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Three-dimensional shear wave velocity imaging by ambient seismic noise tomography

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SUMMARY

3-D shear wave velocity images are of particular interest for engineering seismology. To obtain information about the local subsoil structure, we present a one-step inversion procedure based on the computation of high-frequency correlation functions between stations of a small-scale array deployed for recording ambient seismic noise. The calculation of Rayleigh wave phase velocities is based on the frequency-domain SPatial AutoCorrelation technique. Constitutively, a tomographic inversion of the traveltimes estimated for each frequency is performed, allowing the laterally varying 3-D surface wave velocity structure below the array to be retrieved. We test our technique by using simulations of seismic noise for a simple realistic site and by using real-world recordings from a small-scale array performed at the Nauen test site (Germany). The results imply that the cross-sections from passive seismic interferometry provide a clear image of the local structural heterogeneities and the shear wave velocities are satisfactorily reproduced. The velocity structure is also found to be in good agreement with the results of geoelectrical measurements, indicating the potential of the method to be easily applied for deriving the shallow 3-D velocity structure in urban areas and for monitoring purposes.

Key words: Interferometry; Surface waves and free oscillations; Seismic tomography.

1 INTRODUCTION

The shear wave velocity, $v_s$, is a key parameter related to the assessment of the local amplification of ground motion and is most commonly used in engineering seismology. Examples include empirical predictions of strong ground motion (e.g. Boore et al. 1997), site coefficients for building codes (BBSC 2004), liquefaction potential characterization (e.g. Stokoe & Nazarian 1985; Andrus & Stokoe 2000) as well as input for numerical simulations of the response of sedimentary basins (e.g. Bao et al. 1998; Pitarka et al. 1998; Pilz et al. 2011). Within this context, the shear wave velocity values are usually measured in situ by means of various invasive (borehole) or non-invasive (shear wave refraction and reflection studies) techniques. Although these techniques provide accurate and well-resolved values of $v_s$, they suffer from several drawbacks, such as increasing costs with required depth or only pointwise estimates.

In recent years, passive seismic techniques have received considerable attention. The success of these applications is explained by the fact that these methods are based on surface waves, which are by far the strongest waves excited by ambient seismic noise. Many recent studies on passive seismic processing have focused mainly on two applications: The simplest one, the single horizontal-to-vertical (H/V) technique, requires one station’s recordings only to derive local site conditions and uses the vertical component as a reference. If a strong impedance contrast between the sediments and the underlying bedrock is present, the peak of the H/V spectral ratio appears at the fundamental resonance frequency of the soil and is correlated to the shear wave velocity and the thickness of the low-velocity layers. For a known thickness of sediments, the shear wave velocity structure of the subsoil can then be obtained (e.g. Fäh et al. 2001, 2003; Scherbaum et al. 2003; Parolai et al. 2006; Pilz et al. 2010).

Since the beginning of this century, another procedure, seismic interferometry, has rapidly become popular in a variety of applications (see reviews by Campillo 2006; Curtis et al. 2006; Wapenaar et al. 2008; Schuster 2009; Snieder et al. 2009). One of the most intriguing applications of the method, shown both theoretically and experimentally, is that a random wavefield has correlations, which, on average, take the form of the Green’s function of the media (e.g. Rickett & Claerbout 1999; Lobkis & Weaver 2001; Weaver & Lobkis 2001; Snieder 2004; Wapenaar 2004; Sabra et al. 2005a, b). Between pairs of receivers, the Green’s function can be extracted from cross-correlations of ambient noise recorded at both receivers, in turn allowing an estimate of the propagation delay between the stations.

Such traveltime measurements of Rayleigh waves reconstructed from seismic noise have been used at lower frequencies to produce high-resolution images on continental (Yang et al. 2007; Bensen et al. 2008) and regional (Shapiro et al. 2005; Kang & Shin 2006; Yao et al. 2006; Lin et al. 2007; Moschetti et al. 2007) scales. However, traveltime inversion is a non-linear problem since the ray path is velocity-dependent. Consequently, small changes to the interface position might consequently also produce changes in ray trajectories. In contrast to global tomographic studies we are not seeking small perturbations to an established reference model, but
we rather attempt to locate clear velocity differences on a smaller local scale. Within this context, the applicability of the method has also been shown for higher frequencies at local scales (Chávez-García & Luzón 2005; Brenguier et al. 2007; Picozzi et al. 2009). Since, keeping in mind the level of errors of the input data for shallow seismic surveys, small deviations of the paths from a straight line can be tolerated. However, the use of high-frequency seismic noise interferometry requires a better understanding of the origin of seismic noise and of the spatial and temporal distribution of its sources (e.g. Pederson & Krüger 2007; Halliday & Curtis 2008, 2009). In particular, it remains important to establish reliable conditions under which noise can be considered to be well randomized.

