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Assessment of $P$ and $S$ wave energy radiated from very small shear-tensile seismic events in a deep South African mine

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We discuss requirements for reliable estimates of radiated seismic energy in $S$ and $P$ phases and derive ratios of $S$-to-$P$ radiated energy ($E_S/E_P$) of 539 seismic events with local magnitudes $-5.23 \leq M_L \leq -2.41$ (subdecimeter size) recorded by high-frequency acoustic emission (AE) sensors of the JAGUARS seismic network in the Mponeng deep gold mine, South Africa. The analyzed events are aftershocks of a $M_{P}1.9$ earthquake, and the recording AE sensors are located within about 40 m of the events. A shear-tensile model is used to simulate the radiation pattern of $P$ and $S$ phases from a family of rupture models ranging from pure shear to pure tensile failure. The calculations include correction factors for energy estimates associated with given source-receiver geometries and expected focal mechanism with possible tensile component. Synthetic calculations are used to assess the effects of limited observed frequency band and attenuation on the estimated $E_S/E_P$ ratios. The model calculations provide guidelines on when different approximations may be used. The obtained $E_S/E_P$ ratios for the analyzed events are relatively low (median value < 5) for the full range of model parameters tested, suggesting that significant number of the events display a tensile component. Events with very small ratios (e.g., <1) may reflect enhanced $P$ radiation associated with rock damage in the source volumes.


1. Introduction

The energy radiated from earthquakes and other sources of brittle instabilities is a fundamental quantity of the failure process with important implications for energy partitioning under different conditions and event types, seismic shaking hazard, and multiple other topics [e.g., Abercrombie et al., 2006, Shi et al., 2008, and references therein]. However, the observational constraints on the radiated seismic energy are limited because of large uncertainties in the data analysis process. These are associated primarily with imprecise knowledge of source mechanisms and medium properties, incomplete station coverage, and limited recorded frequency range. Estimates of radiated energy by different investigators for the same earthquake often differ by more than an order of magnitude [Singh and Ordaz, 1994; Mayeda and Walter, 1996]. In comparison, the uncertainty of seismic moment estimates involving only the low-frequency part of the radiated energy typically does not exceed a factor of 2 [e.g., Prieto et al., 2004].

In general, measuring radiated energy accurately requires data with high signal-to-noise ratio over a wide range of frequencies [Singh and Ordaz, 1994], and a number of sensors well surrounding the seismic source (i.e., sufficient directional coverage of the focal sphere) to account for radiation pattern and directivity effects. Obtaining the necessary bandwidth is particularly difficult for small events, due to the low amplitudes of the seismic radiation and the rapid attenuation of the involved high-frequency waves. Insufficient frequency bandwidth typically results in a severe underestimation of the radiated energy [e.g., Di Bona and Rovelli, 1988; Ide and Beroza, 2001]. Significant errors in estimation of radiated energy may also be caused by inappropriate attenuation correction and sensor decoupling/site effects.

The ratio between the radiated energy in the $S$ and $P$ waves, $E_S/E_P$, is an important parameter of the failure mechanism, which may indicate the existence of tensile components of faulting [e.g., Gibowicz and Kijko, 1994]. Estimating the contribution of tensile faulting to the radiation can have profound implications for many aspects of the physics of earthquakes and other brittle instabilities including the heat flow paradox [e.g., Brune et al., 1993; Ben-Zion, 2001; Ou, 2008], investigating fault complexity, implications for landslide mechanics, and processes in volcanic eruptions [Julian et al., 1998], the role of high pressure fluids in the seismogenic process [Fischer and Guest, 2011], and damage-related radiation produced by dynamic changes of elastic moduli [Ben-Zion and Ampuero, 2009; Castro and Ben-Zion, 2013].
[5] For a pure point shear source in a Poissonian solid \(V_P = \sqrt{3}V_S\), the expected \(E_S/E_P\) ratio is 23.2 [Venkataraman and Kanamori, 2004]. However, the finiteness of the fault, rupture velocity, and directivity effects significantly affect the expected \(E_S/E_P\) ratios [e.g., Molnar et al., 1973; Sato and Hirasawa, 1973; Madariaga 1976]. For the simple kinematic fault model of Haskell [1964], the expected \(E_S/E_P\) ranges are, respectively, 30.4–54.0 and 27.4–39.1 for in-plane and antiplane shear faults, assuming Poissonian solid and \(V_R = 0.8V_S\). Sato and Hirasawa [1973] obtained \(E_S/E_P = 17.9 - 24.4\) for a circular shear crack model and \(V_R = (0.5 - 0.9)V_S\). Observational studies associated with shear-dominated earthquakes report typically a broad variety of \(E_S/E_P\) ratios ranging between about 10 and 30. As examples, Prieto et al. [2004] estimated \(E_S/E_P\) value of 9 ± 1.5 for a cluster of over 400 microseismic events recorded by the Anza seismic network in Southern California. Boatwright and Fletcher [1984] reported \(E_S/E_P\) value of 27.3 ± 3.3 based on detailed analysis of nine small earthquakes recorded in Monticello, South Carolina.

