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Broadband Ground-Motion Simulation Using Energy-Constrained Rise-Time Scaling

by Katrin Kieling, Rongjiang Wang, and Sebastian Hainzl

Abstract  The simulation of realistic ground motions associated with large earthquakes is of utmost importance for engineering concerns, for example, in testing the seismic performance of buildings. It is also needed to estimate the level of ground shaking for future earthquakes. In this paper, we focus on two aspects of ground-motion modeling. First, we use deterministic wave propagation for the entire frequency range but account for source variability by implementing self-similar slip distributions and rough fault interfaces. Second, we scale the rise time of the source time function so that the modeled waveforms represent the correct radiated seismic energy. The method is validated for the 2008 Wenchuan, China, earthquake and the 2003 Tokachi-Oki, Japan, earthquake. We demonstrate the robustness of our approach with regard to changes of the assumed rupture velocity. We find the fine source discretizations combined with the small scale source variability ensure that the high frequencies (1–8 Hz) are satisfactorily introduced, justifying the deterministic wave propagation approach even at high frequencies. The adjustment of the rise time via the radiated seismic energy permits a simulation without making assumptions on the stress drop, as is usually done by other modeling approaches. By varying different parameters, such as the energy magnitude, the simulation approach could be especially useful for estimating the range of possible ground motions issuing from a distinct fault in a future earthquake. The rise-time scaling may also be used for seismogram modeling in the context of early-response systems as the energy magnitude, which is the parameter needed for the scaling, is available shortly after the earthquake.


Introduction

Engineering applications require estimates of strong ground motions for wide distance and magnitude ranges. Unfortunately, existing records of seismic networks only represent a small subset of possible earthquake scenarios. Modeling of broadband ground motion seems to be an ideal aspirant to satisfy engineering needs. For the determination of structural responses and damage estimation, information on ground-motion amplitude and phase, as well as duration, is of significant importance. In addition, realistic synthetic strong ground motions are of seismological interest. For example, a kinematic modeling approach may be used as a synthetic data generator for testing earthquake early warning or early response systems (Zollo et al., 2009; Picozzi et al., 2013).

Much attention was devoted to the task of providing ground motions that match observations in a regime where they are strongly influenced by the source process. Several different methods have been developed in the past to meet this challenge. Stochastic methods, such as used by Boore (2003) and Motazedian and Atkinson (2005), perform well at predicting amplitude and frequency content at large distances over a wide frequency range. However, they do not provide correct phase information or good representation of near-fault ground motion in the time domain.

Deterministic methods, using either numerical (e.g., Hartzell and Helmberger, 1982) or semi-analytical methods (e.g., Heaton and Helmberger, 1979), result in correct phases and amplitudes as long as the subsurface structure is appropriately modeled. However, they need a dense grid of Green’s functions along the fault to model high frequencies correctly. Furthermore, the kinematic properties of the fault have to be prescribed on this fine grid, of which we often do not have sufficient knowledge. Hanks and McGuire (1981) state that it is virtually impossible to deterministically synthesize high-frequency ground motion. They claim that it requires at least one stochastic parameter, as it is done in hybrid simulations (Pacor et al., 2005; Graves and Pitarka, 2010).
Another important issue of kinematic ground-motion modeling is the amount of energy radiated in the high-frequency band. This is fixed by the corner frequency or the rise time of the source time function. Usually this is done by either empirically constraining the rise time (Graves and Pitarka, 2010) or by fixing the stress drop and relating the rise time to the subfault size (Zeng et al., 1994). In both cases, either the rise time itself or the stress drop has to be fixed empirically or by best guess.

In this study, we concentrate on the integral approach, because we want to show that broadband ground-motion simulation from permanent displacement up to a frequency of 10 Hz is possible with a nonhybrid approach. Such a fully deterministic wave propagation allows for a study of the influence of different source parameters on the final ground motion. Using an analytical approach and a simple layered Earth model, we reduce the computational efforts for calculating Green’s functions. Complex characteristics of ground motions at high frequencies and at near-fault sites are reproduced by considering small-scale slip variability and fault roughness. Therefore, we include stochastic features in the source description.