Even if the origin of seismic noise is not fully known, Aki (1957) observed that cross-correlations between stations aligned in different directions did not differ substantially, concluding that ‘we may regard the microtremor as being propagated in every direction, each with almost uniform power’. Subsequently, Asten (2006), Bensen et al. (2007) and Harmon et al. (2010) stated that the effect of inhomogeneous source distributions on isotropic velocity estimates will be likely to be within the error bars of isotropic velocity studies.

Among several studies, there are two approaches in the interpretation of the correlation of seismic noise between pairs of stations. The first one, based on diffusion theory, has shown that it is possible to estimate the time domain Green’s function in a medium from measurements of ambient seismic noise (Snieder 2006). The second approach is based on the Spatial AutoCorrelation (SPAC) method (e.g. Aki 1957, 1965, Cox 1973).

Several authors (e.g. Chávez-García & Luzon 2005; Sánchez-Sesma & Campillo 2006; Chávez-García & Rodríguez 2007; Yokoi & Margaryan 2008) have demonstrated the equivalence between the results obtained by both techniques for a small-scale experiment at a site with a homogeneous subsoil structure. Further, observational studies of ground motion (Prieto & Beroza 2008) and attenuation (Prieto et al. 2009; Lawrence & Prieto 2011) have presented an optimistic picture of this ability to exploit real-world measurements of the amplitude of ambient noise. However, for non-homogeneous subsoil conditions (2- or 3-D structure), the SPAC method might only provide an average estimate of the S-wave velocity structure.

On the other hand, one can expect that, similarly to what is obtained over regional scales, local heterogeneities will definitively affect the noise propagation between sensors, and hence, can be retrieved by analysing the Green’s function estimated by the cross-correlation of the signals recorded at these stations.

Nowadays, retrieving the local 3-D S-wave velocity structure quantitatively by the analysis of ambient vibrations has gained considerable attention as a low-cost tool (e.g. Picozzi et al. 2009; Renalier et al. 2010). In these studies, however, the 3-D S-wave velocity structure is obtained in a two-step approach by first calculating fundamental group or phase velocity dispersion curves. The second step involves an inversion of the dispersion curves for obtaining local 1-D S-wave velocity-depth profiles, with the lower frequency component constraining deeper structures than the high-frequency parts of the waves. Using this approach the final 2- and 3-D S-wave velocity models can only be obtained by interpolation between the individual 1-D S-wave velocity profiles.

In this paper, we propose a new method for a one-step tomographic inversion scheme based on seismic noise that allows one to obtain an approximate 3-D S-wave velocity model that is useful for seismic engineering, as well as microzonation and monitoring purposes. This is feasible because the S-wave velocity is the dominant property for the fundamental mode of high-frequency Rayleigh waves. Since traditional sensitivity kernels are rather problematic in 3-D (Marquering et al. 1999), we solve this problem by a weighted 3-D inversion procedure. Hereunto, we will first describe how Rayleigh wave phase velocities are calculated. To validate the procedure, a synthetic example is presented, including a discussion on some of the problems that can be encountered in tomographic inversion. We further test the performance of the technique using real data collected at Nauen test site in northern Germany.

2 MODELLING OF SEISMIC NOISE

The expression ‘ambient seismic noise’ is a generic term for both low- and high-frequency signals. Cultural activities are the origin of the high-frequency signals, also called microtremors, whereas oceanic and atmospheric disturbances result in the low-frequency signals or microseisms (Longuet-Higgins 1950). For the following analyses, we only consider cultural noise higher than a few hertz.

Such synthetic wavefields of the cultural noise can be modelled as a distribution of impulsive point forces located at the surface or subsurface with random force orientation and amplitude (Lachet & Bard 1994). Following Coutel & Mora (1998), the force term in the wave equation can be written as,

\[ f_r(x, t) = a(x, t) \sin[2\pi \theta(x, t)] \]

with \( \theta \) being the orientation of the force relative to the vertical and \( a \) being the amplitude of the force. To simulate a random distribution of forces (both in direction and amplitude), \( \theta \) and \( a \) are taken as uniformly distributed random numbers between 0 and 1.

For our first test, noise synthetics were computed using the spectral element code GeoELSE, developed by the Center for Advanced Studies, Research and Development, in Sardinia and the Department of Structural Engineering of the Politecnico di Milano (Faccioli et al. 1997). The code was developed for the analysis of various wave propagation phenomena, including man-made vibrations and allows for sources and receivers at shallow depths.