[6] Tensile source processes can enrich the P radiation [e.g., Vavryčuk, 2001; Ben-Zion and Ampuero, 2009; Ou, 2008] and lead to lower \(E_S/E_P\) values. Haskell [1964] predicted \(E_S/E_P \approx 3.17 - 4.67\) for a rectangular tensile fault with constant rupture velocity \(V_R = 0.8V_S\). Sato [1978] estimated \(E_S/E_P = 0.90 - 1.05\) for a circular tensile fault assuming rupture velocity ranging from 0.9 \(V_S\) down to 0.1 \(V_S\). Various observational studies reported enriched P wave radiation and relatively low \(E_S/E_P\) ratios [e.g., Gibowicz and Kijko, 1994; Vavryčuk et al., 2008; Fischer and Guest, 2011; Castro and Ben-Zion, 2013], typically in association with small seismic events in mines, fluid injection, and regions without simple preexisting fault. Gibowicz et al. [1990] found that the majority of mine tremors recorded in the Ruhr Basin, Germany, display \(E_S/E_P < 10\). Similar results were reported for nanoseismicity detected in the Underground Research Laboratory (URL) in Canada [Gibowicz et al., 1991]. In both cases, the events with low \(E_S/E_P\) ratio were interpreted as tensile fractures or shear fractures combined with significant tensile component. Garcia-Garcia [2004] observed \(E_S/E_P\) ratios in the range 1–10 (mean value 5.5) for 43 micro-earthquakes recorded in the Granada Basin in Southern Spain. Castro and Ben-Zion [2013] found enhanced high-frequency PP wave radiation from aftershocks of \(M_{\text{w}}\)7.2 earthquake that might reflect isotropic radiation generated by rock damage in the source region. Collins and Young [2000] reported that about 10% of picoseismic activity \(M_{\text{w}}\) ranging −4.2 to −2.9 recorded at the URL have \(E_S/E_P < 10\).

[7] In this work, we perform a detailed study of \(E_S/E_P\) ratios of pico- and femtoseismicity [Bohnhoff et al., 2010] involving source sizes down to a centimeter scale \((-5.23\leq M_{\text{w}}\leq -2.41)\) recorded during the JAGUARS project in the Mponeng deep gold mine, South Africa [Nakatani et al., 2008]. Using spectral fitting and spectral ratio techniques, Kwiatek et al. [2011] investigated source processes and parameters of seismic events induced by blasting related to the mining activity and aftershock sequence of \(M_{\text{w}}\)1.9 earthquake [Yabe et al., 2009]. They reported \(E_S/E_P\) ratios as low as 1 with a median of value 5.5 and speculated that the low \(E_S/E_P\) ratios are related to tensile failures that exist in the stope environment together with shear failures [van Aswegen, 2008]. In the present study, we attempt to resolve whether the low \(E_S/E_P\) ratios of events in the mine reflect actual source processes or may result from uncertainties in estimating the radiated energy. We first discuss theoretically the influence of the radiation pattern, attenuation, and limited frequency band on \(E_S/E_P\) ratio estimates in cases involving shear and tensile faulting. Using the theoretical considerations, we then analyze a subset of minute high-quality seismic events following the \(M_{\text{w}}\)1.9-induced seismic events [Yabe et al., 2009] that occurred in a close proximity to the JAGUARS network.

2. Estimation of Radiated Energy

[8] The radiated energy flux from the source may be calculated using:

\[
J_c = \int_0^\infty \left| \frac{\omega \Omega(\omega)}{\Omega_c} \right|^2 d\omega, \tag{1}
\]

where \(\Omega(\omega)\) is the root mean squared (RMS) ground displacement spectrum of either P or S wave. In the ground displacement model of Boatwright [1980]:

\[
\Omega(\omega) \propto \sqrt{\frac{1}{1 + \left(\omega \Omega_0 \omega_c \right)^4}}, \tag{2}
\]

where \(\omega_c = 2\pi f_c\) is the corner frequency, and \(\Omega_c\) is the spectral level (proportional to the seismic moment) of either P or S waves. As a consequence, the radiated energy flux depends on the spectral level and corner frequency. Since the moment determined from a given wave type \((P\text{ or } S)\) is scaled by the cube of the wave velocity [Keilis-Borok, 1960], the spectral level of P waves is scaled down by a factor of \((V_P/V_S)^3\) compared to the spectral level of S waves.

[9] For spatially stationary seismic or explosive circular source, the corner frequency of \(P\text{ or } S\) phases \(f_S\) is scaled by a factor \(V_P/V_S\) in comparison to the corner frequency of \(S\) phases \(f_S\) [Hanks and Wyss, 1972]. In this case, the corner frequencies are simply defined by the interference of radiated waves with wavelengths greater than a certain critical value and depend exclusively on source dimension and wave velocities in the source region. The stationary approximation ignores the influence of the rupture velocity and directivity effects [e.g., Randall, 1973; Sato and Hirasawa, 1973], which reduce the ratio of \(f_S/f_S\) and increase the ratio of \(E_S/E_P\). The inequality \(f_P \geq f_S\) still holds for circular shear fault models that incorporate rupture velocity and directivity in the typical case when the source does not radiate for a long time compared to the time for seismic waves to traverse the source [e.g., Molnar et al., 1973]. However, the resulting \(f_S/f_S\) is lower than the \(V_P/V_S\) ratio that characterizes the stationary source. As examples, Sato and Hirasawa [1973] obtained \(f_S/f_S = 1.26 - 1.39\) (averaged over the focal sphere for a circular shear fault and rupture velocities in the range \(V_R = 0.5 - 0.9V_S\), and Madariaga [1976] estimated \(f_S/f_S = 1.52\) for a circular shear fault and \(V_R = 0.9V_S\). For long and thin faults, even lower values of \(f_S/f_S\) are expected [e.g., Molnar et al., 1973; Savage, 1972]. For instance, the expected corner frequency ratio for the source model of Haskell [1964] with \(V_R = 0.9V_S\) is \(f_S/f_S = 0.77\). However, circular fault models are likely to better describe small events [Molnar et al., 1973; Gibowicz and Kijko, 1994].

