Topic	**Assessment of theoretical approaches to seismic network optimization (with PPT Tutorial)**
Author	**Peter Bormann**, formerly at Helmholtz Centre Potsdam, GFZ German Research Centre for Geosciences, Telegrafenberg, D-14473 Potsdam, Germany; now E-mail: pb65@gmx.net
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1 Introduction

The United Nations Decade on Natural Disaster Reduction (IDNDR; 1990-99) but also the terrible sequence of subsequent devastating earthquakes with often heavy death tolls (Izmit-Dücze, Turkey 1999; Boumerdez, Algeria 2003; Sumatra-Andaman 2004, Pakistan-Kashmir, 2005; Wenchuan, China 2008; Haiti and Chile 2010; Fukushima, Japan 2011) has in a dramatic way increased the world-wide awareness of the tremendous disaster potential of earthquakes and the urgent need for improved seismic hazard assessment, mitigation and preparedness measures. Part of these efforts is the improved monitoring and analysis of seismicity in the countries at risk, which includes improved event detection, location, magnitude determination and assuring catalog completeness down to as small as possible earthquakes (e.g., Woessner and Wiemer, 2005; Schorlemmer and Woessner, 2008; Schorlemmer et al., 2010; Kraft et al., 2011; Mignan et al., 2011).

Accordingly, the number of seismic stations and networks deployed, especially in earthquake-prone developing countries, has grown tremendously. Connected with these often costly investments in hard and software as well as qualified manpower is the expectation to get a maximum of useful information out of these “owned” physical networks, also by taking additionally into account already existing data acquisition and sharing facilities in neighbouring countries and by accessing “virtual” regional and global networks (see Chapter 8 and IS 8.3 of this Manual). In a similar way, this urgent need for fast and best quality data acquisition and information extraction also applies to the International Monitoring System (IMS) of the Comprehensive Nuclear Test-Ban Treaty (CTBT) Organization with its associated national and international components, e.g., for onsite inspection. Accordingly, related publications on the optimization of seismic networks on a local (e.g., Uhrhammer, 1980; Hardt and Scherbaum, 1994; Steinberg and Rabinovitz, 2003), regional (e.g., Kijko, 1977; Steinberg et al., 1995; Kraft et al., 2011) and global scale (e.g., Soriau and Woodhaouse, 1985) have been flourishing during the last few decades.

The theoretical basis for such studies is provided by the statistical theory of optímal experimental design (e.g., Kiefer and Wolfowitz, 1960; Kijko, 1977; Silvey, 1980; Rabinovitz and Steinberg, 1990; Steinberg and Rabinovitz, 2003). This theoretical approach aims at maximizing the determinant det (A^TA) with A the N·p matrix of partial derivatives of observables with respect to the parameters that characterize the object under investigation, called the *D-criterion*. In the case of event location, e.g., these are the observable t (travel-times) with respect to the 4 unknown source parameters latitude (, longitude (, source depth z and origin time t_0. Then the D-criterion is (det of the information matrix of the hypocenter parameters and a D-optimal network configuration maximizes the information matrix of these hypocenter parameters.

Travel-time observations may also be used for improving the Earth´s velocity model (tomography) or for joint improvement of both event location and velocity structure (see IS 11.1). And other observables allow to derive improved information about other “objects” of interest, e.g., spectral amplitudes of different types of seismic waves for calculating different types of earthquake magnitude, or the 6 components of the seismic moment tensor, or observations of the distribution of P-wave polarities or amplitude ratios of P to S waves in their dependence on source azimuth and distance to derive fault plane solutions. Thus, different combinations of relevant observables with respect to looked-for parameters of the seismic sources, or of the propagation medium such as seismic velocity distribution, inhomogeneity, and anisotropy, or wave attenuation and scattering properties are in principle possible.

