Originally published as:


DOI: 10.1029/2010GL044572
Earthquake scaling characteristics and the scale-(in)dependence of seismic energy-to-moment ratio: Insights from KiK-net data in Japan

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Received 2 July 2010; revised 17 August 2010; accepted 23 August 2010; published 8 October 2010.

[1] We investigate earthquake source characteristics and scaling properties using the results of a spectral inversion of more than 29,000 accelerometric borehole recordings from 1,826 earthquakes (M\textsubscript{JMA} 2.7–8) throughout Japan. We find that the calculated source spectra can be well characterized by the omega-square model and show on average self-similar scaling over the entire magnitude range, with median stress drops of 1.1 and 9.2 MPa for crustal and subcrustal events, respectively. The seismic energy-to-moment ratio, as theoretically expected if the omega-square model is valid, shows a strong dependence on stress drop only, which, in conjunction with data selection practice in some studies to cope with limited recording bandwidth, can explain the often observed apparent scale-dependence. Our observations suggest that there is no significant deviation from similarity of the energy radiation in the investigated magnitude range and that the observed scatter is mainly related to the scatter in stress drop. Citation: Oth, A., D. Bindi, S. Parolai, and D. Di Giacomo (2010), Earthquake scaling characteristics and the scale-(in)dependence of seismic energy-to-moment ratio: Insights from KiK-net data in Japan, Geophys. Res. Lett., 37, L19304, doi:10.1029/2010GL044572.

1. Introduction

[2] Amongst the most fundamental and, at the same time, most heavily debated topics in modern seismology, the question in how far earthquake source parameters scale with earthquake size is of prime importance, since it has significant implications for seismic hazard assessment. In particular, the energy radiated during seismic faulting is of highest interest since it directly reflects the dynamic characteristics of the rupture process [e.g., Kanamori and Rivera, 2004]. Consequently, a scale-dependency of the seismic energy-to-moment ratio (also termed as scaled energy) \( \tilde{e} = E_R/M_0 \) would imply that the rupture dynamics of small and large earthquakes differ [e.g., Kanamori and Heaton, 2000], a finding that in turn would have profound implications both for our understanding of the physics of earthquakes and strong ground motion prediction.

[3] Yet a fundamental controversy still exists upon the scaling characteristics of seismic energy-to-moment ratio. Over the past two decades, a significant number of studies provided persuasive evidence for an increase of \( \tilde{e} \) with moment [e.g., Abercrombie, 1995; Mayeda and Walter, 1996; Izutani and Kanamori, 2001; Mori et al., 2003; Mayeda et al., 2005, 2007; Takahashi et al., 2005], sometimes even in conjunction with self-similar static scaling (i.e., \( M_0 \propto f_C^3 \), \( f_C \) being the corner frequency).

[4] In contrast, other studies cast doubt on these findings. Ide and Beroza [2001] and Ide et al. [2003] suggested that the often-observed scale-dependency of \( \tilde{e} \) may be due to a systematic underestimation of energy resulting from limited recording bandwidth and too simple attenuation corrections. Prieto et al. [2004] found compelling evidence for approximately constant apparent stress and most recently, Baltay et al. [2010] came to the same conclusion from the analysis of seismic coda of four earthquake sequences in western North America.

[5] Thus the issue of whether or not \( \tilde{e} \) is scale-dependent and how it is related to the scaling characteristics of other source parameters remains elusive. In order to obtain new insights on these issues, we use source spectra of a large set of earthquakes in Japan recorded by the KiK-net network [e.g., Okada et al., 2004], obtained from spectral inversion of S-waves, and we investigate the relation between energy-to-moment ratio and stress drop and in how far specifically the former is scale-dependent.