[10] The energy flux at a given position on the focal sphere depends strongly on the radiation pattern, and also on...
directivity and rupture velocity. Neglecting directivity and rupture velocity, the energy recorded at a given station that is radiated by a given wave type from a seismic source can be expressed as [Boatwright and Fletcher, 1984]:

\[ E_C = 4\pi\rho V_C \langle R_C \rangle^2 \left( \frac{R}{R_C} \right)^2 J_C, \quad (3) \]

where \( \rho \) is rock density, \( V_C \) is the wave velocity, \( \langle R_C \rangle \) is the \( P \) or \( S \) wave average RMS radiation pattern correction coefficient [Boore and Boatwright, 1984], \( R_C \) is the correction for the radiation pattern of either \( P \) or \( S \) waves at a particular station (which depends on takeoff angle, azimuth from the seismic source and focal mechanism), and \( R \) is the distance between the source and receiver.

The energy flux may be estimated from waveforms by integrating the squared ground velocity seismogram, or equivalently using the ground velocity amplitude spectrum [Andrews, 1986; Snoke, 1987]. This requires correcting the input data for the attenuation along the path between the source and receiver. In spectral-based calculations, the energy flux is calculated using the following formula:

\[ J_C = \frac{2}{\pi\rho} \int \left[ \tilde{u}_C(f) \exp \left( \frac{\pi f R}{V_C Q_C} \right) \right]^2 df. \quad (4) \]

Here the raw ground velocity spectrum \( \tilde{u}_C(f) \) of either \( P \) or \( S \) waves is multiplied by an exponential factor, describing in this case a frequency-independent attenuation represented by the quality factor \( Q_C \), and the integration is performed over the limited available frequency band. The upper and lower bounds of the integral in equation (4) depend on the limited sampling rate of the acquisition system, transfer function of the sensor, and the filters applied. The limited frequency band should be accounted for [e.g., Snoke, 1987; Di Bona and Rovelli, 1988; Ide and Beroza, 2001] while calculating the energy flux.

2.1. \( E_S/E_P \) Ratio

If the spectral shape is constant over the focal sphere (e.g., for a stationary circular crack) with a corner frequency ratio \( f_C/f_S = V_p/V_S \) (i.e., corner frequencies depend only on source dimension and wave velocities), the ratio of radiated energies in the \( P \) and \( S \) waves is [Randall, 1973]:

\[ \frac{E_S}{E_P} = \frac{R_S^2 V_S^3}{R_P^2 V_P^3} \quad (5) \]

Using mean radiation pattern correction coefficients for shear faulting integrated over the whole focal sphere of \( \langle R_p \rangle^2 = 4/15, \langle R_S \rangle^2 = 2/5 \) [Boore and Boatwright, 1984], and Poisson's solid \( (V_p = \sqrt{3}/V_S) \) leads to radiated energy ratio \( E_S/E_P = 4.5 \) for the stationary circular shear source. More generally, if the RMS \( P \) wave spectral shape is a scaled version of the RMS \( S \) wave spectral shape, we can write [Boatwright and Fletcher, 1984]:

\[ \frac{E_S}{E_P} = \frac{R_S^2 V_S^3}{R_P^2 V_P^3} y^\gamma, \quad (6) \]

where \( y = f_p f_S \sim V_p/V_S \) is the ratio of the RMS \( P \) wave corner frequency to the RMS \( S \) wave corner frequency. As pointed out before, shear models accounting for different fault geometries, various rupture velocities, and directivity effects lead to lower \( y \) values, which will increase the \( E_S/E_P \) ratio according to equation (6). The circular shear crack model of Sato and Hirasawa [1973] gives \( E_S/E_P = 24.4 \) for \( V_p = 0.9 V_S \); the ratio drops down with decreasing \( V_p \) and stabilizes at \( E_S/E_P \approx 17.8 \) for \( V_p < 0.7 V_S \). The elongated rectangular shear fault models of Haskell [1964] generate even larger values of the energy ratio. In contrast, the circular tensile crack model of Sato [1978] gives much lower ratio \( E_S/E_P = 1.1 \), which is practically independent of rupture velocity.

Based on the discussed theoretical and observational results, we assume that the lower bound for \( E_S/E_P \) ratio for shear faulting is 4.5, which characterizes a simple stationary source [Boore and Boatwright, 1984] that ignores various physical effects (e.g., directivity) or is associated with the unlikely case of supershear faulting [Sato and Hirasawa, 1973]. In the context of our study, rupture directivity is unlikely to play in most cases a significant role given the very small source sizes (subdecimeter scale). In the following sections, we perform synthetic calculations and data analysis based on the simple stationary model associated with equations (5) and (6). Having \( E_S/E_P \) ratios for the analyzed mine events that are lower than 4.5 would provide an indication for tensile components of faulting.

2.2. Influence of Radiation Pattern

The radiation pattern coefficient \( R_C \) used in equation (3) is a function of wave type \( (P/S) \), source mechanism, azimuth of observation, and takeoff angle. For shear faulting, it is sufficient to specify the strike, dip, and slip rake along the fault plane to calculate \( R_C \) for given takeoff angle and azimuth. Having the radiation of either \( P \) or \( S \) waves, one can follow the methodology of Boore and Boatwright [1984] to calculate the RMS average radiation pattern coefficients \( \langle R_C \rangle \) and use them together with estimates of the energy flux at a station to assess the radiated energy. To account for radiation from a tensile source, we use the radiation pattern formula of Ou [2008] for shear-tensile source model (STSM) governed by two additional parameters: tensile angle \( \alpha \) and Poisson's ratio \( \nu \) (see supporting information for details). The tensile angle is measured between the vector along the slip direction projected on the fault plane and the actual direction of the fault movement, and is positive for opening and negative for closing motions. When the tensile angle is \( \alpha = 0 \), the radiation from the STSM corresponds to the classical shear source, while for \( \alpha = 90^\circ \), the radiation corresponds to pure tensile opening of the fault. The Poisson's ratio in the source volume changes generally during faulting [e.g., Ben-Zion and Ampuero, 2009 and references therein], but may be calculated for our purpose using the prefaulting \( P \) and \( S \) wave velocities.