In recent years also algorithms of simulated annealing (SA) have been applied for calculating optimization scenarios, e.g., by Hardt and Scherbaum (1995), both for minimizing the volume of the error ellipsoid of the **linearized** earthquake location problem as well as for optimizing network configurations for best source mechanism estimates and velocity tomography studies. SA is a discrete inversion technique, such as genetic algorithms, is fast, easy to program and not restricted to the linearity of the problem. It also allows optimizing of seismic networks for difficult and combined tasks. Kraft et al. (2011), describe SA in more detail. They also extended the algorithm of Hardt and Scherbaum that was restricted to 1D layered velocity models and upgoing direct P and S waves, by

- calculating travel-times of seismic body waves using a finite difference ray tracer and the 3D velocity model of Switzerland;
- calculating seismic body-wave amplitudes at arbitrary stations using the Brune (1970) source model and using scaling relations recently derived for Switzerland, and
- estimating the noise level at arbitrary locations within Switzerland by using a first-order ambient seismic noise model based on satellite-imagery-derived land-use classes and open GIS data.

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However, the complexity and degree of non-linearity of the problem increases with the number of observables and parameters to be optimized. Also, in most countries, detailed information about the local and regional velocity structure, topography, signal-to-noise conditions and other crucial boundary conditions are not yet available. This limits the full exploitation of potential benefits, which are offered by the application of more advanced optimization algorithms. Therefore, even rather recent publications, such as Steinberg and Rabinovitz (2003) still apply the simple “layers-above-half-space” model and assume that the layer thicknesses and velocities are known. They refer to Mitchel´s (1974) DETMAX algorithm for the problem of finding D-optimal networks when constraints on the use of possible station sites do not permit the direct use of the “pure” theoretical results. They also have implemented these ideas in the software program OPTINET (Shimshoni et al., 1992). Thus they derive first bonds on the efficiency of any proposed network but leave the application of more complex algorithms deliberately for the time when a more detailed model parameterization is available.

This has also been our concept for the complementary PPT tutorial linked to this information sheet.

Note, that the D-criterion is applicable only for linearized design problems. Geoscientific surveys, however, are usually of a more complex non-linear nature. To handle large-scale non-linear optimization problems is still beyond current possibilities, however, respective efforts for small-scale industrial surveys are already promising. E.g., Coles and Curtis (2011) presented a new method, termed D_N optimization, for a fully non-linear Bayesian statistical experimental design. It is based on a generalization of the D criterion, takes advantage of efficient linear methods and has lower computation and data storage cost than other non-linear algorithms. When comparing their results for an optimal seafloor microseismic sensor network to monitor a fractured petroleum reservoir they could demonstrate reduced spatial bias and hypocenter uncertainty in comparison with the results of a network designed on heuristic principles and of another one based on a linearized Bayesian design. For a more general overview paper on recent advances in optimized geophysical survey design we refer to Maurer et al. (2010). However, dwelling more on non-linear design optimization is beyond the scope of this Information Sheet.

2 The optinet.ppt tutorial

The tutorial annexed to this information sheet outlines the very essentials of the two most common approaches to network optimization and presents some basic results derived by applying these algorithms to simplified models and sometimes restrictive boundary conditions. None-the-less, these results are more or less representative for initial network optimization calculations based on still insufficient knowledge of the seismicity, environmental, logistic and other conditions in the considered country or area of network deployment. But most importantly, the PPT aims at demonstrating that the optimization result strongly depends on both the main aim or the multiple tasks of the network as well as on the "ambient" conditions under which the network has to be installed and operated. Accordingly, **one cannot expect a unique multipurpose optimal network solution** from any theoretical approach to network optimization**.**

However, in order to avoid overloading the PPT presentation about principal optimization goals, algorithms and results with text and comments related to more sophisticated approaches and results we have elaborated on this in the following section of **Comments.** They are numbered in the order as they appear in red as **Comment 1, Comment 2, etc.** on the respective slides of the PPT presentation. These comments aim at qualifying or complementing some of the statements in the PPT presentation and hint to more recent (yet still linearized) optimization procedures and results of their application. Therefore, reading these comments will ease proper understanding and assessment of the PPT presentation.

The PPT tutorial can be opened from here via hyperlink by right mouse click with the cursor set in front of the file name **optinet.ppt.** The presentation can also be downloaded via the summary listing *Download Programs & files* (see Overview on the NMSOP-2 cover page and follow related instructions).

3 Comments

Comment 1 on slide 4

The idealized assumption of the **first bullet point** is not an inherent requirement of the D-criterion. Rather it has to be qualified depending on the task, environmental conditions, infrastructure, data transmission links, logistic facilities, maintenance requirements, cost and other factors. When the aims and boundary conditions are better known one can “preset” a suitable or affordable number of possible station sites and then run under these refined conditions the search for an D-optimal network.

