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Receiver function summation without deconvolution

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SUMMARY
The separation of the structural effects below a seismic station from other effects like structures far away or from source–time functions is the fundamental problem of the receiver function technique. Two solutions of this problem have been suggested in the early days of the receiver function analysis. One is to model the complete wavefield with synthetic seismograms for a given source–time function with the knowledge of complete source and receiver structures. The other is the deconvolution of the SV component by the P component which removes effects of the source and near source structure. The latter does not require knowledge of the structure at the source or the source–time function. Its disadvantage is that the complete P component, including the P response of the receiver structure, is assumed to be the source signal. For improving the signal-to-noise ratio many traces are summed after deconvolution. The deconvolution technique is almost exclusively used nowadays. Here, we suggest a simple technique to estimate the three component response at the receiver site without deconvolution or similar methods. Our technique preserves the P component unlike receiver functions. It consists of the summation of many records from different source regions at one station only after amplitude normalization but no source equalization. The agreement with deconvolved receiver functions is astonishing. P scattered waves are preserved on the P component (in contrast to the deconvolution technique). We found P-to-P reflected phases from the crust–mantle boundary and from the lithosphere–asthenosphere boundary (LAB) in the data of the station HYB (Hyderabad) in India. Another advantage of the new technique is that it avoids systematic amplitude distortions of some phases (like PpPs crustal multiples) on the Q (or SV) component which are caused by deconvolution. A remaining advantage of the deconvolution technique is the better signal-to-noise ratio after summing the same records.

Key words: Time series analysis; Wave scattering and diffraction.

1 INTRODUCTION
Converted seismic waves are used since the beginning of seismology. They attained, however, their full impact only since a few decades as receiver functions. The slow start of the receiver function technique began perhaps with Phinney (1964). He computed the spectral ratio of the radial and vertical components for an estimation of the crustal structure below the receiver. The assumption was that the vertical component contained the source–time function and the P response of the structure near the source, including source depth effects and the P response of the structure near the turning point. By dividing the radial component by the vertical component, all these effects should hopefully disappear and only the S response of the receiver structure should remain. The vertical component, however, contains also the P response of the structure below receiver. The convolution of the vertical component with its inverse is ideally a spike which means that also the P response of the receiver struc-
of unmodified records from many earthquakes at very different source locations at the same station is generating the response of the recording site. Only amplitude and sign equalizations are required. As in traditional deconvolution receiver functions, effects of source structure, source depth, and structure at the ray turning point or the source–time function are suppressed, because they vary from record to record, whereas only the influence of the structure at the receiver site remains the same in each record. The new technique preserves the vertical component in P receiver function and allows detection of phases like P-to-P reflected multiples from the crust–mantle or the lithosphere–asthenosphere boundaries. Also distortions of the SV (or Q) component introduced by deconvolution are avoided. It should be mentioned that a very similar approach of using plain summation, without deconvolution, has been used by Shearer (1991) for long period seismograms, but in broad-band receiver functions, deconvolution was still used (Lawrence & Shearer 2006). It should be mentioned that Gurrola et al. (1995) have used a technique half-way between standard deconvolution technique and plain summation. We show that a plain summation technique similar to Shearer’s technique works for the receiver functions as well when broad-band records are used, not only for long period records.

We like to mention that the scope of this paper is the comparison of different techniques for time-series summation within the receiver function technique. Especially it is not intended to describe in detail problems of inversion techniques for transforming time-series into earth models and geodynamic problems.

2 EXAMPLES

The efficacy of the suggested approach has been tested at the two permanent broad-band stations HYB (GEOSCOPE network, located on the Indian craton) and RAYN (Global Seismic Network, GSN, located on the Arabian shield). See Fig. 1 for location of the stations and the epicenters of the earthquakes used. The stations are recording data since 1989 and 1996, respectively, and provide a large number of teleseismic events suitable for both P- and S-receiver function analysis. Azimuthal coverage is not homogeneous since most events are located in the eastern quadrant. The crustal structure at HYB is revealed by P-receiver function studies from Saul et al. (2000) and Kumar & Bostock (2008) and at RAYN from Kumar et al. (2002) and Leahy & Collins (2009) and references therein. The LAB was observed by S-receiver functions at both stations by Kumar et al. (2007) and at RAYN by Mohsen et al. (2006) and Hansen et al. (2007).

