Originally published as:

Time-dependent analysis of the earthquake rates in the Dead Sea Fault Zone

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Abstract

The aim of this study is to analyse the temporal changes of the seismicity rate of large earthquakes ($M_w \geq 6$) in the Dead Sea Fault Zone (DSFZ). Therefore, four parametric statistical distributions, including Weibull, Gamma, Lognormal and Brownian Passage Time (BPT), are applied as well as the Poisson process (exponential distribution) as the classically applied time-independent model. The next step is to find which model can explain the seismicity rate better. In order to estimate the model parameters, we use a modified Maximum Likelihood Estimation (MLE). This method considers both inter-event times and censored time in the estimation process. These data are extracted from the earthquake data file assembled at the GFZ (cf. Grünthal and Wahlström, this volume). The last step is the best model choice and validation. This is performed using two models, the Bayesian Information Criterion (BIC) and the Kolmogorov-Smirnov goodness-of-fit test. The results of our study show a significant time-dependency, at least for the northern part of the DSFZ.

Introduction

The seismicity of the Middle East coupled to the Dead Sea Fault Zone (DSFZ) and other tectonic features is significant (Fig. 1). The list of earthquakes, which extends far back into history, includes many cases of severe destructions. The classical Poissonian based (time-independent) analysis of earthquake rates applied for Probabilistic Seismic Hazard Assessment (PSHA) has been repeatedly applied in the past for the DSFZ.

The idea of time-dependent earthquake modeling is traditionally more involved with the aftershock modeling. Omori’s law (Omori 1894) is probably the first time-dependent model for describing earthquake rates. Utsu (1961), Reasenberg and Jones (1989) and Ogata (1988) made use of Omori’s law for their new methods to model aftershock sequences.

In order to estimate the rate of large repeating earthquakes on a single fault or fault segment, time-to-failure modeling is the mostly used statistical method. This method considers the earthquake process as a renewal process, such that the earthquake inter-event times are independently identically distributed. The most commonly used parametric distributions for the time-to-failure modeling of mainshocks are Weibull (Nishenko 1985), Gamma, Lognormal...
(Nishenko & Buland 1987; Michael & Jones 1998) and inverse Gaussian - known as Brownian Passage Time, BPT – (Ellsworth et al. 1998). Recently, several studies have combined these statistical parametric models, as well as pure physical models like the rate-and-state model (Dietrich 1992; 1994). Parsons (2004, 2005) presented a combination of the rate-and-state model and a BPT distribution, and Gomberg et al. (2005) applied the rate-and-state model and a lognormal distribution. Hardebeck (2004) discussed the superposing or double counting of the two models, and proposed a method to calculate interaction probabilities.

In this study four distributions, Weibull, Gamma, Lognormal and BPT as time-dependent models, are tested besides the Poisson process, as time-independent model, to estimate the rate of earthquakes with \( M_w \geq 6 \) in the DSFZ. Then, the results obtained from each model are compared and the time-dependence of the earthquake rate in the study area is discussed.

**Method**

The parametric statistical distributions used in this study are:

- **Weibull distribution** (with \( \beta \) and \( \eta \) as the scale and the shape parameters)
  \[
  f_{\beta, \eta}(t) = \left( \frac{\beta}{\eta} \right) \left( \frac{t}{\eta} \right)^{\beta-1} \exp \left( - \left( \frac{t}{\eta} \right)^{\beta} \right)
  \]

- **Gamma distribution** (with \( k \) and \( \theta \) as the scale and the shape parameters)
  \[
  f_{k, \theta}(t) = t^{k-1} \frac{\exp \left( - \frac{t}{\theta} \right)}{\theta^k \Gamma(k)} \quad , \quad \Gamma(k) = \int_0^\infty x^{k-1} \exp(-x)dx
  \]

- **Lognormal distribution** (with \( \sigma \) and \( \mu \) as the scale and the shape parameters)
  \[
  f_{\mu, \sigma}(t) = \frac{1}{t \sigma \sqrt{2\pi}} \exp \left( - \frac{(\ln t - \mu)^2}{2\sigma^2} \right)
  \]

- **BPT distribution** (with \( \lambda \) and \( \mu \) as the scale and the shape parameters)
  \[
  f_{\mu, \lambda}(t) = \left( \frac{\lambda}{\sqrt{2\pi \mu^3 t}} \right) \exp \left( - \frac{\lambda(t-\mu)^2}{2\mu^2 t} \right)
  \]

- **Exponential distribution** (Poisson process, time-independent, with \( \lambda \) as the rate and \( 1/\lambda \) as the scale parameter)
  \[
  f_\lambda(t) = \lambda \exp(-\lambda t)
  \]

The first four distributions are time-dependent. It means that while the rate functions corresponding to these distributions are functions of elapsed time since the last mainshock, the rate of the exponential distribution is time-independent.