For the calculation of noise synthetics, we consider a simple physical model representative of a soft-material site. It consists of a homogeneous layer, in which a second layer dipping at 20° characterized by a much higher S-wave velocity is embedded (Fig. 1). For the wavefield generation, a random force is applied at each time step at 168 different locations, arbitrarily distributed over the entire modelling surface (Fig. 2). The final data set is composed of 30 min of noise synthetics; such a duration is similar to that usually considered in real-world experiments. Examples of noise synthetics, bandpass filtered between 2 and 25 Hz and recorded at one receiver, are shown in Fig. 3(a). Fig. 3(b) presents the vertical Green’s function and its envelope calculated for a central frequency of 8 Hz. It can be seen that, in general, the Green’s functions are found to be rather symmetric, pointing out an almost uniform noise source distribution surrounding the two stations, which is required by the theory.

3 CALCULATION OF RAYLEIGH WAVE PHASE VELOCITY

We first determined the Rayleigh wave phase velocities, given that the uncertainty in the phase velocity measurement is much less than that of the measurement of group velocity (Bensen et al. 2008). The calculation of the phase velocities is based on the frequency-domain SPAC technique of Aki (1957), which also allows the inclusion of very long wavelengths for a given station spacing. If the microtremor
wavefield is stochastic and stationary in both space and time, the real portion of the azimuthally averaged correlation function between two stations $\rho(r, \omega_0)$ is related to the phase velocity by

$$\rho(r, \omega_0) = J_0 \left( \omega_0 c(\omega_0) r \right),$$

(2)

where $r$ is the interstation distance, $c(\omega_0)$ is the phase velocity for an angular frequency, $\omega_0$, and $J_0$ is the zeroth-order Bessel function of the first kind. Eq. (2) clearly indicates that if one measures $\rho(r, \omega_0)$ for a fixed $r$ and for various $\omega_0$, the phase velocity dispersion curve can be obtained by equating the zero crossings of the real part of the spectrum with the zero crossings of the Bessel function.

In particular, if the distribution of noise sources is azimuthally uniform, the imaginary part of the spectrum is zero and close to zero for modest azimuthal noise variations (Cox 1973; Webb 1986).

Recently, it was shown (Weaver & Lobkis 2004; Chávez-García et al. 2005) that it is possible to substitute an average over time of the correlation in the frequency domain between two stations for a fixed interstation distance instead of the azimuthal average of the correlation required by SPAC, a concept already mentioned by Aki (1957). The basic idea of this hypothesis is that the average of the cross-correlation between two stations for a long-enough time window averages all the different directions of the waves composing the microtremors, that is, a single station pair can be seen as being
equivalent for an azimuthal average of many station pairs with the same interstation distance. Although this theory is valid only if the sources of ambient vibrations or the subsoil structure do not impose a predominant direction of propagation, Derode et al. (2003) showed experimentally that an inhomogeneous source distribution will have a lower effect on the calculated traveltimes of the waves than on their signal-to-noise ratios. Even if the majority of the noise sources are clustered in a narrow azimuthal range, Picozzi et al. (2009) demonstrated that for frequencies of some hertz, the calculated traveltimes are stable if there is still sufficient noise signal from all other directions. Therefore, even if the strongest noise emerges only from a few directions, we are sure that the correlation function can still be calculated reliably. As an example, Fig. 4(a) shows the resulting correlation function plotted against frequency taking two stations of the synthetic data set with an interstation distance of 37 m.

Although the amplitude of the real part of the spectrum depends on the background noise spectrum, the locations of the zero crossings in the spectrum should be insensitive to variations in the spectral power of the background noise, unless there are no strong transient signals. We choose to use the locations of the zero crossings to derive dispersion values using a series expansion to calculate the zero crossings of the Bessel function (Elbert 2001). If $\omega_n$ denotes the frequency of the $n$th observed zero crossing and $z_n$ represents the $n$th zero of $J_0$, the corresponding phase velocities (blue dots in Fig. 4b) are obtained by,

$$c(\omega_n) = \frac{\omega_n r}{z_n}.$$

However, in real-world data the association of a given zero with a particular zero crossing of $J_0$ can be difficult since the noise may cause missed or additional zero crossings. Following Ekström et al. (2009), one may distinguish the positive-to-negative and the negative-to-positive zero crossings by generating two dispersion curves that can be assessed for consistency. The difference between the phase velocities of the two curves at each sampled frequency may be used as a quality criterion.