Fig. 2 describes the principle of the plain summation method we used. 30 individual traces are displayed as example. They are low pass filtered with 1 Hz corner frequency. The summation traces at the top of the figure are the result of summing 745 individual traces. The traces have been rotated into the local P, SV and SH ray coordinate system. Theoretical backazimuths are used to compute radial and transverse components. Incidence angles are determined by minimizing the amplitude of the P phase on the SV component in an automated way. Rotation is done before application of different summation techniques, which means possible errors introduced by rotation influence both techniques equally. The maximum amplitude (the sign is reversed if the absolute value of the minimum is larger than the maximum) in the P-wave group on the P component of each trace is normalized to one and all traces are lined up along that maximum P amplitude. This is the same procedure like in the deconvolution technique except that in the latter the P waveforms are simpler and more similar, that means more spike like. The shifts applied to line up the traces are in both techniques not necessarily the same since deconvolution spikes are not necessarily at the time of the maximum (or minimum) amplitude. This point has, however, no consequences since absolute arrival times are not used in receiver functions. Only differential times between the large signal on the P component and the weaker signal on the SV component are used. Zero time in receiver function figures (see Fig. 2) is not the P arrival time but the time of the maximum of the P group; the same is true for summation of deconvolved traces.

The next step before summation is the moveout correction which is required to correct for the distance dependence of the differential times of P and the converted phases (Yuan et al. 1997, 2006). This is a normal step in controlled source seismics (NMO). The IASP91 model (Kennett & Engdahl 1991), with the reference epicentral distance fixed at 67° (for P waves) or at a slowness of 6.4 s deg⁻¹ (for P and S waves) was used for the moveout correction (Yuan et al. 1997). All delay times of P- and S-receiver functions refer to a slowness of 6.4 s deg⁻¹ and are therefore directly comparable.

Figure 1. Location of seismic stations HYB (India) and RAYN (Arabia). Epicentres of events used for P (red dots) and S (blue dots) receiver function studies are shown.
Figure 2. Example of plain summation of seismic records for determination of the response at HYB (comparable to P receiver functions). 30 individual traces are displayed as example, however the summation traces result from summation of 745 traces. Signals from the Moho and crustal multiples are marked. Ordinates of the summation traces are the percentage amplitudes. The amplitudes of the individual traces in (a) are divided by four relative to the individual traces in (b) and (c).

Figure 3. Same as in Fig. 2 for S plain summations. 30 traces are lined up according to the maximum of the S group on the respective SV components. Timescale is reversed since desired signals are precursors of S. Signals from the Moho and the LAB (lithosphere–asthenosphere boundary) are marked. The top traces result from the summation of 413 traces.
Figure 4. Comparison of $P$, SV and SH receiver functions obtained with deconvolution (red) and plain summation (blue) at station HYB. Signals from the Moho, from the LAB, crustal multiples and a possible signal of unknown origin (X) are marked. All stacked traces have been generated after moveout correction for converted waves with a reference slowness of 6.4 s deg$^{-1}$, therefore other phases, like multiples, are weakened here.

The moveout correction is applied identically to the summation of plain traces and deconvolved traces, which results into identical errors if an incorrect model is used. Optionally moveout correction can also be applied for multiply reflected phases in order to enhance the multiples. Experience has shown that moveout correction for PP crustal multiples is very practical, because it improves the alignment of the crustal multiples and damages the Moho conversion only insignificantly. Finally, all traces are summed to get the receiver site response due to the incident $P$ waves. The same time shifts are used for SV and SH components that used for the $P$ component. Moveout correction is only required if we apply summation in time domain. If we use migration, which means transforming the data from time domain into depth domain with a known velocity model, moveout correction is not required.

Figure 5. Same as Fig. 4 for the station RAYN showing $P$ (RF/GF) and SV (RF/GF); red: receiver functions, blue: plain summations (GF).
The selection of this amplitude criterion for Moho signal has been made in deconvolved traces and the same selected events were also done in deconvolved traces. The deconvolved $S$ component (Q) component differs significantly from the theoretical $SV$ component. The $P_{s}P$ phase has disappeared on the deconvolved $SV$ component, as expected. However, the $P_{p}Ps$ Moho multiple reflection has a significantly distorted amplitude. This is a systematic error introduced by deconvolution. That means we must expect distortions also of the deconvolved $SV$ components in recorded data. Plain summation avoids such problems and preserves in addition the $P$ component.

