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Determination of dispersive phase velocities by Complex seismic trace Analysis of Surface Waves (CASW).

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Abstract
A method for deriving experimental dispersion curves of surface waves from active source recordings is presented. The method is based on the Complex seismic trace Analysis of Surface Waves (CASW) and is applicable when only two receivers are available. Reliable phase velocities are obtained when keeping the geophone interval smaller than one $\lambda$, allowing both a velocity structure as local as possible to be derived and to avoid long geophone spreads which are difficult to handle in urban areas. A large number of velocity estimates for each frequency can be estimated, even when using only 2 sensors, allowing statistical validation of the results, and providing a statistically defined uncertainty interval to be used in the dispersion curve inversion. The method is tested using synthetic seismograms and applied to real-world data, showing that it provides reliable estimates of apparent phase velocities. Although a final conclusion cannot yet be drawn, its application to observed data suggests it has the potential to be a useful method for distinguishing different modes.

1. Introduction
In recent years, methods based on the analysis of the dispersive properties of surface waves have found an increasing application in geophysical and geotechnical engineering site investigations. In particular, studies related to the assessment of the local amplification of ground motion following an earthquake require detailed knowledge of the S-wave velocity structure below a site. Standard invasive geotechnical methods that require the drilling of boreholes are quite expensive and therefore cannot be used to cover large urban areas. By contrast, non-invasive methods based on the analysis of the dispersive characteristic of surface waves are relatively low-cost and allow large areas to be covered, while still attaining a reasonable depth of investigation. Such methods can be divided into two categories, depending upon the signal source required. Active source methods, that is those that require explosives, vibrators or drop weights, include Spectral Analysis of Surface Waves (SASW) [1] and Multichannel Analysis of Surface Waves (MASW) [2], while passive source methods, that is those that require the use of seismic noise, include Spatial Autocorrelation (SPAC) [3] and Refraction Microtremors (ReMi) [4].
Recently, many efforts have also been made in improving the inversion procedure by considering non-linear inversion schemes [e.g. 5, 6, 7] and combining the inversion of the fundamental and higher modes [e.g. 8, 5, 9, 10, 11].

The SASW method suffers mainly from the problem of error propagation during the phase-unwrap procedure [12, 13], while MASW allows one to solve such problems, but requires a large number of geophones.

In this paper we proposed an alternative and rapid method for estimating the phase velocity of surface waves using active source recordings. The method, based on complex trace analysis [14, 15] also works when only two receivers are available and provides reliable phase velocities within geophone intervals of one $\lambda$, allowing both the derived velocity structure to be as local as possible and to avoid long geophones spreads, which are difficult to handle in urban areas. A large number of velocity estimates for each frequency can still be found when using only 2 sensors, allowing the statistical validation of the results.

We show the effectiveness of the method using synthetic seismograms and an application to real data collected during an experiment in the Bonn area (Germany).

2. Method

The phase velocity of surface waves generated by an active source and traveling between two sensors can be obtained after calculating for the original seismograms their corresponding complex analytic traces. In this paper only vertical recordings, and therefore Rayleigh waves, are considered, but similar considerations can be made for the transverse components and hence, for Love waves.

2.1 Basic definitions

The complex seismic trace [14, 15] is given by:

$$c(t) = r(t) + iq(t)$$  \hspace{1cm} (1)

where $r(t)$ is the real seismic trace, $i$ is the square root of -1, and $q(t)$ is the quadrature trace obtained as the Hilbert transform of the real trace.

The complex trace is obtained from the real trace by the following steps: (1) the trace is zero-padded to a power of two sample length greater than or equal to twice the original length; (2) transforming the real trace by Fast Fourier Transform (FFT), zeroing the amplitudes of negative the frequencies and doubling the amplitudes for positive frequencies; and (3) applying an inverse FFT.

The instantaneous amplitude and phase of the trace are given by:
\[ A(t) = [r^2(t) + q^2(t)]^{1/2} \]  
and 
\[ \theta(t) = \arctan[q(t)/r(t)] \]  
(2)  

It is worth noting the time dependence of both parameters and, therefore, the retention of local significance.

### 2.2 Phase velocity calculation: Complex seismic trace Analysis of Surface Waves (CASW)

In order to obtain the phase velocity at a certain frequency, the traces recorded at two different sensors are each first filtered using a Gaussian filter [16, 17, 18]: 
\[ H(\omega_0, \omega) = e^{-[(\omega - \omega_0)/(\alpha \omega_0)]^2} \]  
(4)

where \( \omega_0 \) is the central frequency of the filter, \( \alpha \) is the relative bandwidth and \( \omega \) the frequency. Filtering is performed by first computing the Fourier transform of the trace, then multiplying the obtained complex spectrum by the Gaussian filter and then calculating the inverse Fourier transform of the filtered complex spectrum.