For the lowest analysed frequency, the calculated velocities are more scattered than for higher frequencies (Fig. 4b). When the wavelength becomes too large with respect to the interstation distance, the phase difference between the stations becomes negligible. This trend is particularly evident when the geophone intervals are small. Increasing the frequency shows a decrease in the estimated velocities, demonstrating the dispersive character of Rayleigh waves.
As a general constraint, the SPAC technique establishes a formal relationship between the correlation coefficients and the phase velocity dispersion curve which needs to obey the limits of Henstridge (1979), where it was shown that correlation functions can only be calculated correctly when $2 < \lambda r < 15.7$ (Henstridge criterion), where, $\lambda = 2\pi c(\omega_0)/\omega n_0$, is the wavelength of the wave considered. However, although in our case the left inequality is not always fulfilled, Tsai & Moschetti (2010) pointed out that the errors are mostly less than a few per cent, even for relatively small $\lambda r$, if the distribution of noise is uniform enough, which is likely to be the case in reality. In the following, since the $S$-wave velocity is the dominant property of the fundamental mode of high-frequency Rayleigh waves, we perform inversions for a $S$-wave velocity model estimation while considering the observed velocities in the 3-D frequency-dependent tomography as being representative of Rayleigh wave phase velocities.

### 4 THE ALGORITHM FOR 3-D SURFACE WAVE TOMOGRAPHY

To determine the traveltimes between the sensors, the first step of data processing consists of preparing waveform data from each individual station. Since our interest lies in detecting local variations in $S$-wave velocity for the shallowest part of the subsurface and since the distances of the array only span a range of some metres to around 100 m, we are strongly interested in high-frequency noise (higher than a few hertz). As demonstrated by Chávez-García & Rodríguez (2007), the local velocity structure and the interstation distance strongly affect the frequency range for which the phase velocities can be determined reliably. Chávez-García & Luzón (2005) have shown that Love waves dominate the records in the lower frequency range (lower than a few hertz), whereas Rayleigh waves are prevalent in the higher frequency band of their recordings. For our analyses, we consider only the vertical component of the cross-correlation functions, which are strongly dominated by Rayleigh waves. Therefore, a second-order Butterworth high-pass filter was applied to these data using a corner frequency of 0.9 Hz, with the aim of removing high-amplitude low-frequency irregularities that tend to obscure noise signals. From the continuous data sets, 60 noise time windows, each 30 s long, were extracted. Picozzi et al. (2009) have shown that this duration is sufficient for obtaining a reliable estimate of the Green’s function between two sensors for distances of some tens of metres. After removing the linear trend from each window, a 5 per cent cosine taper was applied at both ends.

After preparing the time-series, one-bit normalized data were used to calculate the cross-correlations, following Campillo & Paul (2003) and Bensen et al. (2007). Although some interstation distances may not be suitable to obtain reliable results, we perform cross-correlations between all possible station pairs. This yields a total of $n(n-1)/2$ possible station pairs, where $n$ is the number of stations. For each of the chosen frequencies and for each pair of receivers, all calculated cross-correlations were stacked together which, on average, improves the signal-to-noise ratio (Bensen et al. 2007). Finally, the traveltimes between all stations were calculated from the estimated phase velocities using the known distances between the sensors.

In the second step, the estimated traveltimes were inverted by a tomographic approach to calculate a 3-D velocity model. In a general form, the traveltime, $t$, between the source and a receiver along a ray path $L$ along a line element $dl$ for a continuous slowness, $s$, (inverse velocity) is given in its integral form as,

$$t = \int_L s dl. \quad (4)$$

This means that the traveltime is based on a linear combination of slowness. However, since the ray path is velocity-dependent, traveltime inversion is a non-linear problem, although, in media with slight velocity anomalies the deviation of the paths from a straight line will either be of the same order as the dimension of the blocks or will be less than a quarter of the wavelength. On the one hand, deviations smaller than the block size will not introduce significant errors and, on the other, deviations smaller than a quarter wavelength are within the limits of image resolution. Hence, following Kugler et al. (2007) and keeping in mind the level of errors of the input data for shallow seismic surveys, we are sure that a bias of a few percent can be tolerated keeping the solution linear, which will be discussed in more detailed later. This means that eq. (4) can be expressed in a simple discrete matrix form to be used in practical applications, $t = L_s s$, \quad (5)

where $L_s$ is an $RF \times MN$ matrix with $R$ being the number of rays for $F$ different frequencies crossing the medium that is subdivided into a reasonable number of $M$ cells in each of the $N$ horizontal layers. Since deviations smaller than the block size will not introduce significant errors and since differences less than a quarter wavelength will be within the limits of image resolution, the phase velocity is assumed to be constant within each of the cells. Therefore, the propagation paths can be considered to be straight rays within the cells.