Figure 1 presents graphical visualization of the radiation patterns of \( P \) and \( S \) phases for different tensile angle values assuming \( \nu = 0.25 \) [cf. Favryčuk, 2001]. It is clearly seen that the amplitudes of the \( P \) and \( S \) radiations in different directions are changing dramatically even for relatively small values of \( \alpha \). Other interesting features are that the nodes in \( P \) wave radiation pattern are suppressed with increasing tensile character of faulting and that for \( \alpha = 90^\circ \) the radiation maxima of \( P \) and \( S \) waves are in the same direction, rather than the orthogonal relation for \( \alpha = 0 \). See also Favryčuk [2001] and Shi and Ben-Zion [2009]. These results have serious
Figure 1. Influence of the tensile angle on the shape of radiation pattern of (top) $P$ and (bottom) $S$ waves assuming Poisson's ratio 0.25 as a function of the tensile angle (increasing values of tensile angle from left to right). The fault is shown as transparent black plane with slip vector marked by yellow arrow and the normal to the fault plane is shown using blue arrow. The directions of the slip vector and its projection to the fault surface (black arrow) coincide only for pure shear motion ($\alpha = 0^\circ$). For $P$ waves, the color reflects the amplitude of radiation pattern coefficient including sign information. For $S$ waves, the sign information is not included as the $S$ radiation pattern is calculated from SV and SH components using $R_S = \sqrt{R_{SH}^2 + R_{SV}^2}$.

Figure 2. The relation between the average RMS radiation pattern coefficients of $P$, $S$, SV, and SH waves as a function of the tensile angle. The RMS radiation pattern coefficients were averaged over the whole focal sphere. (a) The $R_P$ coefficient is plotted for three values of Poisson's ratio (0.20, 0.25, and 0.29, drawn with dashed, solid, and dotted lines, respectively). The $R_P$ coefficient is appropriate for an arbitrary focal mechanism as it is averaged over the whole focal sphere. (b) The $R_S$ coefficient is plotted using black solid line and is independent of the focal mechanism. The average RMS radiation pattern coefficients of SH and SV phases, $R_{SH}$ and $R_{SV}$, are shown using dotted and dashed red/blue/green lines, corresponding to the different focal mechanisms.
implications for the reliability of calculated radiated energy from P or S phases using equation (3) when a limited number of stations is used. If the station location is unfavorable (e.g., close to the nodal planes), small values of the correction factor $R_P$ in equation (3) producing large corrections to the calculated energy flux may result in significant overestimation of the radiated energy. This problem may be overcome by using the “water level” approach [Gibowicz and Kijko,

Figure 3. The relation between the average RMS radiation pattern coefficients of $P$, $S$, $SV$, and $SH$ waves as a function of the tensile angle. The RMS radiation pattern coefficients were averaged over local distances (takeoff angle 120°–180°) and the color reflects different focal mechanism. (a) The $R_P$ coefficient for three values of the Poisson’s ratio (0.20, 0.25, and 0.29, drawn with dashed, solid, and dotted lines, respectively). (b) The $R_S$ coefficient plotted using solid line and $R_{SH}, R_{SV}$ shown with dotted and dashed lines. The $R_S$, $R_{SH}$, and $R_{SV}$ values are independent of Poisson’s ratio.

Figure 4. The expected value of $E_S/E_P$ ratio assuming invariance of the spectral shape (cf. equation (5)) over the focal sphere as a function of the tensile angle for various values of Poisson’s ratio (0.20, 0.25, and 0.29 drawn with dotted, solid, and dashed lines, respectively). (a) The RMS radiation pattern coefficients averaged over the whole focal sphere (cf. Figure 2). (b) The RMS radiation pattern coefficients averaged over takeoff angles 120°–180° (cf. Figure 3).
Figure 5. The expected depletion in energy flux due to ignoring (a) attenuation and (b) attenuation and limited frequency band of observation in comparison to the energy flux at the source (see text for details). The black cross in Figure 5b denotes the expected depletion in energy flux measurement for $t^* = 3.3 \times 10^{-5}$ (e.g., $Q_C = 440, V_C = 6770$ m/s, $R = 100$ m).

Figure 6. Bias introduced to estimated $S$-to-$P$ energy ratios when (a) only attenuation is not taken into account and (b) both attenuation and limited frequency band are not taken into account. The calculations were made assuming $V_P/V_S = f_P/f_S = 1.73$ and $Q_P/Q_S = 9/4$. 

1994] to suppress outliers or using a certain weighting scheme [Boatwright and Fletcher, 1984].