In order to estimate the parameters of each model, a modified Weighted Maximum Likelihood
Estimation (WMLE) is used for two reasons. The first is the intrinsic uncertainty of historical earthquake data. Uncertainties in the earthquake parameters, especially for the older ones, are neither recognizable, nor calculable. The second reason is the fact that earthquake rates probably change with time. Therefore, the estimation process requires higher weights for earthquakes occurring in the recent time. In the WMLE method, the role of the more recent earthquakes is considered to be more important than that of the older ones. The log-likelihood function of WMLE reads as:

$$l(\Theta|T, t_c, W, w_c) = \log(L(\Theta|T, t_c, W, w_c)) = \sum_{i=1}^{n} w_i \log(f_\Theta(t_i)) + w_c \log(S_\Theta(t_c)),$$

where $$T = \{t_1, ..., t_n\}$$ is the set of inter-event times, $$W = \{w_1, ..., w_n\}$$ is the set of corresponding weights, $$t_c$$ is the censored time; i.e. the time since the last event, $$w_c$$ is the weight corresponding to the $$t_c$$, $$\Theta$$ is the set of parameters of the corresponding distribution, and $$f_\Theta$$ and $$S_\Theta$$ are probability density and survivor functions of the corresponding distribution:

$$S_\Theta(t) = \int_t^{\infty} f_\Theta(x) dx.$$

The best parameter estimates result from the solution of the following optimization problem:

$$\hat{\Theta} = \arg \max_{\Theta} l(\Theta | T, t_c, W, w_c).$$

Two different methods are applied to select the best distribution. The first is the Bayesian Information Criterion; $$BIC = k \ln(n) - 2 \ln(L)$$ (Volinsky et al. 1999), where $$n$$ is the number of records involved in the estimation process, $$k$$ is the number of parameters and $$L$$ is the maximum likelihood of the corresponding distribution. The model with smaller corresponding $$BIC$$ fits the data better. The other method is the Kolmogorov-Smirnov goodness-of-fit test (KS-test, Chakravart et al. 1967). A modified weighted version of one-sample KS-test is used in this study.

Data

The dataset (Fig 1a) is extracted from available special studies and catalogues for the Middle East area according to the procedure described in Grünthal & Wahlström (2009 and in this volume). It contains, amongst others, all known earthquakes with $$M_w \geq 6$$ along the DSFZ, with a width of about 20 km (Fig 1b). The completeness of this dataset starts at about 300 AD. A declustered dataset is used.

The study area along the DSFZ is divided into three sub areas, a southern, a central and a northern area (Fig 1a). The approach is not applied for the southern part of the DSFZ, because in this part there is just one mainshock with $$M_w \geq 6$$ since 300 AD. This is of course not sufficient for a rational statistical analysis. In the followings, earthquakes of $$M_w \geq 6$$ are referred to as large earthquakes.
Results

The methods are applied for both the central and the northern sub areas. For the central area there could be found no significant time-dependence for the rate of the big earthquakes. Here it is preferred to apply a Poisson process to estimate the earthquake rate, because the Poisson process (with inter-event times exponentially distributed) is simpler than the time-dependent distributions. For the northern area, there is a significant time-dependence for the earthquake rate (Table 1).
### Table 1. Results of the method for the northern part of the DSFZ

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Model</th>
<th>Scale parameter</th>
<th>Shape parameter</th>
<th>Log-ML</th>
<th>BIC</th>
<th>KS-test P-value</th>
<th>Probability of exceedence during the next 100 years</th>
<th>Weight in the combined model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time-independent</td>
<td>Exponential</td>
<td>111.64</td>
<td>-100.10</td>
<td>203.15</td>
<td>0.06</td>
<td>0.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time-dependent</td>
<td>Weibull</td>
<td>61.70</td>
<td>0.90</td>
<td>93.79</td>
<td>193.47</td>
<td>0.94</td>
<td>0.97</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>Gamma</td>
<td>85.86</td>
<td>0.91</td>
<td>93.63</td>
<td>193.15</td>
<td>0.49</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lognormal</td>
<td>41.68</td>
<td>3.39</td>
<td>92.55</td>
<td>191.00</td>
<td>0.91</td>
<td>0.92</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>BPT</td>
<td>86.36</td>
<td>28.37</td>
<td>92.36</td>
<td>190.60</td>
<td>0.99</td>
<td>0.92</td>
<td>0.70</td>
</tr>
</tbody>
</table>

From Table 1 and Fig. 2, it is clear that the BPT distribution is the best model; however two other distributions, the Weibull and Lognormal, provide good fits as well. All these three models have very significant KS-test p-values, which results in significant estimates. Among these three models, only BPT, as a physical based model, considers the steady tectonic loading process which is superposed by Brownian perturbations (Matthews et al. 2002). In order to estimate the probability of exceedence of a big earthquake in the northern area, a combined model is designed using the three models. Because of the physical base of the BPT and the higher KS-test p-value and lower BIC, the weight for the BPT in the combined model is considered as 0.7, as well as 0.15 for each Weibull and Lognormal estimates. Fig. 3 shows the probability of exceedence of a big earthquake within a generic time \( t \) since the last earthquake based on each of the three mentioned models as well as the combined model and the time-independent model (Poisson process).
Conclusion

The rate of the earthquakes with $M_w \geq 6$ in the northern part of the DSFZ is significantly time-dependent. Not only the higher KS-test p-values and the lower $BIC$ corresponding to the time-dependent models, but also the very low KS-test p-value and the high $BIC$ corresponding to the exponential distribution confirm the time-dependence of the earthquake rates in the northern part of the DSFZ.

On the other hand, the cumulative seismic release curve for this area reconfirms this result (Fig. 4). This curve indicates an obviously periodic behavior of seismicity with time in this area. For the central part of the DSFZ, we could not show a significant difference between time-dependent models and the time-independent one, i.e. the Poisson process is as good as the time-dependent models. Because of lack of data in the southern part the time-dependence of earthquake rates could not be investigated in this segment.

Acknowledgement

We appreciate the support from the German Research Centre for Geosciences, GFZ. This study is part of the DEad Sea Integrated Research project, DESIRE, funded by the Deutsche Forschungsgemeinschaft (German Research Foundation).
References


