pulse is well controlled, one can measure a distance with sub-wavelength accuracy, even for pulses emitted from the laser at different times. In practical terms
this is achieved by comparing the position and the phase of pulses from the comb laser with those reflected from an object (such as another satellite) in a Michelson-type setup. A generalized scheme of such a technique is shown in Figure 1. Several approaches have been suggested for Michelson-type long distance measurements based on comb lasers and here we have compared their projected performance and suitability for a typical near earth surveying mission. Based on a formation flight scenario with up to 500 km distance. Apart from the Michelson itself there would be two major technical components in such a system, namely the comb laser unit itself and the detection unit. The careful selection of these components will be crucial for system performance in outer space.

2. Frequency comb laser source

Regarding the comb laser itself, the most commonly used systems employ Ti:sapphire, Cr:forsterite, Cr:LiSAF, Er:fibre or Yb:fibre gain media. However, there are several examples of other comb sources, most notably Cr:LiSAF and Cr:forsterite, and there are even more potential candidates of laser materials from which frequency combs can be generated. For a comparison of femtosecond comb generators with respect to future space missions, a list of parameters is given below:

- Power efficiency (diode pumped systems)
- Uncritical alignment and robustness (few-element, integrated configuration)
- Possibly all-waveguide configuration
- Compactness (small size and weight, including the pump set-up)
- Low noise operation
- Power scalability (boost amplification)
- Space environment compatible, radiation hardness
- Lifetime of critical elements, e.g. pump diodes, gain materials, actuators ...

A very general comparison of all-solid-state femtosecond sources on a reduced set of criteria, leaving out the aspect of radiation hardness and mission specific aspects which are to be discussed elsewhere, shows that Cr:LiSAF and erbium as well as ytterbium lasers are particular promising. The electrical-to-optical efficiency is particularly important for space applications because of limitations in power consumption and heat dissipation. The electrical-to-optical efficiency should be particularly good for directly diode-pumped systems with a low quantum defect, which gives Cr:LiSAF, Er:fibre and Yb:fibre lasers an advantage over Ti:sapphire, Cr:forsterite and Cr:YAG lasers. Also, the pump diode lifetime in the 940-980 nm range is much longer than in the 660-690 nm range. From the engineering point of view, fiber laser oscillators are superior to free space optics oscillators, because of their mechanical stability and optical guidance properties. It appears also obvious that the Ti:sapphire laser, a commonly used laboratory »work horse« for optical frequency combs, is unlikely to be a candidate for space missions, mainly because of the pump wavelength requirement of 532 nm (or 514 nm). Also, the critical alignment makes Kerr-lens mode-locking less attractive than Soliton-SESAM mode-locking. Of the all-solid state bulk lasers, Cr:LiSAF looks more promising...

Figure 2: Stabilization of the offset-frequency $f_0$ and the repetition rate $f_r$ in a standard fiber laser with a free-space section. The cavity length is tuned with a piezo transducer (PZT) and translation stage for coarse adjustment and locking of the repetition rate, while the laser diode (LD) pump power is used for locking of the offset frequency. (OC is the output coupler, PBS a polarizing beam splitter and QWP a quarter-wave plate)
because of direct diode pumping. A portable battery-powered version of this comb generator has already been demonstrated by Menlo Systems. With respect to all-waveguide confined lasers the technical readiness of Er:fiber is presently slightly more advanced than Yb:fiber. We expect, however, that Yb:fiber comb lasers will be developed substantially over the upcoming years, and that they will be finally more attractive to remote space applications because of better efficiency and radiation hardness. A generalized set-up of a typical fiber comb laser is shown in Figure 2.

3. Distance detection scheme

The other choice, that has to be made, is the method for detection of the pulse position beyond the interferometer. For accuracies down to the 10 µm level, it is sufficient to use fast photodetectors and timing detection via electronic mixers for a Time Of Flight measurement (TOF) [2,3]. For sub-wavelength accuracy in the nanometer range we have singled out spectral interferometry (SI) [4,5] (for schematics see also Figure 3) as the preferred method over the alternative method of temporal SHG-interferometric autocorrelation (IA) [6,7]. A direct comparison on various technical specifications between these three different approaches is given in table 1. A further approach based on comblocked multi-wavelength interferometry has been suggested recently [8], and is based on the MSTAR technique [9]. Compared to SI, however, the use of stabilized selected wavelengths makes such approaches always inferior to the use of the full comb bandwidth. The advantages of SI over IA include fast single-shot detection, no pulse-overlap required, relaxed dispersion requirements, and a much bigger tolerance for movement during distance measurement on the order of km/s. A possible issue with spectral interferometry is the use of a spectrometer. Because the position is measured using interference fringes in the spectral domain, the accuracy of the spectrometer frequency axis influences the precision of the measurement. However, this only plays a role at the highest precisions (< 1/10th of the laser wavelength). The required spectrometer accuracy needed in that case (better than a few times 10⁻⁴) is still quite feasible. One could for this purpose include in-flight calibration of the spectrometer with the comb laser itself.

