Virtual Experiments in 3D TEM Forward Modelling including Topography

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1 Motivation

Volcanic activity as a natural hazard is a threat to life and infrastructure. Hence, investigation and monitoring of volcanoes is crucial in mitigating volcanic risk. Such mitigation measures are based on a solid understanding of volcanic processes, especially those occurring prior to an eruption. In this regard, processes inside volcanic conduits are of prime interest as they control the final rise of magma prior to an eruption. However, monitoring such processes is challenging as direct measurements inside volcanic conduits are impossible.

The transient electromagnetic method (TEM) aims for a wide range of penetration depths. In order to verify whether it is capable of imaging changes in conductivity within a volcanic building we will simulate TEM measurements on a 3D model of Stromboli volcano including real topography and features inside the volcanic building.

In the following we introduce our simulation workflow inspired by Zehner et al. (2015). Based on this workflow, future parameter studies will be carried out to investigate to what extent TEM is able to display conductivity changes in volcanic buildings and whether or not it is capable of imaging conduit structures. These findings will serve for configuring a future field experiment.

2 Model Creation and Mesh Generation

Since adequate modelling is crucial for generating reliable synthetic data we use a digital elevation model (DEM) of Stromboli volcano which is available in the data pool of NASA LP DAAC. The resolution is 1 arc second both in longitude and latitude.

Based on this data set we create a 3D model of Stromboli volcano and its surrounding area. Manual or automatic modifications are performed easily once the 3D model is set up.

The DEM comes as a set of point data which needs to be transformed into a 3D grid before running any simulation. TetGen [Si (2011)] is a quality mesh generator which is able to create an unstructured tetrahedral mesh from given point data. However, this includes a large amount of computational memory because not only the surface topography but the underlying conductive space, the upper air space and some surrounding area have to be created and meshed. In Fig. 1 we present a workflow which avoids this problem. It is
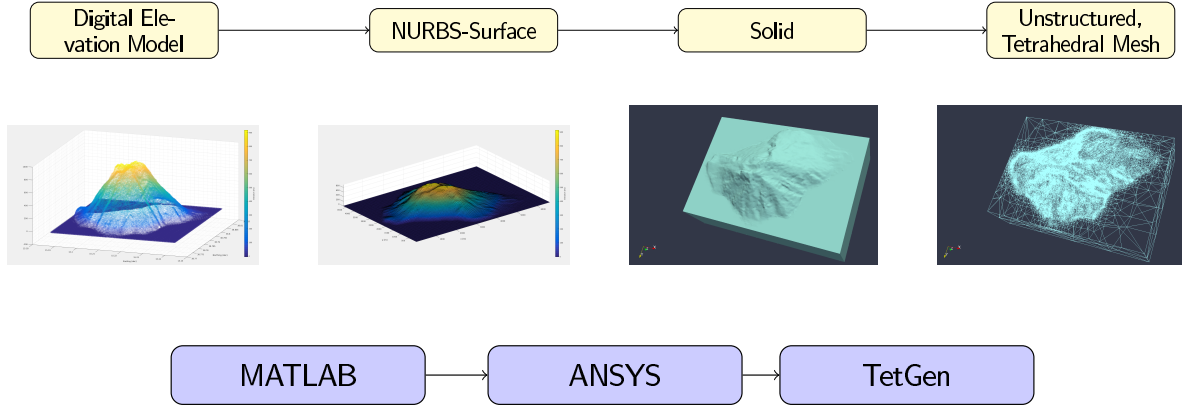
Figure 1: Modelling workflow, modelling stages and corresponding software. The air half-space is neglected for illustrational purposes.

referred to as the Ansys workflow in following sections.

To reduce memory we describe the triangulated topographical surface by a non-rational uniform B-spline (NURBS) and generate a NURBS-surface using MATLAB. This format is common in engineering science and contains nodes and rational coefficients only. Importing these data to ANSYS, a commercial computer-aided engineering (CAE) program, allows us to generate a watertight body called solid composed of conductive Earth and overlying air. Given the polygonal boundary representation of this solid TetGen can generate a tetrahedral mesh easily. This way the tetrahedralization needs much less nodes and memory than using TetGen in a brute force manner. Furthermore, subsurface structures can be inserted less laboriously.