We model the average RMS radiation pattern coefficients of $P$, $S$, SH, and SV waves versus the tensile angle of the STSM following the Monte-Carlo sampling procedure presented in Boore and Boatwright [1984]. Two cases are considered: one with radiation pattern coefficients averaged over the whole focal sphere (takeoff angles 0–180°, Figure 2) and the other with takeoff angles only in the range 120–180° suitable for local seismicity recorded by sensors at the surface (Figure 3). Three values of Poisson's ratio equal to 0.20, 0.25, and 0.29 are considered in the calculations, corresponding to $V_P/V_S$ ratios of 1.63, 1.73, and 1.83, respectively. As the average radiation pattern coefficients are in general a function of...
Figure 7. (a) The 3D view of the JAGUARS site. The separation between the tunnel system where the seismic network is located and the exploitation level (located about 90 m above the tunnel system) is clearly visible. The AE sensors are located in boreholes of various depth and oriented upwards and downwards. (b): The map view of the site including the aftershock activity of Mw1.9 event (star), shown as gray points. The activity is concentrated on the fault plane but it also caused the outburst of activity at the exploitation level (dotted black polygons). The PG dyke boundaries are plotted at the tunnel level. The subset of the aftershock activity investigated in this study is shown using dots with depth color coded. (c) The magnification of selected subset of aftershock activity together with JAGUARS network. The dyke's surfaces are shown with dashed lines.

The dyke's surfaces are shown with dashed lines. The magnification of selected subset of the aftershock activity investigated in this study is shown using dots with depth color coded. (c) The magnification of selected subset of the aftershock activity together with JAGUARS network. The dyke's surfaces are shown with dashed lines.

For the focal mechanism, three typical faulting styles were considered as well: vertical strike slip, 30° dip slip, and 45° oblique slip. Figure 2a shows how the tensile angle affects the calculated $R_P$ values. With Poisson's ratio $\nu = 0.25$, the $P$ wave radiation pattern correction coefficient for pure tensile faulting is $R_P = 1.75$, while for pure shear faulting, it is the standard value $R_P = 0.52$ that is reported by Boore and Boatwright [1984] and widely used. The average radiation of $P$ waves is increasing with increasing Poissons' ratio or $V_P/V_S$ ratio. The radiation of $S$ waves is not sensitive to changes in Poisson's ratio (Figure 2b), and $R_S$ is changing slowly from $R_S = 0.63$ to $R_S = 0.72$ for pure shear and pure tensile cases, respectively. The average radiation pattern coefficients of $SV$ and $SH$ waves depend not only on the tensile angle, but also on the faulting style (Figure 2b). Relations between $P$ and $S$ wave radiation pattern coefficients calculated with limited range of takeoff angles, more appropriate for local epicentral distances and sensors located on the surface, are shown in Figure 3.

We can use the average radiation pattern coefficients to calculate the expected $E_S/E_P$ ratio from a stationary source (cf. equation (5)) with no directivity effects as a function of the tensile angle. This is presented in Figure 4 using the RMS radiation pattern coefficients averaged over the whole focal sphere (Figure 4a) and takeoff angles 120°–180° (Figure 4b). As expected, for pure shear source, the energy radiated in a Poisson's solid as $S$ waves is 4.5 times larger than that radiated as $P$ waves (cf. Figure 4a). The $E_S/E_P$ ratios are generally decreasing with increasing tensile angle; however, for 30° dip slip and the range of takeoff angles 120°–180°, the maximum $E_S/E_P$ ratio is expected for $\alpha = 6°$. To conclude, the value of the $E_S/E_P$ ratio is controlled strongly by the tensile angle. However, for nonuniform coverage of sensors of the focal sphere (e.g., locations only on the surface), the observations may also depend significantly on the focal mechanism.

### 2.3. Influence of Attenuation and Limited Frequency Band

As described in equation (4), calculations of the radiated energy flux from the squared ground velocity spectra require correcting the energy flux for attenuation and limited frequency band before integration. Typically, the spectra are corrected for attenuation using the exponential term $\exp(-\pi FR/Q_C V_C)$, where $Q_C$ is a frequency-independent quality factor estimated either from the seismograms or independent measurements. Uncertainties in $Q_C$ estimation can produce significant errors in the calculated energy flux and radiated energy, especially for small earthquakes with considerable high-frequency radiation. The limited frequency band of observation can also seriously affect the estimation of source parameters [e.g., Boore, 1986; Di Bona and Rovelli, 1988; Ide and Beroza, 2001]. The energy flux can be significantly underestimated when high frequencies are not recorded due to the upper limit imposed on the recorded frequencies. Ide and Beroza [2001] estimated the depletion of radiated energy due to the limited frequency band as well as appropriate correction factors for the Brune and Boatwright source models. Their results indicate that the upper bound of the energy flux integral in equation (4) should be 10 times higher than the estimated corner frequency of the seismic event to approach 90% of the radiated energy.
Figure 8. P wave radiation pattern of the main event plotted using the lower hemisphere stereographic (equal angle) projection, calculated assuming (a) pure shear and (b) pure tensile faulting on the fault. The locations of AE sensors are plotted over the radiation pattern for each AE event investigated (plus, dot and x symbol for sensors AE13, AE11, and AE6, respectively). Low absolute values of expected P wave radiation pattern ($R_{P}^{\text{abs}}$ < 0.15) are surrounded with green circles indicating AE sensor-AE event pair where the energy flux estimation may be strongly biased (see discussion in the text).

[21] To illustrate how the attenuation and finite frequency band affect the estimated $E_{P}/E_{P}$ ratio, we compare the energy flux based on the model equations (1)–(2) with the energy flux affected by different limited frequency band and attenuation. Figure 5 displays the reduction of energy flux due to attenuation (Figure 5a) and attenuation together with limited frequency band (Figure 5b) as a function of the attenuation operator $R^{*}=R(Q_{P}/V_{C})$ and the ratio of the corner frequency to the upper frequency band limit $f_{\text{max}}$. As an example for a seismic event with $f_{c}=0.1V_{\text{max}}$ located 100 m from a sensor in a medium characterized by $Q_{P}=440$ and $V_{C}=6770$ m/s ($R^{*}=3.3 \times 10^{-5}$), neglecting the limited frequency band of observations and attenuation will lead to estimated energy flux that is about 60% (cf. black cross in Figure 5b) of the true radiated energy.