However, to ensure the accuracy of the inversion, the setting of the cell size and the number of rays travelling through every cell is not a negligible issue. On the one hand, too few rays in each cell will cause the inversion not to converge or to converge too slowly to reflect the real velocities. By contrast, a too-large number of rays will lead to a waste of data, resulting in a lower resolution. Moreover, since the spatial resolution will also depend on the frequency range used, the quarter-wavelength criterion suggests that for a frequency range higher than a few hertz and low-velocity sediments, a cell size of several metres will provide balanced ray coverage.

The basic problem of tomography is to solve eq. (5) for the a priori unknown slowness. In general, uncertainties in measured traveltimes can degrade the solution and can produce spurious velocity anomalies. Since traditional sensitivity kernels are rather problematic in 3-D (Marquering et al. 1999), we adopted an iterative procedure for solving eq. (5) using damped least squares or singular value decomposition, following Long & Kocaoğlu (2001). For the first iteration, we start with a homogeneous 3-D model $s_{0}$ (i.e. the same velocity for all the cells) and calculate the new solution $s_{n+1}$ by solving

$$W\Delta t_{k} = L_s \Delta s_{k}, \quad (6)$$

in which $\Delta t$ is the vector of the normalized misfit between the observed and theoretical traveltimes, $[t_0 - t_{k+1}] / t_k$. Accordingly, $\Delta s$ is the vector of the normalized slowness modification $[s_{k+1} - s_{k}]/s_k$. To reduce the risk of divergence and to stabilize the iteration process, the adaptive bi-weight estimation (Tukey 1974), which is a kind of the maximum likelihood estimation, is applied by introducing a diagonal weighting matrix, $W$, with $RF \times RF$ elements. The design matrix, $L_s$, in eq. (6) consists of two blocks,
namely,
\[ L_2 = \begin{bmatrix} \mathbf{W} & \mathbf{I} \\ \mathbf{K} (\varepsilon^2) & \mathbf{M} \end{bmatrix}, \tag{7} \]
in which the upper-block elements represent the ray path segment matrix \( L_1 \) weighted by \( \mathbf{W} \). In particular, the measured slowness contains information about the underlying structure to depths corresponding to approximately one-third to one-half of the wavelength of each frequency.

The lower elements \( \mathbf{K} \) and \( \mathbf{M} \) in eq. (7) describe constraints on the solution, making use of the damping coefficient \( \varepsilon \). As pointed out by Marquardt (1963) and by Ammon & Vidale (1993), the damping factor significantly controls the speed of convergence and further acts as a constraint on the model space (Tarantola 1987). After some trial and error testing of different values, we fixed \( \varepsilon \) to 0.7 to give the finest resolution without causing instabilities in the inversion.

The \( \text{MNF} \times \text{MNF} \) matrix, \( \mathbf{K} \), weights the data depending on the number, length, orientation and vertical penetration depth of each ray path segment crossing each cell. According to Yanovskaya & Ditmar (1990), for the 3-D problem the weights turn out to be representable as a product of two functions, one depending on the horizontal coordinates and one depending on the depth, that is, the frequency. For the first function, the singular values \( (a_1, a_2) \) of the ray density matrix were used to calculate its ellipticity \( \sqrt{\lambda_2^{\text{ray}} / \lambda_1^{\text{ray}}} \), based on a proposal of Kissling (1988). If several rays with different azimuths cross the cell, the ellipticity is close to 1 and a good resolution is achieved. Therefore, the horizontal weights were computed by multiplying the ellipticity for the number of rays crossing each cell.

The second function calculates the vertical weights to account for different penetration depths of the different frequencies. Using a homogeneous starting model for the first iteration, the vertical weights are based on the analytical solution of displacement components, \( u \), for Rayleigh waves in a half-space, following Hill (2010):
\[ u(z) = \int_0^\infty \frac{\partial}{\partial z} u(z) dz - s e^{\sigma z}, \tag{8} \]
with \( z < 0 \) being the depth below the surface; \( k = \omega/c_R \) is the Rayleigh wavenumber, in which \( c_R \) is the Rayleigh wave phase velocity as given in eq. (3) and \( \omega \) the angular frequency. Since we can assume the Rayleigh wave velocity to be around 0.92 \( v_s \), which is realistic for typical values of Poisson’s ratio \( (0.2 < v < 0.4) \), this leads to \( t = 0.8475 \), \( p = 0.3933 \) and \( s = 1.4679 \) (see Bullen 1963; Lay & Wallace 1995). The corresponding vertical weights, \( w \), are calculated as the normalized average values of \( u \) within each layer,
\[ w = \frac{\int_0^{d_2} u(z) dz}{\int_0^{d_1} u(z) dz}, \tag{9} \]
with \( d_1 \) and \( d_2 \) being the lower and upper bound of each layer, respectively.

