Topic	**Moment-tensor determination and decomposition**
Authors	Frank Krüger, Potsdam University, Institute of Geosciences, 14415 Potsdam, E-mail: kruegerf@geo.uni-potsdam.de Günter Bock ✞ (formerly GeoForschungsZentrum Potsdam, Germany)
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1 Aim

The tasks in this exercise are aimed at making you more familiar with the use and meaning of the basic equations and matrix formalisms in seismic moment-tensor presentation and decomposition.

2 Formulae used

The following formulae are used:

$$d_s(\mathrm{x}, t) = M_{kj}\,[G_{sk,j}\,(\mathrm{x},\xi,t) * \mathrm{s}\,(t)] \qquad (1)$$

which is identical with Eq. (3.70) in Chapter 3, with $d_s(\mathrm{x}, t)$ - ground displacement at position x and time t. (1) simplifies to $d_s(\mathrm{x}, t) = M_{kj}G_{sk,j}(t)$ if the source-time function $\mathrm{s}(t) = \delta(t)$ is a needle (spike) impulse.

$$M_{kj} = A\{\Lambda \nu_i s_i \delta_{kj} + \mu\,(\nu_k s_j + \nu_j s_k)\} \qquad (2)$$

which describes the seismic moment tensor for an isotropic medium in the most general way with Λ - elastic Lamé-parameter, μ - shear modulus, A – area of fault rupture, s - slip vector on the fault and ν - normal to the fault plane. Note that the scalar seismic moment $M_o = \mu A |s|$. The term $(\nu_k s_j + \nu_j s_k)$ forms a tensor D describing a double-couple source. In case of an explosion it is zero. And the equations:

$$\begin{aligned}
M_{xx} &= -M_o(\sin\delta\cos\lambda\sin2\phi + \sin2\delta\sin\lambda\sin^2\phi)\\
M_{xy} &= M_o(\sin\delta\cos\lambda\cos2\phi + 0.5\sin2\delta\sin\lambda\sin2\phi)\\
M_{xz} &= -M_o(\cos\delta\cos\lambda\cos\phi + \cos2\delta\sin\lambda\sin\phi)\\
M_{yy} &= M_o(\sin\delta\cos\lambda\sin2\phi - \sin2\delta\sin\lambda\cos^2\phi)\\
M_{yz} &= -M_o(\cos\delta\cos\lambda\sin\phi - \cos2\delta\sin\lambda\cos\phi).\\
M_{zz} &= M_o\sin2\delta\sin\lambda
\end{aligned} \qquad (3)$$

with ϕ - strike direction and δ - dip angle of the rupture plane, and λ - slip direction (rake angle).

3 Tasks

Task 1:
By using Equations (2) and (3) above, respectively, determine the Cartesian moment tensors for:

a) an underground nuclear explosion;
b) a double-couple focal mechanism with strike $\phi = 0°$, dip $\delta = 90°$, and rake $\lambda = 0°$;
c) a double-couple focal mechanism with $\phi = 0°$, $\delta = 45°$, and $\lambda = 90°$;
d) a double-couple focal mechanism with $\phi = 0°$, $\delta = 90°$, and $\lambda = 90°$.

Task 2:
Determine the moment tensor for a tension crack in the direction normal to the fault plane in a homogenous isotropic medium. Use Equation (2) of the exercise.

Task 3:
The relation Equation (3) between moment tensor and parameters of a shear dislocation can be expressed as a weighted sum of 4 elementary moment tensors:

$$M = \cos\delta \cos\lambda \, M_1 + \sin\delta \cos\lambda \, M_2 - \cos 2\delta \sin\lambda \, M_3 + \sin 2\delta \sin\lambda \, M_4.$$

Derive the elements of M_1, M_2, M_3, and M_4 and discuss the shear dislocations that are represented by the elementary moment tensors.

4 Solutions

Task 1:

a) $$M = M_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b) $$M = M_0 \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

c) $$M = M_0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

d) $$M = M_0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

Task 2:

$$M = \begin{pmatrix} \Lambda s_3 & 0 & 0 \\ 0 & \Lambda s_3 & 0 \\ 0 & 0 & (\Lambda + 2\mu) s_3 \end{pmatrix}$$

Task 3:

$$M_1 = M_0 \begin{pmatrix} 0 & 0 & -\cos\phi \\ 0 & 0 & -\sin\phi \\ -\cos\phi & -\sin\phi & 0 \end{pmatrix} \rightarrow \delta = 0°;\ \lambda = 0°, \text{ i.e., horizontal slip on horizontal fault}$$

$$M_2 = M_0 \begin{pmatrix} -\sin 2\phi & \cos 2\phi & 0 \\ \cos 2\phi & \sin 2\phi & 0 \\ 0 & 0 & 0 \end{pmatrix} \rightarrow \delta = 90°;\ \lambda = 0°, \text{ i.e., strike slip on vertical fault}$$

$$M_3 = M_0 \begin{pmatrix} 0 & 0 & \sin\phi \\ 0 & 0 & -\cos\phi \\ \sin\phi & -\cos\phi & 0 \end{pmatrix} \rightarrow \delta = 90°;\ \lambda = 90°, \text{ i.e., dip slip on a vertical fault}$$

$$M_4 = M_0 \begin{pmatrix} -\sin^2\phi & \frac{1}{2}\sin 2\phi & 0 \\ \frac{1}{2}\sin 2\phi & -\cos^2\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \delta = 45°;\ \lambda = 90°, \text{ i.e. dip slip on a 45°dipping fault.}$$