2. Data and Analysis

[6] We applied a non-parametric inversion scheme to separate the Fourier amplitude spectra of S-waves into source spectra, attenuation characteristics and site response, similar to the original works of Andrews [1986] and Castro et al. [1990]. In this approach, the functional form of the attenuation operator is not pre-defined, thus providing a means for reliable attenuation correction without strong a-priori assumptions. The attenuation-corrected spectra were separated into their source and site contributions, setting the average site response of all borehole sensors to unity. A bootstrap analysis of 100 consecutive inversions indicates that the technique provides highly stable results. The details of the data processing and inversion methodology are given by A. Oth et al. (Spectral analysis of K- and KiK-net data in Japan: II. On attenuation characteristics, source spectra and site response of borehole and surface stations, submitted to Bulletin of the Seismological Society of America, 2010).
model = 2), we use C0/2.72.70 km and d (respectively moment magnitude M = 2 and fit equation (1) to the inverted spectra using non-linear least-squares to determine M0 (respectively moment magnitude M_W [Hanks and Kanamori, 1979]) and f_c. For large events with M_JMA ≥ 5, where f_c is likely to be smaller than the lowest frequency in our analysis, we constrained M0 to the value given in the GCMT catalogue (www.globalcmt.org). Two examples of the source spectra are shown in Figure 1, with the best fitting $\omega^2$ source spectrum indicated as dashed line, showing good agreement between observed and fitted spectra.

[8] Stress drop estimates $\Delta\sigma$ are computed following Hanks and Thatcher [1972]:

$$\Delta\sigma = 8.5M_0 \left( \frac{f}{V_S} \right)^3, \quad (2)$$

and we calculate the radiated energy $E_R$ from the inverted S-wave source spectra as [e.g., Izutani and Kanamori, 2001] (neglecting the contribution of P-waves, compare also with equation (1)):

$$E_R = \frac{4\pi}{5\nu V_S} \int_0^\infty |Cf^{-1}S(f)|^2 df, \quad \text{with} \quad C = \frac{\nu^2 R_0}{4\pi^2 V_F} \cdot (3)$$

3. M_JMA − M_W Scaling

[12] Fukushima [1996], starting from the assumption that M_JMA can be considered to result from peak horizontal displacement at 5 seconds period and that the $\omega^2$ model holds, derived a relationship between seismic moment and M_JMA (in the range 4–8, similar to our dataset) of the form $\log_{10}(M_0^{1/3} + 10^{-17} M_0^{1/2}) = C_1 \cdot M_{JMA} + C_2$, where M_0 is given in dyn·cm. Figure 2a shows the determined moment magnitudes M_W versus M_JMA. At first glance, a linear relationship seems to hold between M_JMA and M_W, and the relation of Fukushima [1996] does not provide an appropriate description for earthquakes smaller than about M_JMA 4.5–5 (Fukushima’s dataset contained however predominantly events with M_JMA in the range 5–8). We performed a linear orthogonal regression to determine the optimal linear fit and, accounting for potential saturation effects of M_JMA (however not noticeable from Figure 2a by eye), a nonlinear least squares regression including a quadratic term (Table 1). Both the linear and quadratic fits provide comparable overall rms residuals of 0.22 and 0.21 magnitude units, respectively. However, for small events (M_JMA ≈ 2.7–3.5), the linear fit systematically underestimates M_W (rms residual 0.24), and the quadratic fit provides better estimates of M_W in that M_JMA range (rms residual 0.19), while the opposite is true for large M_JMA. An F test between the linear and quadratic models provides statistical evidence that over the entire M_JMA range, the quadratic fit can be considered more appropriate.

4. Stress Drops and Static Scaling Relationship

[13] If the principle of self-similarity holds between small and large earthquakes, then the well-known scaling relation provide an appropriate representation of the source spectrum of small and moderate earthquakes [e.g., Izutani and Kanamori, 2001]. Since the inverted acceleration source spectra show an increase approximately proportional to $f^2$ at low frequencies and a plateau at high frequencies (which is only the case if $n = 2$), we use $n = 2$ and fit equation (1) to the inverted spectra using non-linear least-squares to determine M0 (respectively moment magnitude M_W [Hanks and Kanamori, 1979]) and f_c. For large events with M_JMA ≥ 5, where f_c is likely to be smaller than the lowest frequency in our analysis, we constrained M0 to the value given in the GCMT catalogue (www.globalcmt.org). Two examples of the source spectra are shown in Figure 1, with the best fitting $\omega^2$ source spectrum indicated as dashed line, showing good agreement between observed and fitted spectra.