Using these equations will provide a first approximation of the behaviour of surface wave penetration at depth. Since eqs (8) and (9) strongly depend on the underlying Rayleigh wave phase velocities, the vertical weights for all the cells and accordingly the matrix \( \mathbf{K} \) are updated after each inversion. Eq. (9) further implies that the vertical weights vary smoothly, from which it follows that there is no practical limitation of the vertical resolution.

Matrix \( \mathbf{M} \) in eq. (7) constrains the solution to vary smoothly in the horizontal domain, that is, the slowness of each cell is related also to the slowness of all the surrounding cells with weights depending on the cell’s location. The smoothness constraints are implemented by adding a system of equations to the original traveltime inversion problem, following Ammon & Vidale (1993). Note, however, that these constraints do not act as a low-pass filter in the classical sense; therefore, not all sharp contrasts in the model are necessarily smoothed.

Finally, we solve eq. (6) using the singular value decomposition technique (e.g. Golub & Reinsch 1970; Arai & Tokimatsu 2004). The decomposition of the matrix, \( L_2 \), leads to a product of three matrices, \( \mathbf{U} \mathbf{A} \mathbf{V}^T \), in which \( \mathbf{U} \) and \( \mathbf{V} \) are \( (\text{RF} + \text{MNF}) \times (\text{RF} + \text{MNF}) \) and \( \text{MNF} \times \text{MNF} \) matrices, respectively and \( \mathbf{A} \) is a \( (\text{RF} + \text{MNF}) \times \text{MN} \) matrix that has the singular values of \( L_2 \), \( a_1 \), in the diagonal elements of the matrix. Decomposing \( L_2 \) in eq. (6) and utilizing the orthogonal property of the matrices \( \mathbf{U} \) and \( \mathbf{V} \), we obtain,
\[ \Delta a_{i1} = \mathbf{V} \mathbf{A}^T \mathbf{W} \Delta \mathbf{a}_{i1}. \tag{10} \]

If some of the eigenvalues of \( \mathbf{A}^T \) are small, errors in the data could cause strong fluctuations in the solution. To suppress those undesirable effects in solving eq. (10), the diagonal components of the matrix \( \mathbf{A}^T \), \( a_1^{-1} \), are replaced by \( (a_1 + \varepsilon/a_0)^{-1} \) (Levenberg 1944; Marquardt 1963) with the damping factor, \( \varepsilon \) (Fletcher 1971).

From eq. (10), the slowness vectors are updated after each inversion step until a reasonable compromise between the reduction of the rms error between the observations and the predictions and the norm of the solution is reached.

The final slowness \( s \) is used to calculate the 3-D shear wave velocity, \( v_s \). Slowness is related to the shear wave velocity by,
\[ s = \frac{1}{c v_s}, \tag{11} \]
that is, \( c < 1 \). For typical values of Poisson’s ratio \( (0.2 < v < 0.4) \) and a homogeneous half-space, \( c \) takes values \( 0.9 < c < 0.95 \). As already mentioned, we used \( c = 0.92 \) for all calculations. We will discuss later that this approximation will hold for engineering purposes.

## 5 Validation of the Proposed Inversion Scheme

### 5.1 Synthetic example

For evaluating to what degree the initial model can be restored and to check the resolution of the results, we used the synthetic data set with a known subsurface structure as described in Section 2. Since the spatial resolution depends on both the frequency and the inversion technique and following Nolet (1987) and Jensen et al. (2000), a horizontal resolution of 10 m can be achieved for the selected frequency range (4.5 Hz \( f \leq 13 \) Hz). For the vertical resolution we chose a value of 5 m to calculate the average velocities for each layer.

Fig. 5 shows the inversion results and the average S-wave velocities for the indicated depths using five different frequencies (4.5, 6, 8.3, 10.1 and 13 Hz) to calculate the interstation traveltimes estimated by the correlation function. The comparison between the input structure and the calculated velocities shows that the structure is well reproduced. For the uppermost layer the velocity is almost constant, whereas for deeper layers the widening high-velocity block can clearly be identified. Also, in terms of absolute values, the S-wave velocities are fairly well reproduced, whereas, the velocity contrasts are sometimes less sharp. This is based on the fact that we used a smoothing parameter in the solution (matrices \( \mathbf{K} \) and \( \mathbf{M} \) in eq. 7).
Due to the difficulty in calculating meaningful uncertainty estimates of the weighted and smoothed inversion procedure, modern statistical techniques, such as bootstrap, (Efron 1979; Koch 1992) have been proposed over the last few decades. Here, we propose the application of the bootstrap technique to the tomographic algorithm presented in this work. Bootstrap methods work by repeated inversions of the bootstrap data set obtained by randomly resampling the original data set. The resampling is carried out by replacing a row of the matrix with another one, with both rows randomly chosen. Each bootstrap sample has the same size as the original data set and the bootstrap estimate of the standard error is given by the standard deviation of the distribution of the reconstructed velocity models (called bootstrap replications) obtained for each bootstrap sample. Finally, the standard deviation for the velocity model obtained by the inversion of the original data set is estimated from the values of the bootstrap standard error and the number of data.