Using spectral interferometry, the detector (such as a CCD camera or photodiode array) will record an oscillatory pattern as a function of the frequency components present in the comb pulses:

\[ \phi(\omega) = (1) \]

The phase in Eqn. 1 consists of three parts. The distance information is encoded in the first term, \( \Delta \omega t \), where \( \omega \) is the optical (angular) frequency.
quency, and \( t \) is the time delay between the arrival of the pulses. The delay can be written as \( \tau = \frac{2(D-L)}{c} \), if the time is taken modulo the repetition time of the comb laser. The second term accounts for the carrier-envelope phase shift between the pulses that interfere, while the third term represents the noise on the phase of the pulses. The delay dependent phase term can be rewritten:

\[
\phi(\nu) = \phi_0 + \frac{2(D-L)}{c} \cdot \frac{1}{f_{\text{rep}}}.
\]

It means that the distance can be obtained by determining the slope of the phase as a function of the optical frequency. In Figure 4 the principle for a distance measurement is demonstrated by means of a simulation. Part (A) shows the raw data in the form of the recorded spectral fringes. The first step involves Fourier-transformation (FFT) of the interferogram to the time domain, as shown in part (B) of Figure 4. The time-domain representation has two sidebands of which only one is shown here, and a central component at \( \nu = 0 \). If the spectrum is smooth enough compared to the oscillation period of the spectral fringes, then the sidebands are well separated from the central peak. This condition is essential for a proper reconstruction of the phase evolution as a function of the frequency. One practical consequence of this measurement scheme is that the pulses do not need to overlap, and that the interference pattern can be obtained instantaneously without scanning the reference arm or \( f_{\text{rep}} \) of the comb laser. Variations in the repetition rate have therefore far less consequences for spectral interferometry than for temporal interferometry because CCD and diode array detectors can capture the whole interferogram at once. It merely results in a different reading for the position for each measurement. After analysis of the distance from the pattern, the results can be averaged, so that the long-term stability of frequency comb lasers can still be exploited fully.

An interesting approach has been recently suggested by Coddington and coworkers [5]. They measured the spectral interference by use of two coherent frequency combs with slightly detuned repetition rate. In this way they continuously scan one comb over the other and achieve some nanometer level of precision with an ambiguity range of 1.5 m within 60 ms, at low light levels and with high immunity to spurious reflections. It is suggested that the ambiguity range could be easily extended to 30 km, by use of a smaller frequency difference. If another factor 10 in the ambiguity range could be achieved, remains to be determined. An ambiguity range of 500 km would make the system very well adapted to formation...
flight applications. In addition, the time resolved signal of the dual comb method allows for measurements between multiple reference planes in a single beam path. This host of features is presumably unavailable in any other single system discussed so far.

As a main result we have recently developed a software in order to be able to fully simulate SI based distance measurements, taking into account the properties of any selected laser system and various other effects. Here we show the result of a HAALDM system based on an Er-fibre comb. The total analysis can be quite complicated, as can be seen from an example of the full analysis in Figure 5. The individual uncertainty components are: ‘timing’: uncertainty due to pulse timing noise; ‘phase’: likewise due to pulse phase noise; ‘analysis’: accuracy due to the evaluation of the interferogram; ‘clock’: frequency reference induced; ‘sub-total’: sum of the effects mentioned above; ‘spectr.’: induced uncertainty due to spectrometer calibration; ‘ref. arm’: uncertainty of the reference arm in the Michelson interferometer; ‘total’: all components combined; ‘+’: distance error from the interferogram analysis when 30 measurements are averaged. Furthermore, two dotted lines are visible that indicate the maximum expected error based on single pulse timing deviations (upper dotted line), and phase fluctuations of the pulses (lower dotted line). As a major result of our analysis we find a step-wise increase in the total error indica-

Figure 4: (A) Simulated spectral interferogram, together with (B) the time-domain analysis (FFT from panel A), and (C) the reconstructed phase from which the pulse distance is extracted (Eqn. 2), and (D) the deviation of the phase from a linear fit.

Figure 5: An example of the full analysis of the single (individual) measurement accuracy of HAALDM with a RF-stabilized fibre comb (see text for an explanation of the symbols and lines).
ting the critical distance where the contrast of the interferogram is no longer sufficient (due to accumulated timing and phase noise) to perform a proper analysis. This timing and phase noise ultimately stems from timing jitter of the comb itself, and depends on accuracy of the RF clock that locks the comb laser repetition rate. In our study we have assumed a typical RF clock precision of $10^{-13}$. Better results can be obtained with high precision frequency standards from atomic or quantum clocks ($10^{-16}$, $10^{-18}$). Present ground based clocks are reaching an accuracy of $10^{-17}$, corresponding to sub-µm accuracy at distances up to 109 m. The technical development of precision frequency standards for space applications is currently under way. Nevertheless, from our simulations we find that even the simple additional introduction of a high-finesse cavity into the comb laser may be sufficient to improve the timing and phase noise to an accuracy level that would be reaching 50 nm at 10000 m.

3. Conclusion

We conclude that frequency combs can provide unprecedented absolute length measurements on a sub-micrometer scale over a wide range of distances. We have reviewed different schemes for measurements, and selected spectral interferometry as the method of choice for long distance measurements in space. Of the two main methods to obtain sub-wavelength resolution, temporal and spectral interferometry, the later one has the big advantage that no scanning or overlapping pulses are required. Because the spectral interferogram can be recorded single-pulse if necessary, it also allows to measure fast changing distances (e.g. between two or more moving satellites). From our simulations of spectral interferometry which are presented here we conclude that the uncertainty in determining a distance of 500 km could be better than 50 nm, provided that sufficiently accurate RF clocks are available in space.

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