[22] For the purpose of this study, it is also instructive to investigate the ratio of S-to-P energy flux when the effects of attenuation and limited frequency band of $P$ and $S$ phases on the source spectra are ignored. Figure 6a presents the expected bias introduced to the $J_{P}/J_{P}$ ratio when the energy fluxes from $P$ and $S$ waves are corrected for the limited frequency band, but not corrected for the attenuation. The calculations were made assuming the knowledge of corner frequencies of $P$ and $S$ phases. Figure 6b shows the bias introduced to the $J_{P}/J_{P}$ ratio when the energy fluxes are corrected neither for attenuation nor finite frequency band. To calculate the expected biases in both cases, we use standard relations between $S$ and $P$ phases: $V_{P}/V_{S}=f_{P}/f_{S}=1.73$ and $Q_{P}/Q_{S}=\sqrt{3}/4$, assuming the relation $Q_{P}/Q_{S}=V_{P}^{2}/V_{S}^{2}$ holds [e.g., Knopoff, 1964; Hauksson and Shearer, 2006]. These assumptions result in relatively low expected biases introduced to the estimated ratios of radiated energy fluxes for close source-receiver distances, even if the attenuation and finite frequency band are totally neglected. However, other assumed values of $Q_{P}/Q_{S}$, $V_{P}/V_{S}$, or $f_{P}/f_{S}$ can change the results. It is important to note that the results shown in Figure 6 are reliable only for relatively low values of $f_{P}/f_{\text{max}}$. For higher values of $f_{P}/f_{\text{max}}$, large parts of the $S$ and $P$ radiated energy are modeled rather than observed and may not reflect the actual energy flux.

3. Data

[23] Having the theoretical background of section 2, we now examine data recorded by the JAGUARS network in the Mponeng deep gold mine, South Africa [Nakatani et al., 2008]. The sensors are located at depths of 3261–3272 m, 90 m below the exploitation area [see, e.g., Kwiatek et al., 2011] and centered on a vertically inclined Pink Green (PG) dyke composed of a diorite (Figure 7). Here we analyze data of acoustic emission (AE) sensors installed in boreholes of various depths (6–15 m) to avoid the highly attenuating zone around the tunnels. The AE sensors are a special construction designed for the project by the GMuG company, Germany. They display very wide sensitivity between 1 kHz and 170 kHz; however, their frequency response is generally not flat [Manthei, 2005] as they are not calibrated in an absolute sense [cf. with the calibration procedure presented in Kwiatek et al., 2011]. The most reliable recording frequency band of the AE sensors is between 1 kHz and 60 kHz where their sensitivity changes relatively slowly. The sensitivity of the AE sensors also depends on the incidence angle and may be severely affected by the coupling quality [Kwiatek et al., 2011; Plenkers et al., 2012]. Previous data analyses of the JAGUARS project indicate that the coupling quality of several sensors (AE7, AE8, AE14, and AE10) is insufficient for the purpose of this study. The remaining AE sensors with reliable data are AE1, AE2, AE3, AE4, AE5, AE6, AE9, AE11, AE13, and AE15.
coupling (AE11, AE13, and AE6) are used below to estimate the $E_s/E_p$ ratios of pico- and femtoseismic events in the mine.

[34] We focus on aftershocks of the $M_W$ 1.9 seismic event that occurred 30 m above the JAGUARS network [Yabe et al., 2009; Kwiatek et al., 2010; Plenkers et al., 2010; Naoi et al., 2011]. The event possibly nucleated in the central part of the dyke that fractured due to mining activity nearby. The focal mechanism, calculated using the industrial seismic network in the mine, displays normal faulting with slightly oblique slip direction with strike/dip/rake parameters: 348°/56°/−59° [Naoi et al., 2011]. The $M_W$ 1.9 event was followed by more than 25,000 aftershocks with confirmed moment magnitudes down to $M_W$ −4.1 [Kwiatek et al., 2011], and numerous smaller events were detected as well [Kwiatek et al., 2010]. The aftershocks delineated a fault that is planar within the dyke [Naoi et al., 2011] and bends toward higher dip values at the bottom of the generated failure zone, possibly due to the location there of the dyke-host rock contact (cf. Figure 7b).

[25] In the following, we analyze the $E_s/E_p$ ratios of 539 pico and femto aftershocks with the highest quality. The event selection was based on signal-to-noise ratio at the sensors exceeding 20 dB both for $P$ and $S$ waves and locations in the dyke less than 40 m from the center of the network (Figures 7b and 7c). This selection reduces the influence of propagation effects on the recorded waveforms including those involving the dyke-host rock boundary.

Fortunately, all AE sensors with appropriate coupling quality (AE6, AE11, and AE13) are also located in the dyke. As a result, we can assume that the $P$ and $S$ wave velocities and mass density are approximately constant for the investigated seismic events. The geometric parameters of the diorite forming the PG dyke were estimated from core sample measurements to be $V_P = 6770$ m/s, $V_S = 3700$ m/s, and $\rho = 2900$ kg/m$^3$ [Stanchits et al., 2010]. The uncertainties in estimation of $P$ and $S$ wave velocity did not exceed 10% (S. Stanchits, personal communication).

