Topic	**Determination of source parameters from seismic spectra**
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1 Aim

This exercise shows how to estimate the source parameters seismic moment, size of the rupture plane, source dislocation and stress drop from data in the frequency domain only and how the results depend on the underlying model assumptions. These parameters could also be estimated in the time domain. However, for estimation in the time domain the records have to be converted into true ground motion (displacement) records. This may be a problem if the bandwidth of the recording system is limited (e.g., short-period records) or if the phase response of the system is not well known. For estimation in the frequency domain only the amplitude response of the instrument is needed.

2 Data

Figure 1 shows a velocity record (vertical component) of an aftershock of the 1992 Erzincan earthquake (Turkey). Figure 2 shows the corresponding *displacement spectrum* of the P wave. The calculated spectrum was corrected for the amplitude response of the recording system (which includes both response of the velocity seismometer and the anti-aliasing filter of the recorder). Furthermore, the P-wave spectrum was corrected for attenuation, $\exp(i\omega t/2Q_p)$. Q_p had been estimated beforehand from coda Q_c- observations in the area under study assuming that $Q_p = 2.25\ Q_c$. This is a good approximation under the assumption that $v_p/v_s = 1.73$, $Q_c = Q_s$ and the pure compressional Q_κ (κ - bulk modulus) is very large ($\rightarrow \infty$). In Figure 2 also the noise spectrum, treated in the same way as the P-wave spectrum, was computed and plotted in order to select the suitable frequency range for analysis (with signal-to-noise ratio SNR > 3).

At low frequencies typical P- and S-wave spectra approach a constant amplitude level u_o and at high frequencies the spectra show a decay that falls off as f^{-2} to f^{-3}. Plotted on a log-log scale the spectrum can be approximated by two straight lines. The intersection point is the corner frequency f_c. u_o and f_c are the basic spectral data from which the source parameters will be estimated. Event and material data required for further calculations are the epicentral distance Δ, the source depth h, the rock density ρ, the P-wave velocity v_P, and the averaged radiation pattern Θ for P waves. Respective values are given under 3.1 below. Other needed parameters can then be calculated.

Note: The apparent increase of spectral noise amplitudes in Figure 2 for f > 25 Hz and of signal amplitudes for f > 50 Hz is not real but sampling noise due to anti-aliasing filtering of the record. Thus, this increase should not be considered in the following analysis. The same holds for spectral amplitudes f < 1 Hz. Since the analyzed P-wave train is only 2 s long, lower frequencies are not properly represented.

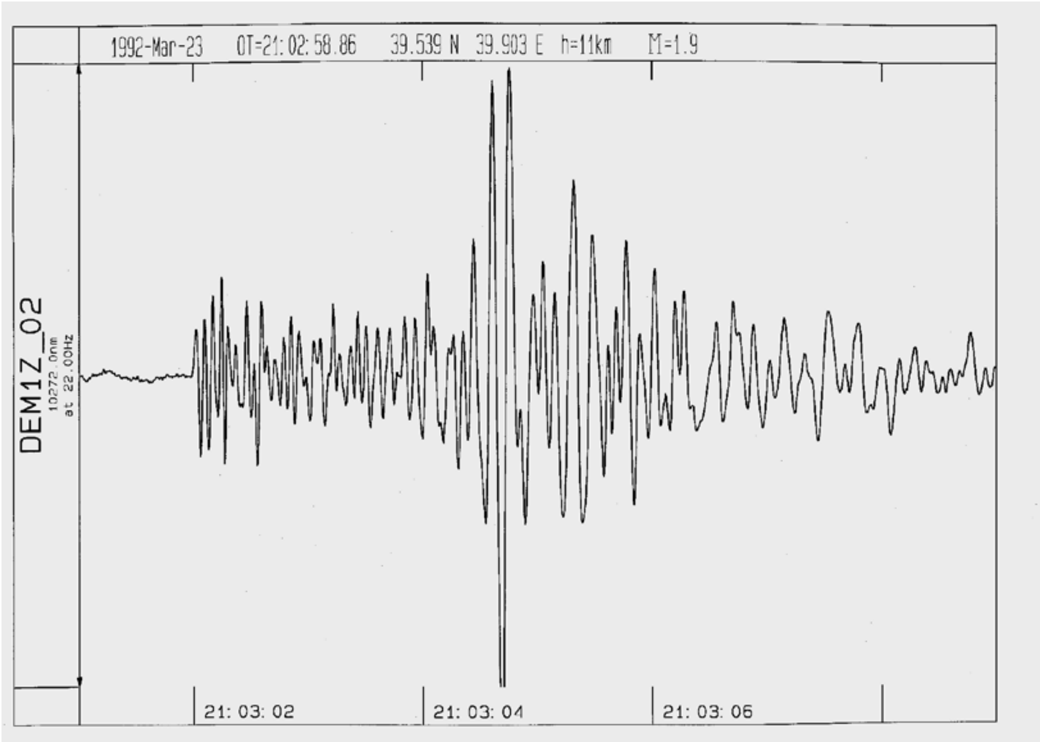


Figure 1 Record of an Erzincan aftershock (vertical component). For the indicated P-wave window the displacement spectrum shown in Figure 2 has been calculated.