We performed 500 bootstrap inversions to estimate the stability of the solution. Fig. 6 shows the standard deviation estimated for the tomographic models for two different depths. The standard deviation is often significantly smaller than 30 m/s and often, especially in parts with high ray coverage, below 10 m/s. We also observe slightly higher uncertainties for deeper parts of the model due to larger variations in the short frequency data. A similar observation was reported by Yang et al. (2006). Nevertheless, the results confirm the reliability of the applied inversion algorithm to obtain accurate S-wave velocity distributions.

**5.2 A real-world example: the Nauen test site**

We now apply the proposed procedure to a real data set, obtained from an array consisting of 21 three component seismological stations deployed at the test site of Nauen (Germany) for one week in 2007 May (http://www.geophysik.tu-berlin.de/menue/forschung/testfeld_nauen/uebersicht). Further details about this experiment can be found in Picozzi et al. (2009). The geology of the Nauen site is representative for large areas of northern Germany with Quaternary sediments overlying Tertiary clays. In the actual area of investigation, these sediments mainly consist of fluvial sands bordered by glacial till. The topography is characterized by flat hills underlain by till and plains comprised of glacial fluvial sands and gravels. Therefore, there was no need to consider the shape of the topography in the inversion. In the studied area, there is an unconfined shallow aquifer consisting of fine to medium sands underlain by an aquiclude of marly and clayey glacial till. North of the site, the glacial till approaches the surface in a nearly E–W strike. Electrical resistivity cross-sections indicating the 2-D subsurface structure and the distribution of the seismic stations are shown in Fig. 7. It is obvious that the array covers structural variations at the site and a good azimuthal coverage exists.

After correcting the noise recordings for the instrumental response, the recorded data were processed using the same technique as described above. Inversion results for four horizontal slices obtained after 200 iterations using four different frequencies (5, 7, 10 and 14 Hz) are presented in Fig. 8. The calculated velocities are shown only for cells which are crossed by rays and a few adjacent
ones uncrossed by rays but influenced by the smoothing constraints imposed on the solution.

The occurrence of strong variations in the site conditions is visible at a first glance. The topmost layer shows an almost constant velocity distribution, suggesting that the medium is quite homogeneous down to 5 m. For the subjacent layer between 5 and 10 m, higher velocities are found in the northeastern part of the study area. Below 10 m, the area of high velocities extends towards the southwest. This is a clear indication of a two-dimensional structure that was also found in two 2-D geoelectric cross-sections (see Fig. 7). Profile A clearly indicates a layer with low electrical resistivity below 5 m in the northeastern part, which is identified to be glacial till. This layer extends towards the southwest and covers the entire study area below \( \sim 17 \) m (Yaramanci et al. 2002).

As can be seen when comparing Figs 7 and 9, this area corresponds well with the absolute depth and the impedance contrast level in our cross-sections. Also, the absolute velocities agree well with Goldbeck (2002) who reported S-wave velocities between 200 and 450 m/s for profile A. Fig. 9 further shows a fair agreement of our interpolated results with the findings of Picozzi et al. (2009). However, the latter authors calculated the 1-D S-wave velocity-depth profiles. Their 2-D S-wave velocity cross-sections could only be obtained after the interpolation of the inversion results of the Rayleigh wave velocity dispersion curves along a profile of 10 cells. In contrast, our proposed inversion method directly allows the calculation of a first-order 3-D S-wave velocity model.

