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2D Boundary Conditions in a 3D EM Finite-Element Forward Modelling

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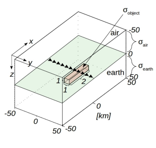
Existing Forward Modelling Code

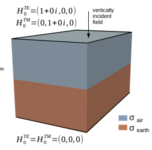
Change 3D FEM fwd modelling code by Ren et al (2013) to be integrated as part of 3D EM inversion algorithm.

Features:

- Discretise 3D volume with earth & air space (with topography) into elements
- 1D boundary conditions
- Source of incident plane EM field at top
- Adaptive mesh refinement based on different error estimators



Schematic model for 1D BC case, where object cannot touch boundaries



1D boundary conditions, fixed values at top and bottom, 1D EM solution in between

Equations:

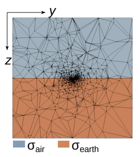
E field in computational volume Ω:

$$\nabla\times\xi^{-1}\nabla\times E+\underbrace{\chi E}_{J}=0$$

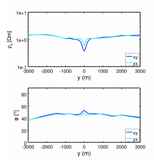
Neumann condition for electric field on boundary dΩ:

$$-\hat{n}\times\xi^{-1}\nabla\times E=\hat{n}\times H_0$$

ξ impedivity	χ admittivity
E electric field	
J total current density in Ω	
$\hat{n}$ outw ptg nml unit vector on dΩ	
H_0 1D pl wave solution on dΩ	



Cross section through 3rd FEM mesh, along the profile, refined along object



Off-diagonal apparent resistivities & phases obtained along the profile for f=1 Hz and conductivities of: σ_{air} = 10^{-10} S/m, σ_{earth} = 1 S/m σ_{object} = 10^{5} S/m

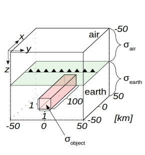
First Alterations:

- 2D boundary conditions
- Total- and scattered field decomposition

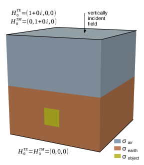
2D Boundary Conditions

Why?

- 1D BC only represent layered half space, boundaries must be placed far enough away from 3D structures
- 2D BC: boundaries can be moved towards structures, touch them
- Advantage:
  - shrink computational domain → less computational cost
  - represent local geology better, e.g. dipping layers, "infinitely" elongated objects, etc
- In inversion, smoothness constraints generate 2D models at boundaries



Schematic model with 2D BC (boundaries moved towards object)



2D Boundary Conditions of domain

Formulation:

- Same as for 1D BC:

$$-\hat{n}\times\xi^{-1}\nabla\times E=\hat{n}\times H_0$$

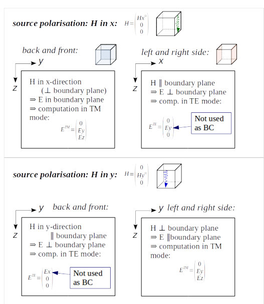
except H_0 = **2D** plane wave solution on dΩ

How to do it:

- Define 2D model for domain boundary
- Boundary mesh computed independently from 3D mesh of volume
- Solve 2D problem for boundary and use recovered H_0 as BC in 3D problem

Modes for 2D solution:

- Solve both source polarisations for vertical incidence (x- and y- direction)
- Solve TM-Mode for sides parallel to oscillational plane, TE for perpendicular ones

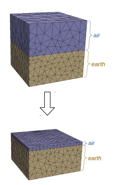


Which mode to use at which boundary to calculate the electric fields for the BC, only components within boundary plane are used as BC

Total- and Scattered Field Decomposition Approach

Why?

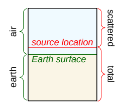
- EM fields are reflected at top boundary and propagate unphysically back to earth
  - → place top boundary far above surface
  - → Air volume very large
- incident field implemented as BC at top → dispersion error as it propagates through air layer
- TSFD: reduce computational cost, avoid dispersion by shrinking air layer

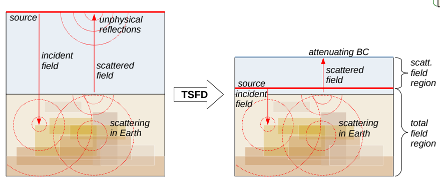


Reduce computational volume by shrinking air layer

How to do it:

- Divide volume into 2 domains: *total* and *scattered* field
- Implement plane wave source at interface (at Earth surface, or just above)
- Incident field propagates into Earth as usual, is scattered by structures, here total field is determined (E^{tot} = E^{sc} + E^{inc})
- Waves scattered by Earth propagating upwards: pass domain interface, above interface: only scattered field, which is damped at top boundary





Sketch of computational domain: as before (left) and with TSFD (right)

Formulation:

Boundary value problem in total field region (similar to Riley et al, 2006):

Volume Ω_T: $\nabla\times\xi^{-1}\nabla\times E^{tot}+\chi E^{tot}=0$

Surface $d\Omega_T$: $\hat{n}\times\xi^{-1}\nabla\times E^{tot}=0$

Scattered field region:

Volume Ω_{SC}: $\nabla\times\xi^{-1}\nabla\times E^{sc}+\chi E^{sc}=0$

Surface $d\Omega_{SC}$: $\hat{n}\times\xi_0^{-1}\nabla\times E^{sc}+Y_{\Omega_{sc}}\hat{n}\times\hat{n}\times E^{sc}=0$

$E^{tot,sc,inc}$	total, scat, incident E-field
$\hat{n}$	outw ptg nml unit vector on dΩ
$Y_{\Omega_{sc}}$	surface admittance
ξ_0	impedivity for zero conductivity

Summary & Outlook

- Reduce computational domain
  - Shrink air layer (TSFD)
  - Move boundaries closer to structures (2D BC)

  → less computational cost
- Represent local geology more accurately (2D BC)
- Allow smoothness constraints in inversion to generate 2D BC

Outlook:

- Finish including the 2D BC into forward modelling code
- Implement *total and scattered field decomposition*
- Include forward modelling algorithm into inversion algorithm

References:

Ren, Z., Kalscheuer, T., Greenhalgh, S., & Maurer, H. (2013). A goal-oriented adaptive finite-element approach for plane wave 3-D electromagnetic modelling. Geophysical Journal International, 194(2), 700-718.

Riley, D. J., Jin, J. M., Lou, Z., & Petersson, L. R. (2006). Total-and scattered-field decomposition technique for the finite-element time-domain method. IEEE transactions on antennas and propagation, 54(1), 35-41.

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