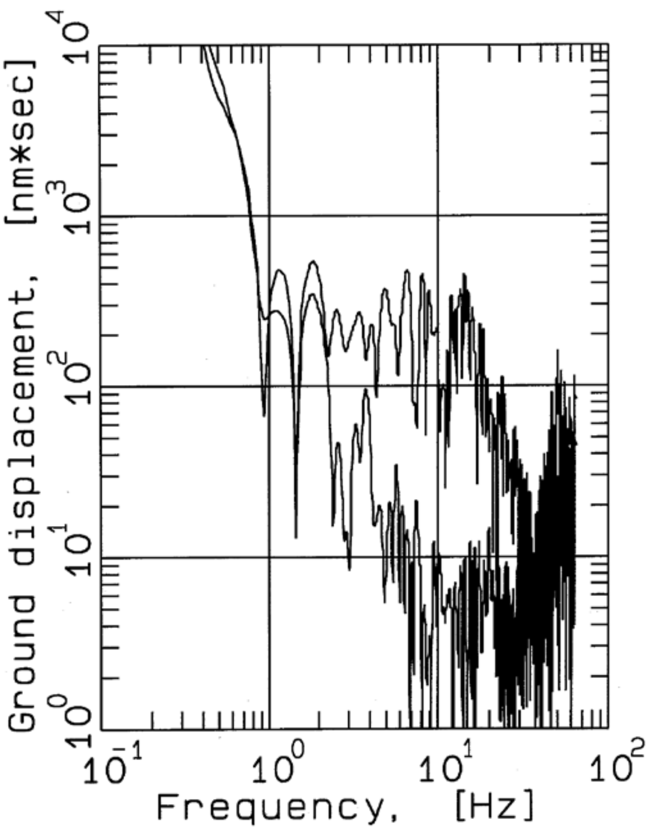


Figure 2 P-wave spectrum (upper curve) and noise spectrum (lower curve) of the record shown in Figure 1, corrected for the instrument response and attenuation.

3 Procedures

The parameters to be estimated are:

- Seismic moment $M_o = \mu\ \bar{D}\ A$ (1)
  (with μ - shear modulus; $\bar{D}$ - average source dislocation; A - size of the rupture plane)
- Source dislocation $\bar{D}$
- Source dimension (radius R and area A)
- Stress drop $\Delta\sigma$

3.1 Seismic moment M_o

Under the assumption of a homogeneous Earth model and constant P-wave velocity v_p, the seismic moment M_0 can be determined from the relationship:

$$M_0 = 4\ \pi\ r\, v_p^3\ \rho\ u_o\ /\ (\Theta\ S_a) \tag{2}$$

with r – hypocentral distance, ρ - density, u_o – low-frequency level (plateau) of the displacement spectrum, Θ - average radiation pattern and S_a surface amplification for P waves.

In the exercise we use the following values:

density $\rho = 2.7$ g/cm^3

P-wave velocity $v_p = 6$ km/sec

source depth $h = 11.3$ km

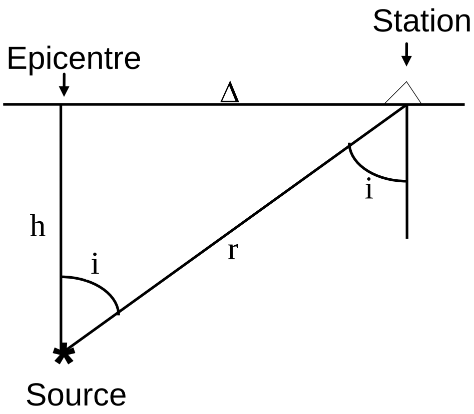
epicentral distance $\Delta = 18.0$ km

hypocentral distance (travel path) $r = \sqrt{(h^2 + \Delta^2)}$

incidence angle $i = \arccos(h/r)$

free surface amplification S_a for P waves

averaged radiation pattern $\Theta = 0.64$ for P waves.



Note the differences in dimensions used! M_0 has to be expressed in the unit Nm = kg m^2 s^{-2}. S_a can be determined by linear interpolation between the values given in Table 1. They were computed for the above given constant values of v_p and ρ (homogeneous model) and assuming a ratio $v_p/v_s = 1.73$. i is the angle of incidence, measured from the vertical.