Also the assumptions made under the **second and third bullet point** are no prerequisites for applying the D-criterion. Rather, they are “set” by the way in which the matrix elements are calculated, e.g., by the algorithm used for the seismic ray-tracing. Kraft et al. (2011) base their search for a D-optimal solution on a 3D velocity model and realistic station heights. But also variable depths of borehole seismometer installations could be handled. Accordingly, also the **warning** at the end of this slide holds only for the case that the algorithm is run under the mentioned simplifying or restrictive conditions.

Comment 2 on slide 6

The v_P/v_S ratio tends to be more stable than the absolute velocities of P and S waves. Therefore, the joint use of P- and S-wave arrivals holds the promise of more reliable hypocentral distance and location estimates. This is usually the case for locations with modest precision requirements. However, in the case of significant variations of the v_P/v_S ratio (found out, e.g., by constructing Wadati diagrams; see IS 11.1) the inclusion of S-waves into the hypocenter location procedure would require the availability also of an independent good S-wave velocity model. Regrettably, the latter is usually not the case because of the dominating trend in both routine interactive and automatic seismogram analysis to pick only P-wave first motions. This notwithstanding, according to Gomberg et al. (1990), the inclusion of S waves is a powerful tool to significantly improve the hypocenter location accuracy and to reduce the trade-off between source depth and origin time. The most important findings of these authors can be summarized as follows:

- In the absence of detailed knowledge of the regional velocity structure, the most certain way to minimize the solution´s sensitivity to model parameter inaccuracies and theoretical simplifications is to record and pick the onsets of S phases within the aforementioned distance range.
- Using an S phase recorded close to the event almost always improves the accuracy of depth determination. E.g., including at least one reliable S-phase recorded at an epicentral distance D within about 1.4 times focal depth h yields depth estimates accurate within approximately (1.5 km. The accuracy will be only slightly inferior when the vertically averaged velocity model assumed in the location procedure differs from the true one by less than a few percent.
- In contrast, when systematic model errors are in the order of 4% and no S phases have been identified close to the source and included into the location procedure then the depth error can be larger than (3 km. This holds even in the case of good azimuthal coverage and the inclusion of several P-phases recorded at D < h.

- The largest relative improvement in focal depth accuracy is achieved in the case of poor azimuthal station coverage.
- **HOWEVER:** Just because of S-wave readings being potentially so powerful a constraint, systematic S-phase timing errors, already as small ones as 0.2 s, may degrade the accuracy of h estimates by several kilometres, even if the azimuthal coverage is good. Moreover, such S-wave timing errors may even misleadingly result in a stable - but wrong - solution with reduced standard error, i.e., improved *precision* of the solution although the actual *accuracy* is worse and may even lead to the construction of an incorrect S-wave velocity model.
- Thus, in order to take full advantage of the potential of S wave readings for improving hypocenter solutions one should aim at an optimal station spacing not larger than three times the most shallow earthquake depth which has to be determined accurately, and further, at assuring correct identification and time-picking of the S-wave onset. The latter can be assured best when readings are made on a transverse component record of sufficient dynamic range to avoid clipping.

Comment 3 on slide 7

Experiments by Kraft et al. (2011) have shown that optimization runs for an N-station network and an N+1 network with free choice of location in both cases still show significant differences in network geometry even in the case of large N. In this sense, the procedure described in the last bullet point on slide 7 is not D-optimal for a “free” N+1 network but only in the sense of adding one station to a given network of already fixed station positions. Yet, in practice, the latter is the usually looked for.

Comment 4 on slide 8

One major problem, which has great influence on the proposed optimal networks, is the normalization of the used objective cost functions and their relative weight with respect to other parameters. Another problem is the dependence on the random numbers and the high degree of non-linearity of systems with more than 10 stations. This requires careful tests and control of the starting conditions. E.g., Kraft et al. (2011) looked for the optimal network for hypocenter location of events down to magnitude Ml = 1 in northern Switzerland by adding between 10 to 35 new stations to an existing net of 67 stations in Switzerland, Germany and Austria. Repeated optimization runs over a grid net of 952 potential station sites, separated by a spacing of some 4.5 km, yielded fluctuating results of the optimal station sites. Although their positions usually did not differ by more than two grid spacings this hints to inherent non-linearity of the problem. This made it advisable to search within these fluctuation zones of about 10 km diameter for the best suited sites. While the overall hypocentral resolution of the network configurations calculated by several optimization runs are about the same, some larger variations in the actual network geometry are possible. Generally, it is advisable to test the stability of solutions by repeated initialization of the optimization procedure.