Successively, for each of the two filtered traces, the complex analytic trace is calculated.

A complex two component trace may be defined from equation (1):
\[ c(t) = r(t) + iq(t) \]  
(5)

where the real trace \( r(t) \) is now a vector quantity defined by the two filtered traces, and each component of the quadrature trace \( q(t) \), is obtained by the application of the Hilbert transform to the corresponding component of \( r(t) \).

The geometric mean \( G(t) \) of the instantaneous amplitude is given by:
\[ G(t) = \left[ r_1^2(t) + q_1^2(t) \right]^{1/2} \]  
(6)

where the subscript 1 and 2 refer to the traces from each sensor.

The instantaneous phase difference \( \phi(t) = \theta_1(t) - \theta_2(t) \), where \( \theta_1(t) \) and \( \theta_2(t) \) are the instantaneous phases, is computed between the two complex traces of the two-component trace by:
\[ \phi(t) = \arctan \left[ \frac{r_2(t)q_1(t) - r_1(t)q_2(t)}{r_1(t)r_2(t) + q_1(t)q_2(t)} \right] \]  
(7)

Equation (7) is used to avoid correcting for phase unwrapping. Assuming that trace 2 is the one recorded at the larger distance from the source, we expect \( \phi(t) \) to be positive.
Phase differences are retained only if calculated at a time \( t \) when \( G(t) \) is greater or equal to 0.7 of the maximum, that is, only in the most energetic part of the seismogram. This reduces the influence of noise on the results, and allows us to calculate the local phase velocity

\[
\nu(t) = \frac{(D/2\pi)/\phi(t)}{D_{\text{max}}}
\]

where \( D \) is the distance between sensors and \( f \) is the frequency. It is worth noting the dependence of \( \nu \) on time and, therefore, the retention of local significance.

In the event of more than two traces being available, the procedure can be repeated by considering all possible combination of recordings. Using this method, only phase differences corresponding to a maximum of one cycle of a given frequency can be correctly retrieved. This limits the maximum geophone interval \( D_{\text{max}} \) to a wavelength \( \lambda \).

The minimum geophone interval \( D_{\text{min}} \) that provides well-constrained velocities can be chosen to be:

\[
D_{\text{min}} = 10\Delta t \nu
\]

where \( \Delta t \) is the sampling rate and \( \nu \) is the phase velocity. This equation shows that \( D_{\text{min}} \) is chosen here arbitrarily as the distance that allows us to sample at least ten times the time difference of the wave arrivals at the two geophones. An uncertainty of one sample will therefore have a limited influence on the reliability of the estimated phase velocity at larger distances. However, \( D_{\text{min}} \) can be much smaller than what is generally required when using the standard SASW technique [19].

In order to ensure that the most energetic part of the seismograms correspond to surface waves and to avoid body wave effects [20], the minimum offset (source-receiver) should be large, that is being at least equal to or greater than the wavelength \( \lambda \) of the lowest frequency of interest.

For each considered frequency, the \( \nu(t) \) obtained for all combinations of available recordings are plotted, and after a rapid visual inspection, the ones obtained from traces recorded at geophone intervals between \( D_{\text{min}} \) and \( D_{\text{max}} \), are selected and presented as a hystogram.

The phase velocity of a certain frequency can then be defined by either using statistical criteria (e.g. mean or median value of the distribution) or by simply picking the value that occurs most frequently (mode).

The procedure described above is summarized in Figure 1.