Table 1 Surface amplification S_a for P waves; i is the incidence angle.

i	S_a	i	S_a	i	S_a
0	2.00	30	1.70	60	1.02
5	1.99	35	1.60	65	0.90
10	1.96	40	1.49	70	0.79
15	1.92	45	1.38	75	0.67

20 25	1.86 1.79	50 55	1.26 1.14	80 85	0.54 0.35

3.2 Size of the rupture plane

For estimating the size of the rupture plane and the source dislocation one has to adopt a kinematic (geometrical) model, describing the rupture propagation and the geometrical shape of the rupture area. In this exercise computations are made for three different circular models (see Table 2), which differ in the source time function and the crack velocity v_{cr}. v_s is the S-wave velocity, which is commonly assumed to be $v_s = v_p / \sqrt{3}$.

Table 2 Parameters of some commonly used kinematic rupture models.

1. Brune (1970)	$v_{cr} = 0.9\ V_s$	$K_p = 3.36$	$K_s = 2.34$
2. Madariaga I (1976)	$v_{cr} = 0.6\ V_s$	$K_p = 1.88$	$K_s = 1.32$
3. Madariaga II (1976)	$v_{cr} = 0.9\ V_s$	$K_p = 2.07$	$K_s = 1.38$

The source radius R (in km) can then be computed from the relationship

$$R = v_s K_{p/s} / 2\pi fc_{p/s} \tag{3}$$

with v_s – shear-wave velocity in km/s, $fc_{p/s}$ – corner frequency of the P or S waves, respectively, in Hz and K_p and K_s being the related model constants and v_s. The differences in K_p and K_s between the various models are due to different assumptions with respect to crack velocity and the rise time of the source-time function. Only K_p has to be used in the exercise (P-wave record!). The size of the circular rupture plane is then

$$A = \pi R^2. \tag{4}$$

3.3 Average source dislocation $\overline{D}$

According to (1) the average source dislocation is

$$\overline{D} = M_0 / (\mu A). \tag{5}$$

Assuming $v_s = v_p/1.73$ it can be computed knowing M_0, the source area A and the shear modulus $\mu = v_s^2 \rho$.

3.4 Stress drop

The static stress drop $\Delta\sigma$ describes the difference in shear stress on the fault plane before and after the slip. According to Keilis-Borok (1959) the following relationship holds for a circular crack with a homogeneous stress drop:

$$\Delta\sigma = 7 M_0 / (16 R^3). \tag{6}$$

The stress drop is expressed in the unit of Pascal, $\mathrm{Pa} = \mathrm{N\ m^{-2}} = \mathrm{kg\ m^{-1}\ s^{-2}} = 10^{-5}$ bar.

4 Tasks

Task 1:
Select in Figure 2 the frequency range f_1 to f_2 that can be used for analysis (SNR > 3):

f_1 = ... Hz f_2 = ... Hz

Task 2:
Estimate the low-frequency level, u_o , of the spectrum by approximating it with a horizontal line. Note in Figure 2 the logarithmic scales and that the ordinate dimension is nm s = 10^{-9}m s.

u_o = ... m s

Task 3:
Estimate the exponent, n, of the high-frequency decay, f^{-n} ; mark it by an inclined straight line.

n = ...

Task 4:
Estimate the corner frequency, fc_p (the intersection between the two drawn straight lines).

fc_p = ... Hz

Task 5:
Calculate from the given event parameters and relationships given in 3.1 and Table 1 the values for:

r = ………km i =………° S_a = ………. M_o = …………Nm.

Task 6:
Using the equations (3), (4), (5) and (6) calculate for the three circular source models given in Table 2 the parameters

a) source radius R and source area A,
b) shear modulus μ and average displacement $\bar{D}$ and
c) stress drop $\Delta\sigma$.