6 DISCUSSION

Our proposed technique aims at estimating a 3-D model of the local S-wave velocity structure at shallow depths. However, the
The calculation of the final $S$-wave velocity model (eq. 11) is an approximation and in a strict sense only valid for a homoge-
neous half-space. However, it is still reasonable because the $S$-wave velocity is the dominant property for the fundamental mode of high-frequency Rayleigh waves. Of course, for a layered structure the Rayleigh wave disperses when the wavelengths are in the range of 1–30 times the layer thickness (Stokoe et al. 1994), but Xia et al. (1999) never found variations of more than 25 per cent in shear wave velocity between this approximation and their true multilayer model. On the other hand, using an alternative procedure by inverting our phase velocity results to obtain $v_s$-depth profiles would involve additional uncertainties, since it would further require information about the densities and $P$-wave velocities introduced by the inversion process. Furthermore, also the inversion of dispersion

Figure 8. Inversion results using data collected at the Nauen test site. The images were obtained after 200 iterations. Yellow lines mark the locations of the geoelectric cross-sections shown in Fig. 7.
curves calls also for homogeneity. If the wave path is horizontally heterogeneous, Kennett & Yoshizawa (2002) and Strobbia & Foti (2006) show that this can cause significant perturbations in the observed velocity. As a consequence, Lin & Lin (2007) point out that artefacts may be introduced in spatially more-dimensional shear wave velocity imaging if lateral heterogeneity is not accounted for.

Hereunto, our 3-D model provides a fair estimate of the prevailing velocity structure that can be used for engineering and monitoring purposes.

To estimate the exact resolution limits of our inversion technique, one has to take into consideration that the horizontal resolution will depend on many site specific factors, such as the velocity structure, the used frequencies and the wave propagation paths. Therefore, no strict formula can be provided. Keeping in mind the quarter-wavelength criterion and the selected frequency range, a horizontal resolution of 10 m is fairly reachable for low-velocity sediments. However, when the size of the cells is too large, our methodology tends to smear sharp discontinuities in velocity and is, therefore, also likely to obscure fine scale velocity structures. Nonetheless, for most of the engineering applications, such small-scale peculiarities are of minor interest.

Since we introduced a smoothing parameter in the solution (matrices $K$ and $M$ in eq. 7) the formulation above assumes that the properties of the medium are only changing smoothly at every point of the medium. Of course, it is easy to generalize the equations to the case where there are discontinuities as explicit parameters of the inverse problem. However, if such an approach is followed, non-existent discontinuities may also be introduced as the iterative inversion proceeds as steep gradients, but, obviously, this is not ideal.

In contrast, since the weights of the individual cells vary smoothly vertically, there will also be only a steady change in the resulting velocities, that is, vertical discontinuities will be resolved less sharply. For deeper parts of the investigated area, only smaller frequencies show non-zero weights, reducing the resolution for these parts. Subsequently, since at each frequency information about the underlying velocity structure to a depth of about one-third to one-half wavelength is provided, the penetration depth only depends on the lowest frequency used. The use of very low frequencies might be problematic due to difficulties in the correct determination of the traveltimes, averaging out smaller scale $S$-wave velocity contrasts. On the other hand, the absolute number of data sets (frequencies) will not have a significant influence on the inversion results as long as a wide enough range of frequencies uniformly encounters the depths of interest. In particular, there appears to be no decrease in accuracy due to reductions in the number of used frequencies, consistent with Rix et al. (1991). Analyses on the number of necessary frequencies have indicated that five or six frequencies equally distributed over the frequency range of interest are sufficient, since neighbouring frequencies provide similar information (not shown here). Of course, more frequencies can be used at the expense of reduced inversion speed.

In addition, to estimate the effect of the initial value selection on the final results, we ran both inversions using many different homogeneous starting models. Within the region of highest ray coverage, the standard deviation is much less than 10 per cent. Only a few isolated spots show deviations up to 20 per cent in areas of sparse ray coverage. The choice of the starting value was found to be critical to the convergence of the inversion procedure only if an unrealistic initial velocity (i.e. the difference to the final velocity was larger than a factor 4) is chosen.

7 Conclusions

We have used synthetic and real-world data sets based on the vertical component of seismic noise recordings to perform a tomographic 3-D inversion for obtaining images of the $S$-wave velocity structure. Based on the correlation of seismic noise recordings, Rayleigh wave phase velocities and corresponding traveltimes have been calculated between each pair of seismic sensors. With only a limited number of seismic stations and recording times of several tens of minutes, detailed images of the local subsoil structure can be obtained. The reliability of the proposed technique was validated using synthetic data sets and bootstrap showing that the results are well constrained.

Using real recordings of seismic noise and comparing the results with independently calculated geophysical results, a good agreement for the position of the main geophysical boundaries is found, highlighting the potential of the method to be used where other geophysical methods might fail. Since reliable traveltime estimates for the frequency range investigated can be obtained in almost real time, the results imply the use of the proposed procedure as an exploration and monitoring tool. In future work, a joint inversion of Rayleigh and Love wave data sets, the latter estimated from horizontal components, should help to further constrain the inversion and to improve the resolution of the model.

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