3. A synthetic seismogram application
In order to test the proposed method, synthetic seismograms were calculated using a semi-analytical method that consists of an improved Thompson-Haskell propagator matrix algorithm that overcomes numerical instabilities by an orthonormalization technique [21]. Synthetic seismograms were generated considering the simple 4 layer model described in Table 1. A source consisting of a single vertical force was located at the surface and the minimum offset to the geophones was set to 50 m in order to both avoid interference from body waves and to satisfy the plane wave assumption [19]. The maximum offset was fixed to 69 m, with a geophone interval of 1 m. The sampling rate was fixed to 1000 samples per second. The resulting synthetic seismograms are shown in Figure 2, and clearly exhibit a strong surface wave component that dominates the seismic energy. The CASW method described in section 2.2 was applied to the traces, considering all the possible combinations. Figure 3 (top) shows the phase velocities estimated for a central frequency of the Gaussian filter of 20 Hz using the 10th (59 m offset) and 16th (65 m offset) synthetic seismogram traces. Phase velocities \(v(t)\) are calculated only for the most energetic part of the traces (black horizontal line in Figure 3 (bottom)), and show very consistent values for all analyzed time. Density plots, depicting the number of occurrences of a certain velocity within a rectangle of 0.5 meter width on the x scale and 10 m/s width on the y-scale, are shown in Figure 4 for some of the analyzed central frequencies. For the lowest analyzed frequency (10 Hz) velocities are more scattered than for higher frequencies, especially, as expected, when the geophone interval is small. Considering equation (9) and the average velocity estimated in the plot (~360 m/s), for the following analysis, at 10 Hz, only \(v(t)\) calculated with geophone intervals larger than 4 m will be considered. Following on from this criteria, smaller values will be allowed for higher frequencies. Increasing the frequency shows a decrease in the estimated velocities, demonstrating the dispersive characteristic of Rayleigh waves. Clear limitations in the method when estimating phase velocities for cases where the geophone interval is larger than one wavelength appear in the 20 Hz, 30 Hz and 40 Hz plots. This allows us to define the thresholds for the maximum exploitable geophone interval. At geophone intervals corresponding to ½ of the wavelength (i.e. 8m for 15 Hz) some scattering appears in the estimated velocity. This might be related to numerical instabilities occurring in the procedure when the numerator in equation (7) gets closer to 0. The selection of inter-sensor distance was carried out using an interactive program, with the user selecting the required distance range (between \(D_{\text{min}}\) and \(D_{\text{max}}\)). It is worth noting how with only one geophone interval, a large number of phase velocity estimates are provided.
In Figure 4 the theoretical Rayleigh wave velocity of the fundamental mode (dashed line) and the apparent velocity (dashed line, as defined by [8], for the used model are shown. The estimates of velocity obtained by CASW agree very well with these values, within the exploitable geophone interval ranges.

A histogram of the selected phase velocities for each frequency is calculated and used to estimate the phase velocity to be assigned to each frequency. Figure 5 shows examples of the histograms obtained for different central frequencies. The phase velocity is estimated as the median value of the distribution (gray line), and the velocities corresponding to the first (Q1) and third (Q3) quartile of the data are also indicated (dashed gray lines).

There is a very good agreement between the median phase velocities and the theoretical apparent velocity calculated from the model (Figure 6). The width of the inter-quartile interval (Q3-Q1), used to represent uncertainties, increases with decreasing frequency as expected, but it is still generally less than 10%.

4. A real-world example

A 48-trace data set was collected at a test-site in Bornheim (Germany) (Figure 7, left), with the original aim of comparing the results obtained by standard techniques such as SASW, MASW and ReMi [22]. Vertical geophones with a resonant frequency of 10 Hz were used. The sampling rate was fixed to 1000 samples per second and the recording time was 2 s. Geophones were spaced regularly at 2 m intervals. A weight drop source of 25 kg was released from a height of 0.5 m with a near offset of 10 m. The CASW method was applied to 11 traces with geophone intervals varying from 40 m to 60 m (Figure 7, right), in order both to avoid interference from body waves and to satisfy the plane wave assumption [19]. The traces show clear surface wave arrivals. Figure 8 shows, as an example, the field data after the application of the filter with a central frequency of 20 Hz.

The density plots, representing the number of occurrence of a certain velocity (y-scale, 10 m/s bin) for each geophone interval (x-axis), are shown in Figure 9 for some of the analyzed central frequencies. Generally, velocities cluster around a well defined value, except for frequencies higher than ca. 32 Hz. As expected, larger scattering is observed for smaller geophone intervals, and for frequencies larger than 30 Hz, the exploitable range of geophone intervals is very limited. Indeed, this is already less than 15 m for 15 Hz, and only 4 m for frequencies equal to or greater than 30 Hz.

Although the acquisition geometry was not meant nor optimized for this method (smaller geophone intervals would have improved its performance by increasing the number of usable
intervals) we see that well constrained phase velocities are also obtained by only considering a few geophone intervals. In addition, starting from 15 Hz, effects similar to those depicted in the synthetic data analysis plots, are observed. The selection of the geophone intervals was carried out as for the synthetic seismogram application.

The local phase velocity distributions are shown in Figure 10. The median as well as the Q1 and Q3 values of the distributions are indicated by the vertical continuous and dashed lines, respectively. In the interval 10 Hz-32 Hz, velocities have a narrower distribution around a well-defined value, indicating that over this frequency range, velocities are well constrained. At lower and higher frequencies, velocities are less constrained due to a lack of energy from the source and the low signal-to-noise ratio for such large source-receiver offsets, respectively. In particular, at frequencies higher than 32 Hz, the velocity distribution is no longer unimodal, and shows several peaks. In these cases, the use of a median value loses its significance. Nevertheless, the median and the inter-quartile interval are also shown for these frequencies to allow easier comparison of the results.