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$M_0 \propto f_C^2$ is expected to apply [Aki, 1967]. Figure 2b shows log $f_C$ versus log $M_0$ for crustal (black dots, red line) and subcrustal events (gray dots, blue line). The slopes are in both cases very close to the expected value of $-1/3$. Considering a potential modification of the scaling relationship to $M_0 \propto f_C^{0+\varepsilon}$ [Kanamori and Rivera, 2004], we find $\varepsilon = 0.12 \pm 0.12$ for the crustal and $\varepsilon = 0.18 \pm 0.08$ for the subcrustal case, which is far smaller than the values of 0.5–1 reported for instance by Mayeda et al. [2007]. Considering the regression errors obtained for $\varepsilon$, we cannot reject the null hypothesis of self-similar scaling for crustal events at 95% confidence level. The subcrustal events seem to show a more robust, but still only slight increase of $\Delta \sigma$ with $M_0$ (which would be consistent with the scaling interpretations of Kanamori and Rivera [2004]), but this deviation from self-similarity is still extraordinary small in view of other uncertainties apart from the regression error of fit of the $f_C-M_0$ relation (e.g., 3D attenuation effects) that are difficult to quantify. Thus, we find no significant deviation from self-similarity over the entire analyzed magnitude range.

In Figure 2c, stress drop is plotted versus $M_W$, and the stress drop distributions are depicted in Figure 2d. The estimates for subcrustal earthquakes are rather tightly clustered and approximately log-normally distributed (median 9.2 MPa), while in the crustal case, the distribution is broader and slightly asymmetric with a prevalence of smaller values (median 1.1 MPa). We performed a Lilliefors goodness-of-fit test of composite normality on both samples (in log-scale), and in the crustal case, the null hypothesis of normal distribution could be rejected at a significance level of 0.1%, while this was not the case for the subcrustal events, where the returned p-value (i.e., the probability, under the null hypothesis, that a value at least as extreme as observed of the test statistic is obtained) of 0.41 clearly forbids the rejection of the null hypothesis at acceptable significance level. The smaller variability in the case of subcrustal earthquakes may result from the fact that these are predominantly linked to the subduction zone (Oth et al., submitted manuscript, 2010) and thus related to a rather uniform tectonic feature, while the broader distribution for crustal earthquakes reflects the stronger heterogeneity of the crustal stress field and fault mechanisms. A two-sample K-S test rejects the null hypothesis that the crustal and subcrustal

Table 1. Functional Forms, Coefficients and RMS Residual of the Three $M_W$-$M_{JMA}$, respectively $M_0$-$M_{JMA}$, Relationships Depicted in Figure 2a

<table>
<thead>
<tr>
<th>Relation</th>
<th>Coefficients</th>
<th>RMS Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Fukushima [1996]}$</td>
<td>$C_1 = -1.10 \pm 0.06$</td>
<td>0.41</td>
</tr>
<tr>
<td>$\text{log}<em>10(M_0^{1/3}) = C_1 \cdot M</em>{JMA} + C_2$</td>
<td>$C_2 = -17.92 \pm 0.42$</td>
<td></td>
</tr>
<tr>
<td>Quadratic nonlinear least squares fit</td>
<td>$C_1 = 0.057 \pm 0.006$</td>
<td>0.21</td>
</tr>
<tr>
<td>$M_W = C_1 \cdot M_{JMA} + C_2 \cdot M_{JMA} + C_3$</td>
<td>$C_2 = 0.455 \pm 0.055$</td>
<td></td>
</tr>
<tr>
<td>Linear orthogonal least squares fit</td>
<td>$C_1 = 1.037 \pm 0.008$</td>
<td>0.22</td>
</tr>
<tr>
<td>$M_W = C_1 \cdot M_{JMA} + C_2$</td>
<td>$C_2 = -0.297 \pm 0.034$</td>
<td></td>
</tr>
</tbody>
</table>
stress drop samples are drawn from the same distribution at a significance level of 0.1%.

5. Scale-(In)dependence of Seismic Energy-to-Moment Ratios

[15] The results presented so far hence support the validity of the self-similar static scaling relationship (i.e. \( M_0 \propto f_C^2 \)) over the entire analyzed magnitude range. Since moreover the source spectral shape is well approximated by the \( \omega^2 \) model, in which case \( \tilde{e} \propto M_0 \) or, following equation (2), \( \tilde{e} \propto \Delta \sigma \), we would expect to see on average a constant value of scaled energy independent of moment [Izutani and Kanamori, 2001; Kanamori and Rivera, 2004], with the scatter from this average being directly related to the scatter in stress drop.