[26] The $P$ and $S$ onsets in the data were picked manually, and the seismograms were analyzed with a window length of 512 samples (approx. 1 ms) including a short period prior to the $P$ or $S$ onsets. The waveforms were high-pass filtered using eight-order Butterworth filter with a cutoff frequency of 400 Hz ($f_{\text{in}} = 400$ Hz in equation (4)). The upper frequency limit for the spectral integration was set to be $f_{\text{max}} = 60$ kHz. The energy flux was estimated for each sensor separately using formula (4), without correcting the spectra for the attenuation effects because of the short source-receiver distances. The associated bias on the results is estimated to be small.

4. Analysis Results

[27] From equation (3), the radiated energy ratio of the data recorded at a particular sensor is:

$$\frac{E_s}{E_p} = \frac{V_S}{V_P} \frac{\langle R_S \rangle^2 \langle R_P \rangle^2 \langle J \rangle}{\langle J \rangle \langle J \rangle} = \frac{\langle J \rangle}{\langle J \rangle}.$$

where $\Theta, \Phi$ are the takeoff angle and azimuth to the sensor, respectively. Given the geometry of the employed sources and receivers (Figures 7b and 7c), we assume isotropic velocities and use the values previously measured in the laboratory. Since the used AE sensors (AE6, AE11, and AE13) as well as investigated events are located in the dyke, $V_S/V_P = 0.54$ is assumed for the entire study volume.

Figure 9. Influence of distance and quality factors of $P$ and $S$ phases on assessment of energy flux ratio when neither limited frequency band nor attenuation is considered while calculating the energy fluxes. The curves are parameterized for different distances using solid ($R = 20$ m) and dashed ($R = 40$ m) lines. The thicker solid and dashed lines show the most pessimistic configuration of quality factors (e.g., high value of $Q_P$ and low value of $Q_S$) and different distances. The curves were calculated assuming $f_{\text{min}}/f_{\text{max}} = V_P/V_S = 1.83$, the value representative for the dioritic dyke.

Figure 10. The distribution of $E_s/E_p$ ratio for AE events analyzed in this study assuming the events involve pure shear (light gray) or pure tensile (dark gray) faulting. The dashed lines mark the median values assuming pure tensile and pure shear faulting.
The fault plane solutions are not available for the events used in this study due to the limited number of stations and unfavorable focal sphere coverage. However, it is likely that the focal mechanisms of most events are generally similar to that of the main shock (a normal fault with strike/dip/rake of 348°/56°/—59°). Such mechanisms are expected from the results of Lucier et al. [2009] on normal faulting stress conditions in the nearby TauTona mine, and that faults dipping 50°–85° to the east-northeast or west-southwest are critically stressed based on the Coulomb failure criterion [Heeckens et al., 2011a, 2011b]. Faults with such parameters are also supported by direct in situ observations at the exploitation level [McGarr, 1971a, 1971b; Cichowicz et al., 1988; van Aswegen, 2008].

We assume that the radiation patterns of $P$ and $S$ waves generated by the examined small events are similar to those expected from the $M_W$ 1.9 main shock and stress regime, and use this information to calculate the RMS average radiation pattern coefficients as well as the radiation pattern corrections for each event-sensor pair (see Appendix B in Kwiatek et al. [2011] for details of the procedure).

The information on the faulting character (shear/tensile) of individual events is also not available. Therefore, when calculating the radiation pattern correction coefficients, we consider two end-member cases: (1) pure shear faulting with $\alpha=0°$ and (2) pure tensile faulting with $\alpha=90°$. The ratios of the average RMS radiation pattern coefficients of $S$ and $P$ waves for pure shear and pure tensile events on the dip-slip fault are $\langle R_S^{\omega=0°} / R_P^{\omega=0°} \rangle = 1.21$ and 0.39, respectively. The calculations assume dynamic Poisson’s ratio $\nu=0.29$ based on the laboratory-derived $P$ and $S$ wave velocities of the PG dyke. For a given AE sensor, the $E_S/E_P$ ratio from equation (7) is now:

$$\frac{E_S}{E_P} < 0.54 \times \begin{cases} 1.21 \left( \frac{R_P^{\omega=0°}(\Theta, \phi)}{R_S^{\omega=0°}(\Theta, \phi)} \right)^2 & \text{if } \omega = 0°; \\ 0.39 \left( \frac{R_P^{\omega=90°}(\Theta, \phi)}{R_S^{\omega=90°}(\Theta, \phi)} \right)^2 & \text{if } \omega = 90° \end{cases} \times \frac{J_S}{J_P} \quad (8)$$

To estimate the influence of the radiation pattern of individual events, we assume again that the fault plane solutions follow generally the mechanism of the $M_W$ 1.9 main shock (348°/56°/—59°). When calculating the radiation pattern correction coefficients, we introduce random deviations ($\pm15°$) to the fault plane parameters (strike, dip, and rake), resulting in radiation pattern correction coefficients of a family of fault planes slightly deviating from that of the main shock. This suppresses the effect of proximity to the radiation nodes for the pure shear radiation, while introducing at the same time some expected level of variability in the focal mechanisms of the small events.