We use the same approach like in Fig. 2 to compute $S$ response (Fig. 3). Original $S$-wave seismograms (in a time window around the $S$ phase) are rotated into $P$–$SV$–$SH$ system. Equivalently to the case in Fig. 2 the incidence angle is determined by minimizing $SV$ energy in a time window $\pm 0.6$ s around the $S$ arrival time on the $P$ component in an automated way. We further limited the selected traces to the ones with indications of Moho signals (within 2–6 s precursor time) greater 0.1 (i.e. 10 per cent of the incident $SV$). The selection of this amplitude criterion for Moho signal has been done in deconvolved traces and the same selected events were also used for plain summation. Since in the case of Sp conversions the incidence angle is larger than in the Ps case, the rotation around the incidence angle is more influential. $S$-wave seismograms with unrealistic incidence angles larger than $50^\circ$ are discarded. Then the sign and amplitude normalizations are done similar to the case as described in Fig. 2. Further we applied the time shifts and moveout corrections to the $SV$ components. The summation traces in the top panel of Fig. 3 is a result of 413 traces.

Fig. 4 shows the comparison of stacked traces between the new approach with the traditional deconvolution technique for $P$, $SV$ and $SH$ receiver functions. We have generated plain summation traces and deconvolution receiver functions for incident $SH$ waves in a similar way as for $SV$ waves since in the case of heterogeneity there could be also precursors on the $P$ component from incident $SH$ waves. Deconvolution and plain summation traces are in all cases very similar. Signals from the Moho, 410 and 660 km discontinuities and also the crustal multiples are clearly seen on the $SV$ component in both, the $P$-receiver functions and in the plain summation traces. These phases are undisputed common phases observed in many $P$-receiver function studies at many stations worldwide. The Moho and the conversion from the LAB are also clearly seen in the $P$ component of the $SV$-receiver functions. In the $SH$-receiver functions none of these well known phases are visible. A perhaps meaningful difference between the two techniques is seen at a signal marked $X$ near 20 s precursor time. This phase is only visible in the plain summation traces (GF in Fig. 4). If this phase is real, it cannot be modelled with a flat-layered isotropic model. It is a general problem when one trying to interpret new phases which are unknown from earlier similar studies. It is a challenging task before interpreter to decide a signal in the category of real phase, noise and or multiples in the case of $P$-receiver functions. A common way to confirm observations of new phases is to check its lateral extension by using a number of neighbouring stations. Since in our case we have only one station available, there is not much chance to discuss about reality or properties of the $X$ phase.

Waveform inversion of the complete seismic trace for a flat layered isotropic structure is relatively easy and produces nearly perfect agreement between data and model (e.g. Kind et al. 1995). However the resulting model is very complex because every wiggle in the seismogram is transformed into a wiggle of the model. Such an inversion is not unique and transforms any heterogeneity or anisotropy effect into vertical model variation. Therefore we prefer to interpret only clearly identifiable signals, which are confirmed at several stations, like Moho, LAB or 410 and 660. New phases from previously unknown discontinuities require sufficient confirmation from distributed data before they can be accepted. The waveform of signals converted at discontinuities should have important information about properties of the discontinuity. However using summation signal for waveform inversion has the problem that topography of the discontinuity contributes to the signal width, which could easily be misinterpreted in terms of discontinuity width. This problem applies for plain summation traces and for receiver function traces alike. Waveforms of converted signals should only be interpreted relative to the waveform of the incident phase. This reduces the problem of the influence of discontinuity topography.

Fig. 4 shows good agreement of the main features of the waveforms of deconvolved and plain summation traces, especially at the $SV$ component of $P$-receiver functions and the $P$ component of the $S$-receiver functions, which are the classical $P$- and $S$-receiver functions. There are, however also differences. Some maybe caused by the generally better signal-to-noise ratio of the traditional receiver functions. Others, like perhaps a signal marked $X$, could be real signals in the plain summation traces, which is destroying by the deconvolution in the receiver functions. This discussion is continued in connection with the Figs 6 and 7. The transverse component of the $P$-receiver function in Fig. 4 has a clear signal at 10 s, which is about two seconds earlier than the LAB signal of the $S$-receiver function. The same signal has also been observed on the $SH$ component of the $P$-receiver functions by Saul et al. (2000) and Kumar & Bostock (2006). It can probably be explained by a relatively thin anisotropic layer at the base of the lithosphere. It should be noted
that this possible anisotropic signal is dominated by events from easterly directions (see Fig. 2). Similarly to station HYB in Figs 4 and 5 shows a comparison of receiver functions and plain summation traces for the station RAYN in Saudi Arabia. The agreement in both techniques is again relatively good; Moho and LAB are clearly seen with both techniques. Differences in detail could be due to differences in the noise suppression in both techniques, or due to cancelling of certain phases by the deconvolution, which are assumed to be due to the source function. For the station RAYN the number of $P$ and $S$ waveforms is 706 and 382, respectively.