Write the respective values in Table 3

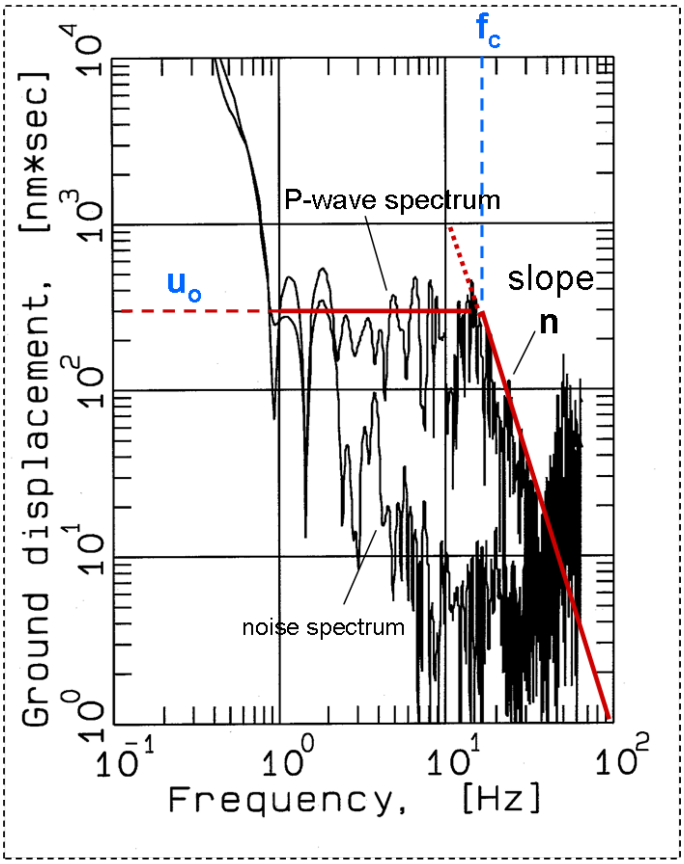
Table 3

Model	R [m]	A [m^2]	$\bar{D}$ [m]	$\Delta\sigma$ [MPa]
1. Brune				
2. Madariaga I				

3. Madariaga II				

5 Solutions

Although individual visual parameter readings from Figure 2 might be subjective, they should not differ by more than about ± 10% from the values given here for tasks 1 to 5 but may be larger for 6. Acceptable average values for the read and calculated parameters are for:



Task 1: $f_1 = 2$ Hz, $f_2 = 30$ Hz

Task2: $u_o = 3 \times 10^{-7}$ m s

Task 3: $n = 3$

Task 4: $f_{cp} = 14.4$ Hz

Task 5: $r = 21.3$ km $i = 58^o$, $S_a = 1.07$, $M_o = 6.8 \times 10^{13}$ N m

Task 6:

a) $R_1 = 129$ m, $A_1 = 5.23 \times 10^4$ m^2
$R_2 = 72$ m, $A_2 = 1.63 \times 10^4$ m^2
$R_3 = 79$ m, $A_3 = 1.96 \times 10^4$ m^2

b) $\mu = 3.24 \times 10^{10}$ kg m^{-1} s^{-2} $D_1 = 4.0 \times 10^{-2}$ m
$D_2 = 1.3 \times 10^{-1}$ m
$D_3 = 1.1 \times 10^{-1}$ m

c) $\Delta\sigma_1$ = 13.8 MPa
$\Delta\sigma_2$ = 79.7 MPa
$\Delta\sigma_3$ = 60.3 Mpa

6 Comments

One should always be aware that the interpretation of seismic source spectra is based on simplified model assumptions of the generally unknown *source geometry* (e.g,, circular fault or rectangular fault with different, also magnitude dependent aspect ratios), as well as unknown *rupture kinematics* (rupture velocity and direction, e.g. uni- and/or bilateral and – directional; see Figure 6 in IS 3.1), and *dynamics* (stress drop). Different model assumption will yield different results, even for identical spectral parameter readings, as demonstrated with the above solutions. They may differ even for the most common models of small earthquakes by a factor of about two for the source “radius” and more than 5 for the stress drop (because of $\Delta\sigma \sim R^{-3}$).

Moreover, also the correction of measured spectra for wave propagation effects (geometric spreading, scattering and intrisic attenuation) is largely model-based and thus error-prone. And the reading from rather noisy empirical spectra of the spectral plateau amplitude - and in particular of the corner frequency -is afflicted with relatively large errors too. All this increases further the uncertainty of source radius and especially stress-drop estimates. The latter may be, in the worst case, uncertain up to about two orders of magnitude.

Therefore, in studying possible systematic differences in source parameters derived from spectral data for events in a given area, or within an aftershock sequence, one should always stick to using one type of model only, even if this might not be the best in absolute terms with respect to the given (but usually not yet well known) specific seismotectonic situation. Yet, one should at least be reasonably sure about the validity of assuming that the events have similar modes of faulting and crack propagation. If later models become available which better seem to describe the seismic source processes in the area and/or in the magnitude range under investigation then earlier, maybe inferior but consistent results, may be “rescaled”.

References

Brune, J. N. (1970). Tectonic stress and the spectra of shear vaves from earthquakes. *J. Geophy. Res.*, **75**, 4997-5009.

Keilis-Borok, V. I., (1959). On the estimation of displacement in an earthquake source and of source dimension. *Ann. Geofis.*, **12**, 205-214.

Madariaga, R. (1976). Dynamics of an expanding circular fault. *Bull. Seism. Soc. Am.*, **66**, 639-666.