[16] A general problem in the estimation of radiated energy resides in the limited bandwidth of seismic recording systems, and in particular, limitations at high frequencies can have a serious impact on energy estimates, since more than 80% of the energy are carried by waves of higher frequency than the corner frequency [Izutani and Beroza, 2001]. Therefore, in order to avoid an underestimation of radiated energy, some authors selected events for the calculation of scaled energy according to the values of their estimated corner frequencies with respect to the analyzed frequency band. For instance, Abercrombie [1995] only used events for which \( 2f_{\text{min}} \leq f_C \leq f_{\text{max}}/5 \), and Mayeda and Walter [1996] limited their analysis to events for which at least 70% of the energy were radiated in their analyzed frequency range. As Izutani and Beroza [2001] pointed out, selection criteria of this type are likely to introduce an artificial trend. Considering the case of a simple \( \omega^2 \) source, they also proposed a simple formulation for correcting estimates of scaled energy for bandwidth limitation.

[17] With this extensive dataset at hand, we explore these issues and investigate the scaling characteristics of \( \tilde{e} \). First, we follow the selection criteria used by Abercrombie [1995] and only consider events with \( f_C \geq 1 \) Hz. Figure 3a shows \( \tilde{e} \) versus \( M_W \) and each data point is color-coded following its estimated stress drop. We applied no correction for missing energy, since its effect is negligible due to this rigorous selection. An apparent increase of \( \tilde{e} \) with increasing \( M_W \) is immediately visible. However, the stress drop color-coding indicates that for a given stress drop level, the estimates of energy-to-moment ratio are remarkably constant, and that the increase of scaled energy is simply related to an increase of stress drop. Figure 3b shows again \( \Delta \sigma \) versus \( M_W \), where the values of discarded events in Figure 3a due to too high or too low \( f_C \) are shown as blue and red symbols, respectively. As speculated by Ide and Beroza [2001], the missing events roughly define a triangle in the upper left corner (\( f_C > 5 \) Hz), but also in the lower right corner (\( f_C < 1 \) Hz). This way a clear artificial scale-dependency is introduced in stress drop, which is then reflected by a scale-dependency of scaled energy as well.

[18] In Figures 3c and 3d, we used all events with estimated corner frequencies in the range 0.05–10 Hz, which represents the largest part of our dataset, and we determined correction factors \( F_{\text{corr}} \) following Ide and Beroza [2001]. Figure 3c shows scaled energy with stress drop color-coding, while in Figure 3d, the color-coding relates to the correction factor \( F_{\text{corr}} \). As can be seen from Figure 3d, \( F_{\text{corr}} \) is generally lower than 1.5–2, with significant impact only for the largest earthquakes where \( f_C \) is smaller than the lowest frequency of analysis (0.5 Hz). Figure 3c clearly confirms our expectations from the fact that the source spectra obey
the $\omega^2$ model with self-similar static scaling, i.e. that the seismic energy-to-moment ratio does not show any dependence on seismic moment. Rather, for each given stress drop level, $\varepsilon$ is remarkably constant, and the overall scatter is only reflecting the scatter in stress drop (compare Figure 3c with Figure 3b).

6. Conclusions

[9] Our results from Japan thus indicate that the $\omega^2$ source model provides an excellent overall description of source spectral shape and that the scaling relationship $M_0 \propto f_\omega^2$ seems to hold over the entire investigated magnitude range. Moreover, the seismic energy-to-moment ratio does not show any significant trend with seismic moment, and the observed scatter can be fully attributed to the scatter in stress drop estimates. While the latter finding may seem trivial with respect to the first result of $\omega^2$ source spectral behavior, it has precisely been a subject of controversial discussion over the recent years and not been shown in this clarity and using such an extensive database up to date. We also clearly demonstrated the suspicion of Ide and Beroza [2001] that selection procedures based on corner frequency indeed introduce artificial trends in the $\varepsilon$ versus $M_0$ scaling behavior and can explain some of the scale-dependency of $\varepsilon$ seen in earlier studies.

[20] Finally, we note that our results were obtained through analysis of earthquakes spread throughout the entirety of the Japanese archipelago, while many of the earlier studies were based on results from specific earthquake sequences respectively mainshock/aftershock analysis. Therefore, it may be possible that within a particular earthquake sequence, a deviation from self-similarity may occur, which however seems not to be the case over such a large area as Japan.

[21] Acknowledgments. We wish to thank the National Research Institute for Earth Science and Disaster Prevention (NIED) for making the KiK-net data available. D. Di Giacomo was supported by a research grant from the European Center for Geodynamics and Seismology and enrolled in the PhD program of the University of Potsdam, Germany. This publication was supported by the National Research Fund, Luxembourg (FRN/10/AM4/45).

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