A general comparison between plain summation traces and deconvolution receiver functions is shown in Fig. 6 using synthetic data. The deconvolved $P$ trace has lost all information except the source signal; even $P$-to-$P$ reflected multiples are also subdued. These phases are considered by the deconvolution technique as part of the source. The deconvolved $SV$ component has lost its $P$ energy (except at zero time), as expected. However, the $PpPs$ Moho multiply reflected phase has too large amplitude on the $SV$ component of the deconvolved receiver function, which is a systematic error. Plain summation avoids such errors on the $SV$ component and preserves in addition the $P$ component. A real data example of the observation of $P$-to-$P$ multiples on the $P$ component is shown in Fig. 7. This figure shows such multiples are not only from the Moho but also from the LAB. These $P$ multiples are not very clear in Fig. 4 because a moveout correction for $P$-to-$S$ converted phases is applied there, which suppresses multiples. No Moveout correction is applied to the data in Fig. 7. The above-discussed points demonstrate clearly the possible principle advantage of plain summation over deconvolution. In many cases, however, the differences on the $SV$ components might be small.

Figs 4 and 5 also show that the noise level of the plain summation traces is generally higher than in the deconvolution traces. We have made experiments with the summation of different numbers of traces. The results are shown in Fig. 8 for the station HYB where summations of 30, 60, 150 and 300 randomly selected traces out of a total of 500 traces are shown. The errors are clearly larger in plain summation traces than in the deconvolution receiver functions in both the cases, $P$- and $S$-receiver functions. This can even better be seen in Fig. 9, where the mean of standard deviation as a function of the number of summation traces is shown for both stations for plain summation method. This figure indicates that for $P$ plain summations about 100 traces would be needed to come relatively close the minimum standard deviation, for $S$ plain summations about 200 traces would be needed. Permanent seismic stations might easily have such numbers of records in their archives.
Figure 8. Station HYB: Bootstrap error estimations for (a) $P$-receiver function, (b) $P$-plain summations, (c) $S$-receiver function and (d) $S$-plain summations using different number of events for stacking (30, 60, 150, 300). The black lines in all the four traces are the mean curves obtained from stacking 745 P-RF/GF and 413 S-RF/GF, and the gray lines are $\pm 1\sigma$ standard deviations. In each figure the lines closest to the black lines are the 300-stack traces, and the outer most ones are the 30-stack traces.

As a final step we have checked the stability of the summed traces as a function of the length of the deconvolution window or the window length which is used to search for the maximum amplitude in the plain summation method. The results are shown in Fig. 10. This figure shows that for window lengths greater than $\sim 30$ s for both the techniques lead to stable results. That means the deconvolution is getting stable for windows greater 30 s and no maximum (or very few) is found after that window in the case of plain summation. We have checked windows lengths up to 300 s and found no variation.

Figure 9. Error in plain summation estimations for HYB and RAYN. Abscissa denotes the various numbers of traces used for stacking. The ordinate represents the standard deviation ($SD$) of the stack. For $P$-wave data the time window from 0 to 80 s and for $S$-wave data the time window from 0 to $-30$ s has been used for estimating the mean in standard deviation.

Figure 10. Squared and summed difference of deconvolved and plain summation traces for various lengths of the time windows used for deconvolution or search of maximum amplitude (station HYB). Both techniques lead to stable results if the time windows reach at least about 30 s beyond the $P$ onset times. We have tested up to 300 s window length and found no change in stability.

3 CONCLUSIONS

We have shown that the approximate response at the site of a broadband seismic station may be obtained without deconvolution. That means no source equalization besides amplitude normalization is really required in the receiver function technique for summation of many traces. The advantage of the new technique is that possible systematic distortions of the $R$ component caused by deconvolution are reduced and information on the vertical component ($P$ reverberations from the Moho and LAB) are preserved. These are significant results leading to improvements in velocity and depth determinations in receiver functions. A remaining advantage of the deconvolution technique is the better signal-to-noise ratio.
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