Comment 5 on slide 9

One basic concept of SA (Kirkpatrick et al., 1983) is to occationally accept "bad" solutions with E(j) > E(j-1) depending on the preset and slowly decreasing virtual temperature T of the system (termed “annealing schedule”). This allows to overrun local minima. The criterion to accept "bad" solutions is: exp(dE/T) ≥ rand (0, 1). The criterion depends on the random number rand (0,1) and is more likely fulfilled if the virtual temperature T is high.

Comments 6 on slide 10

Hardt and Scherbaum (1994) considered only 1D-layered or homogeneous velocity models and stations in the distance range of Pg/Sg first arrivals, i.e., within epicentral distances typically less than about 150 km.-As said already in slide 5, for v = const. the optimal station distance radius r around the source would be infinite, because this would assure the smallest relative error in onset-time picking with respect to the total travel time. In reality the dimensions of the network are finite and determined by the scale of the problem, i.e., by the size of the search grid. Therefore, an optimal network geometry in the finite homogeneous case will consist of stations symmetrically distributed on the circumference of the search grid. This is clearly seen in the 4 solutions presented on this slide.

Further, the D-criterion tends to cluster stations at optimal sites, e.g., along the outer circumference of the search grid, if the number of looked-for stations is large with respect to the scale of the problem such as the size of the earthquake source region. This behaviour can be changed by introducing a weighting function for station spacing (e.g., Rabinowitz and Steinberger 1990). Kraft et al (2011) made use of the clustering feature of the D-criterion to identify important sites in an optimal network geometry. They performed repeated optimizations for the same large scale problem with slightly different starting conditions, i.e. re-initialized random numbers. Although the network geometries of individual runs differed due to the inherent non-linearity of the problem, stations of the different solutions where found to cluster at specific locations, which were therefore considered important optimal station sites for the considered problem. For suitable action see Comment 4 and the final comments.

In contrast to the examples shown in slide 10 with event numbers Ne less or only somewhat larger than the number of looked-for stations sites Ns, Kraft et al. (2011) searched for the best location of 10 to 35 new stations (to be added to the existing 67 stations) for locating 2240 synthetic catalog earthquakes that were equally distributed over the area under investigation. Although equal distribution of earthquakes is surely not the case in reality, this assumption assures an equally representative assessment of the resolving power of the looked-for network over the hypocentral volume covered by the network and within which earthquakes are likely to occur.

Comment 7 on slide 15

See last paragraph of Comments 6. The gain to be expected by later optimization according to the actually observed distribution of earthquakes is considered by T. Kraft (personal communication 2011) as likely being not so large.

Comment 8 on slide 20

The results of optimization strongly depend on the assumed ambient noise model. Therefore, Kraft et al. (2011) developed a first order ambient noise model for Switzerland on the basis of land-use-data derived from satellite imagery. Further, they decided to place, in a first implementation step, only part of the recommended new stations at locations with the lowest noise expected according to the three considered noise classes. With actual noise data at these stations, measured over a representative time window, they plan to update the noise model for the remaining required new station sites and to re-evaluate the initial optimization results.

However, it is often overlooked that low ambient noise does not guarantee an improved signal-to-noise ratio (SNR). Low noise hard rock sites often have also reduced signal amplitudes, whereas sites on soft rock with usually higher ambient noise typically show, especially in the short-period range, increased (up to about a factor of 10) signal amplitudes. Moreover, even for given hard-rock site the short-period SNR may, due to lateral velocity and/or topographic inhomogeneities, significantly depend on source distance and azimuth (see Chapter 4, Figs. 4.34-4.36). Investigating such site-specific signal and noise amplification has meanwhile become a widely used method for microzonation studies (see Chapter 14). Yet, these effect have so far rarely been taken into account in network optimization algorithms, although they affect especially network optimization aimed at assuring as low as possible homogeneous “completeness magnitude” Mc. Therefore, when new information about noise level and SNR conditions become available, even for a small number of the stations, then the optimization procedure should be repeated until the desired hypocentral resolution and detection/magnitude threshold sensitivity of the network is achieved.