The calculated $P$ wave radiation pattern generated by the main event assuming pure shear and pure tensile faulting is shown in Figures 8a and 8b together with the distribution of directions (azimuth/takeoff) between the AE stations and examined events over the focal sphere. Figure 8a shows that sensor AE11 (dots in Figure 8a) is not suitably oriented with respect to the investigated sources as many points are located close to the expected nodal planes (black dots surrounded with green circle in the bottom-right part of Figure 8a). The same situation applies to some events recorded by sensor AE6 (top-left part of Figure 8a). Estimations of the radiated energy from these AE sensor-event pairs may be unreliable due to the low values of the $R_P^{\omega=0°}$ coefficient. Because of this possible instability, event-sensor pairs with low absolute value of $P$ wave correction coefficient for shear faulting ($R_P^{\omega=0°} < 0.15$) were not used in the subsequent estimation of $E_S/E_P$ ratio. The $P$ wave radiation patterns from tensile events as well as the $S$ wave radiation patterns (regardless of the faulting type) do not produce strong nodes (Figure 8b).

The quality factors of $P$ and $S$ waves have been calculated previously for seismic events recorded by the JAGUARS network at the highly damaged exploitation level, more than 100 m from the center of the network to be $Q_P=440 \pm 150$ and $Q_S=640 \pm 162$ [Kwiatek et al., 2011]. In this study, we focus on much closer events located in the dyke, so it is expected that the attenuation plays a minor role on the estimations of energy fluxes. Nevertheless, we use the above values to calculate the bias we expect from neglecting the attenuation correction and the correction for the limited frequency band while calculating the energy flux ratio $J_S/J_P$, following the methodology presented before. The expected bias is shown in Figure 9 as a function of $P$ wave corner frequency $f_P$ scaled to the upper limit of the frequency band considered (60 kHz). The curves are parameterized by different combinations of $P$ and $S$ wave quality factors and distances between the AE event and sensors. Figure 9 shows that the bias is increasing with the distance and it depends on the combination of the quality factors of $P$ and $S$ waves. The bias values are mostly greater than unity suggesting that ignoring the limited frequency band and attenuation will likely produce overestimation of the $J_S/J_P$ ratios. However, for the average values of quality factors estimated by Kwiatek et al. [2011], the bias is not very significant reaching at most about 1.2. Therefore, given the close source-receiver distances for the majority of events investigated in this study, the effect of attenuation and limited frequency band is not significant for calculating the $J_S/J_P$ ratio.

Figure 10 presents the distributions of $E_S/E_P$ ratio for the AE events analyzed in this study assuming the raw energy flux ratio was corrected for attenuation bias and expected faulting character (either pure tensile or pure shear faulting, cf. equation (8)). The results indicate that using corrections for either pure shear or pure tensile faulting leads to similar $E_S/E_P$ distributions, with ratios ranging from 0.2 to 50 and median values of 2.95–4.83 assuming pure tensile and pure shear radiation pattern corrections, respectively.

5. Discussion and Conclusions

We present theoretical and observational results on estimating radiated seismic energy and the $S$-to-$P$ energy ratios in the context of pico- and femtoseismicity recorded in the Mponeng deep gold mine in South Africa. An accurate estimation of the radiated seismic energy is generally difficult to achieve, as the results are strongly sensitive to the quality of the network (focal sphere coverage, operational frequency band), attenuation, radiation pattern (including possible existence of tensile component of faulting), and directivity effects. Results on $E_S/E_P$ values are somewhat less sensitive to imprecise knowledge on source and medium properties, and they contain important information on the character of faulting. In this study, we use the simple stationary source model, for which $E_S/E_P$ ratio of 4.5 may be considered a lower bound for pure shear faulting and lower values indicate events containing tensile components.
We perform basic theoretical calculations to model the influence of radiation pattern, limited frequency band, and attenuation on estimated \( E_p/E_s \) ratios for shear and tensile faulting types. The \( E_p/E_s \) ratio is decreasing dramatically with increasing tensile character of the event. Extending results ofBoore and Boatwright [1984], we provide radiation pattern correction coefficients as functions of tensile angle and takeoff angle. This allows us to correct the recorded energy flux not only for different focal mechanisms but also for the tensile/shear character of faulting.

The attenuation and limited frequency band affect severely the estimation of radiated energy, but have smaller influence on the \( S\)-to-\( P \) radiated energy ratio, especially if close source-receiver distances are considered. Therefore, it is not always useful to correct the ground velocity spectra for attenuation, which is a poorly constrained parameter and may introduce large uncertainties into calculations of radiated energy. A better strategy followed in this work is to ignore the attenuation when calculating the \( E_p/E_s \) ratios, and estimate theoretically the possible bias that may be introduced to \( E_p/E_s \) ratio by the attenuation.

We investigate the \( E_p/E_s \) ratios of 539 high-quality minute aftershocks of \( M_w \) 1.9 main shock, with minimal complications due to propagation effects between sources and receivers. Our precision of estimating the energy flux is limited strongly by the small number of available stations that are not well distributed over the focal plane. We therefore cannot quantify whether a specific AE event is associated with a significant tensile behavior. However, the obtained distribution of \( E_p/E_s \) ratios includes many events (Figure 10) with ratios below 4.5, which is the lower bound for pure shear events with the adopted stationary model. Since adding corrections for rupture directivity and velocity does not affect significantly the \( E_p/E_s \) ratios of tensile events (section 2), most of our events could indeed be considered to have a significant tensile component. Many events have \( E_p/E_s < 1 \) with calculations that assume either pure shear or pure tensile faulting. Such events may reflect additional \( P \) radiation generated by brittle rock damage in the source volumes [Ben-Zion and Ampuero, 2009; Castro and Ben-Zion, 2013].

The methodology used in this paper can provide constraints on the possible existence of tensile component of faulting and \( E_p/E_s \) ratios in situations associated with good coverage of the focal spheres. Observational studies of crustal seismicity should account more carefully for attenuation effects, especially in the vicinity of large active fault zones. This will be done in a future work in the context of earthquakes in California.

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