3   Insights into the Numerical Simulation Routine

The transient electromagnetic method is an induction method. A DC current in a circular or square loop is shut off at time $t = 0\,\mathrm{s}$ which causes eddy currents underground. In time the induced current system decays in intensity and expands in space. The measurand is a normalized response at the Earth's surface in terms of $\partial_t B_z$.

For simulation purposes $\partial_t B_z$ can be easily extracted from Faradays law of induction

$$\partial_t B_z = - \left(\nabla \times \mathbf{E} \right)_z , \tag{1}$$

so a field equation for the electric field intensity is required.

Starting from Maxwell's equations we can eliminate the magnetic field and with neglecting displacement currents we state the time domain formulation of transient electromagnetic induction as an initial-boundary value problem for the electric field intensity [Börner et al. (2015)]

$$\sigma \partial_t \mathbf{E} + \nabla \times \left(\mu^{-1} \nabla \times \mathbf{E} \right) = \mathbf{0} \quad \text{on } \Omega \times (0, \infty) \tag{2a}$$

$$\sigma \partial_t \mathbf{E}|_{t=0} = \mathbf{q} \quad \text{on } \Omega \tag{2b}$$

$$\mathbf{n} \times \mathbf{E} = \mathbf{0} \quad \text{on } \partial\Omega \times (0, \infty). \tag{2c}$$

Using Nédélec elements for spatial discretization the given field equation reduces to an ODE initial value problem reading

$$M\partial_t\mathbf{u}(t) + C\mathbf{u}(t) = \mathbf{0}, \quad t \in (0, \infty), \quad M\mathbf{u}(0) = \mathbf{q}, \tag{3}$$

where $\mathbf{u}$ denotes the coefficient vector of the finite element approximation of $\mathbf{E}(t)$ with respect to the Nédélec basis at time $t \geq 0$, $\mathbf{q}$ the vector of initial values and the matrices M and C denote the mass and stiffness matrix according to the Nédélec basis, respectively. Now the explicit solution of (3) is given by

$$\mathbf{u}(t) = e^{-tM^{-1}C}M^{-1}\mathbf{q} \tag{4}$$

using the matrix exponential function which can be evaluated using the Rational Arnoldi approximation [Börner et al. (2015)].

4 Application to the Homogeneous Half-Space

Both the modelling sequence using Ansys and the simulation routine have been proven within various simulations. However, linking both sequences might cause trouble. Thus, it is necessary to check the integrity of the resulting process.

	TetGen standalone	Ansys workflow
Number of tetrahedral elements	91 721	43 794
Degrees of freedom	579 786	277 266

Table 1: Mesh statistics

Therefore, we compare the simulation results using the Ansys workflow for modelling to a known quantity: the analytical solution over the homogeneous halfspace. In addition to the described Ansys modelling workflow we show a simple tetrahedralization using the TetGen software as a standalone. Both numerical solutions use the described simulation routine with Nédélec elements of order $k = 2$. The simulation characteristics are presented in Tab. 1.

The synthetic time series shown in Fig. 2 were calculated within a cubic modelling domain $[-1000, 1000]^3 \text{m}^3$ in cartesian coordinates, with z pointing downwards. The plane $z = 0$ marks the Earth's surface, i.e. the boundary of conductive halfspace in $z > 0$ with $\rho_{hom} = 100\,\Omega\text{m}$ and the isolating halfspace in $z < 0$ with $\rho_{air} = 10^9\,\Omega\text{m}$. A square loop of side length $20\,\text{m}$ is centered at the origin, on top of the conductive layer. The evaluation location $(200, 200, 0)\,\text{m}$ is located at sufficient distance with respect to the validity of the analytical solution.