For some newer works and results on the variable SNR and magnitude completeness problem see Mignan et al. (2010), Schorlemmer and Woessner (2008), Schorlemmer et al. (2010), Woessner and Wiemer (2005) as well as the final comments.

Final comments related to slide 21

With reference to slide 4 it should be mentioned that also the rather advanced Kraft et al. (2011) optimization approach does not account for correlation errors. Therefore, the resulting D-optimal networks often contained stations that were closely spaced with already existing or newly placed stations. The clustering increases with the number of stations to be placed in a network. This redundancy effect, however, can also be interpreted as an up-weighting of important placement regions for improving the hypocentral resolution. If such a cluster placement region contains already a station but with high noise (or bad average SNR) level one should either find within this placement region a better site for re-installation or consider placement of the seismometer in a shallow borehole.

Also the investigations by Kraft et al. (2011) have shown that achieving optimal network design goals for different parameters requires different network size and efforts. While for their regional network it would be sufficient to add only 10 stations to assure an epicentral resolution of 0.5 km, a minimum of 20 new stations is required to achieve a hypocentral resolution of 2 km. And the envisaged constant magnitude completeness of Mc = 1 for an area of several 100 km^2 could only partially be achieved by adding 26 new stations. This highlights that results from optimization algorithms, when being available already in the planning phase, may help to set more realistic goals of what can reasonably be done and provide useful guidance for final decision making prior to hardware purchases, site selection and installation efforts, depending also on available financial resources.

Although the algorithm by Kraft et al. (2011) had been developed for optimizing a rather large-scale multi-station regional network in Switzerland, it is also applicable to smaller optimization problems, e.g., for small-scale local networks aimed at surveillance of the induced seismicity from geothermal, oil or gas extraction operations, due to high-rise dam impoundment, or for volcano monitoring. Also, the algorithm is especially useful to optimize networks in populated areas with heterogeneous noise conditions, and/or complex velocity structures, as well as for complementing already existing networks. The algorithm is freely available for research and teaching from the author (contact: toni.kraft@sed.ethz.ch)

Finally, we have to emphasize that practically all network optimization cases discussed so far in this information sheet related to short-period high-gain systems, which are usually deployed for monitoring local and regional seismicity. However, several aspects discussed above for assuring high network performance are irrelevant or less important for long-period broadband and global seismic networks. E.g., some sites with significantly improved SNR in the short-period range and thus signal detection from week events may show no SNR improvement at all for long-period signals (see Chapter 7, section 7.2.4.4, Fig. 7.33-7.36). Also, whereas SP sensors are rather insensitive to air-pressure, tilt and temperature fluctuations, high-gain broadband sensors requires very careful installations and shielding efforts to minimize these kinds of disturbances. However, some of the task-dependent optimal site positions identified by the algorithms may not permit to assure very stable broadband sensor placement and optimal shielding. Moreover, the optimization of the configuration of global seismic networks has to be made under rather different boundary conditions. On a global scale, earthquakes are even more inhomogeneously distributed, but also the continents and oceans, where the sensors have to be deployed. Also, the climatic, topographic, logistic and "political" environments differ more, as do the research goals when studying the whole planet Earth. This requires to consider many additional boundary conditions for optimization scenarios (e.g., Souriau and Woodhouse, 1985) as well as implementation strategies.

4. Final recommendations

Embarking on the optimization of seismic network configuration one should practise a step-wise and problem-/task-oriented approach. Theoretical algorithms on design optimization maybe very helpful in such efforts. Yet, one should accept that their importance lies not so much in the accuracy of their results in absolute terms, which is model-dependent and thus usually still insufficient, but rather in their qualitative value. Even answers to questions such as "Which measures may significantly improve or deteriorate the network performance?", without giving precise numbers, may provide already useful guidance for sound decision making, as do expert recommendations based on rich seismological experience and sound intuition. The tutorial and the above comments aim at developing this needed problem understanding.

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