For each analysed frequency, the phase velocity estimated from the median and the uncertainty interval defined by the inter-quartile range are compared in Figure 11 to the normalized dispersion spectrum calculated by MASW [22]. There is a very good agreement between the phase velocities estimated by CASW and the maxima in the dispersion spectrum of MASW. Between 10 Hz and 15 Hz, phase velocity estimates from CASW are clearly affected by secondary maxima in the velocity-frequency plane, with a similar trend observed above 32 Hz, where more than one maximum are observed. However, these maxima correspond to higher mode velocities for the model calculated by the inversion of the apparent velocity curve derived by [22] when combining passive and active source experiment results. Furthermore, the maxima occur (Figure 12) over frequency ranges where the medium response of the higher modes is closer to the fundamental mode. Therefore, a more detailed assessment of the velocity distribution is required. As previously indicated for frequencies higher than 32 Hz, the velocity distribution is multimodal. For example, at 35 Hz (Figure 10) three distinct peaks are observed at around 180 m/s, 270 m/s and 300 m/s, which agree well with the velocities of the fundamental and first two higher modes of the model. A similar behaviour is also seen at other frequencies.

Although these results show the potential of this method to distinguish between different modes, ad-hoc data sets are still necessary before drawing a final conclusion.

5. Conclusions
In this study we proposed an alternative analysis (CASW) for estimating phase velocities from two- or multi-component shot gathers. The method, based on complex trace analysis, provides reliable phase velocities within a maximum geophone interval of one \( \lambda \) and an minimum geophone interval smaller than that necessary for SASW, allowing us to keep the derived velocity structure as local as possible and to avoid long geophones spreads, that are difficult to handle in urban areas. A large number of velocity estimates for each frequency are estimated even when using only 2 sensors. Therefore, the application of CASW does not require large amounts of equipment. The large number of velocity estimates allows both a statistical validation of the results, and, differently from MASW, the calculation of an uncertainty range to be used in the inversion while following defined statistical criteria. The analysis was tested successfully on synthetic seismograms and when applied to a real world case, shows the capability of the method to retrieve apparent Rayleigh wave phase velocity. Our results indicate that this method might have the potential of distinguishing different modes of propagation.

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References


Figure Captions

Figure 1: Flowing chart describing the procedure followed in this work.

Figure 2. Synthetic seismograms calculated using the earth model described in Table 1.

Figure 3: Top. Filtered synthetic seismograms calculated for an offset distance of 59 m (black) and 65 m (gray). The central frequency of the filter is 20 Hz. The horizontal black line indicated the selected time window for local phase velocity calculations. Bottom. Local phase velocity against time.

Figure 4: Local velocity against geophone interval density plots for different central frequency of the filter. The darker the area, the greater the number of velocities estimated for a certain geophone interval. The horizontal continuous line indicates the theoretical apparent phase velocity of Rayleigh waves for the considered model. Horizontal dashed line indicates the theoretical fundamental mode Rayleigh wave velocity.

Figure 5: Histogram (y-axis, frequency percent) of the selected local phase velocities for different frequencies. The vertical continuous line indicates the estimated median value of the distribution, the vertical dashed lines indicate the Q1 and Q3 of the distribution.

Figure 6: Estimated Rayleigh wave velocities found by CASW (black diamonds) with their interquartile interval (black vertical line) against apparent Rayleigh wave velocities (black line), Rayleigh wave fundamental mode (gray line) and first higher mode (dashed gray line) velocities.

Figure 7: Left. 48-channel raw-field data acquired in Bornheim (Germany). Amplitudes are corrected for geometrical spreading. Right. Raw field data, for channel 10 to 16, used in the CASW analysis. Amplitudes are not corrected for geometrical spreading.

Figure 8: Filtered field data for channel 10 to 16. The central frequency of the filter is 20 Hz.

Figure 9: Local velocity against geophone interval density plots for different central frequency of the filter, for the raw shot gather. The darker the area, the greater is the number of velocities estimated for a certain geophone interval.

Figure 10: Histogram (y-axis, frequency percent) of the selected local phase velocities, estimated by CASW analysis of the raw shot gather for different frequencies. Vertical continuous lines indicate the estimated median value of the distribution, while vertical dashed lines indicate the Q1 and Q3 of the distribution.

Figure 11: Median phase velocities (diamonds) and inter-quartile intervals (black vertical lines) estimated by the CASW method, compared to normalized dispersion spectrum calculated by MASW. The fundamental and higher mode theoretical velocities calculated using the model derived by [22], which involved inverting the apparent dispersion curve obtained by combining passive and active source experiment results, are shown with gray lines.

Figure 12: Medium response for the vertical component of Rayleigh waves for a source consisting of a vertical force. The black line indicates the fundamental mode. The lighter the gray shade, the higher the mode.
Table 1: Parameters of the earth model used for synthetic seismograms calculation

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