As expected all synthetic data curves show similar behaviour and same values. Referring to the relative error we achieve an error of less than $5\,\%$ for all but the last time $t_{end} = 10^{-3}\,\text{s}$ where it is below $10\,\%$. At the zero-crossover the relative error was set to zero. Due to numerical instabilities it is large enough to mask all other errors.
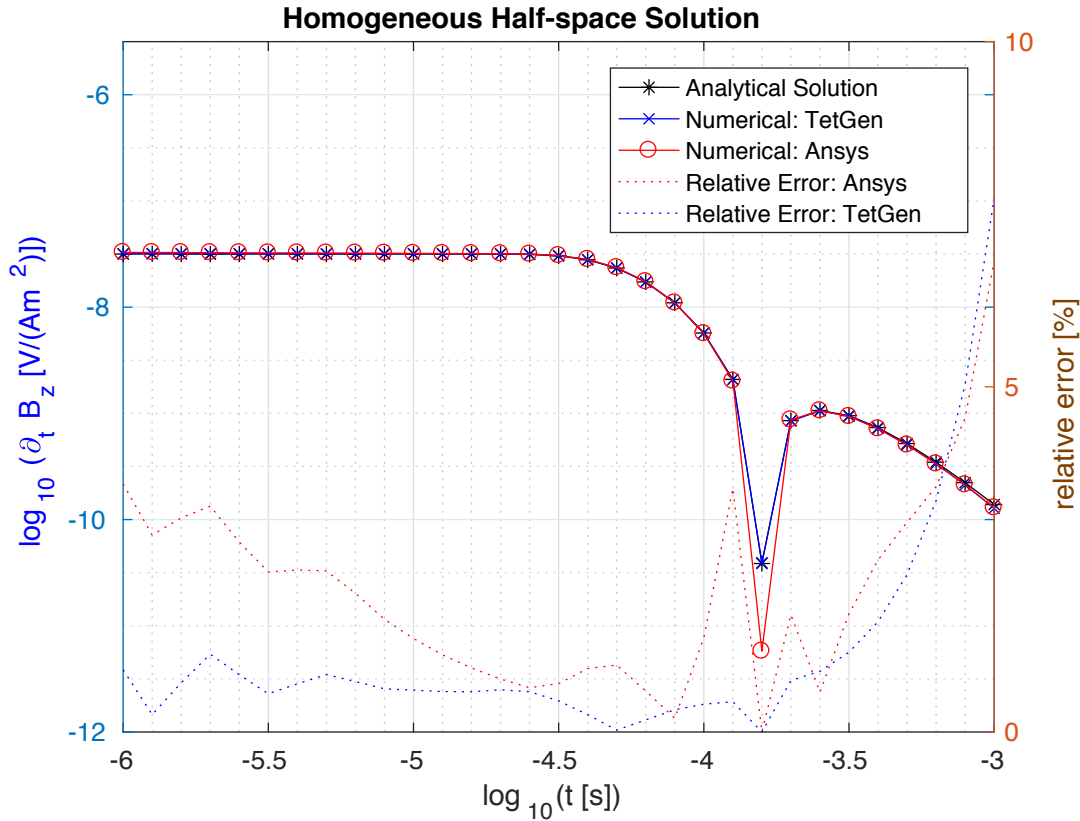
Figure 2: Comparison of simulation results acquired by the presented Ansys workflow, the TetGen standalone and evaluation of the analytical solution. A horizontal, $20 \times 20\,\mathrm{m}^2$ square loop is centered at $(0,0,0)\,\mathrm{m}$, on top of a homogenous halfspace with a resistivity of $\rho_{hom} = 100\,\Omega\mathrm{m}$ The simulated quantity is evaluated at $(200, 200, 0)\,\mathrm{m}$.

5 Conclusion and Outlook

Since the numerical results for the homogeneous half-space indicate the functionality of the described Ansys workflow we are now able to carry out TEM simulations on the digital elevation model presented in section 2. Therefore, sensitivity studies are crucial for reducing the model to a handsome size and proper choice of the computational node distribution. Next, subsurface structures will be added with main focus on a volcanic conduit.

Acknowledgement

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