An important new ingredient of our methodology is the combined use of two different magnitudes for constraining the rise time of the source time function: the moment magnitude $M_w$ and the energy magnitude $M_e$. Thereby, we avoid a guessed or empirically constrained stress drop. For our purpose, it is desirable that the two magnitudes are determined independently but considered jointly during the simulation of the ground motion. In addition, using different (but statistically reasonable) energy magnitudes for the same moment magnitude may be useful when simulating scenarios for future earthquakes, as this could show the range of possible ground motions.

In the following, we first introduce our method in detail. We apply the approach to the 2003 Tokachi-Oki, Japan, earthquake and the 2008 Wenchuan, China, earthquake. Here, we compare simulated peak ground acceleration (PGA) and response spectra with the observations. For those quantities, we also calculate the model bias. We validate the rise-time scaling by changing the rupture velocity of the simulations. Finally, we discuss the simulation results including the merits and drawbacks of the method.

**Methodology**

The presented approach consists of four major steps, which will be described in detail in the following subsections: (1) calculation of Green’s functions, (2) description of the kinematic source model, (3) adjustment of the radiated energy, and (4) convolution and superposition

**Calculation of Green’s Functions**

The wave propagation used in our approach is purely deterministic. We use a layered Earth model without horizontal heterogeneities. The synthetic Green’s functions are calculated using an orthonormalizing propagator algorithm. Details of the calculation are described by Wang (1999). The algorithm delivers full solutions with correct phase arrivals and is applicable to the whole frequency range, including the static permanent deformation.

This is the most time-consuming part of the calculation. However, for a given velocity model and source–receiver distances, the Green’s functions have to be calculated only once. They can be saved in a Green’s functions database and may be accessed for different scenario calculations.

**Kinematic Source Description**

The rupture is represented by a kinematic source description. The necessary input parameters, which have to be provided, are

- moment magnitude $M_w$,
- energy magnitude $M_e$,
- hypocenter,
- rupture velocity $V_r$, and
- fault segments, each defined by its geometry and focal mechanism (strike, dip, and rake).

If a slip distribution is available, we keep the low-wavenumber part of the distribution and add a random high-wavenumber part. If no slip distribution is available, a random slip distribution is created. In either case, the resulting slip distribution is designed to be self-similar; that is, its spectrum shows squared fall-off at high wavenumbers as described by Herrero and Bernard (1994). The amount of slip on the fault plane is calculated such that the seismic moment resulting from the moment magnitude is reproduced. The level of the high-wavenumber part of the slip distribution is set such that it forms a continuous spectrum with the low-wavenumber part. The small scale variations are introduced to reproduce the complex ground motion close to the fault, assuming that real earthquakes show the same roughness in terms of fractal dimensions (Mai and Beroza, 2002). Details of the procedure to create the refined slip model are described in the Appendix.

We introduce variations in strike and dip, such that the fault plane is no longer an exact plane, but forms a fractal surface (Turcotte, 1992). The amplitude of the deflection is proportional to $k^{-\alpha}$ ($k$ is the wavenumber and $\alpha$ the fractal dimension) and the maximum deflection is $\beta L$ with $L$ as the shorter dimension of the fault plane and $\beta$ as the aspect ratio. In the following applications, we use $\alpha = 1.8$ (corresponding to a Hurst exponent of $H = 0.8$ [Renard et al., 2013] with $\alpha = H + 1$) and $\beta = 0.02$. Candela et al. (2012) show that the factor defining the roughness amplitude is strongly variable between different earthquakes, while the Hurst exponent is rather stable 0.8 ± 0.1. The choice of $\beta$ is somewhat arbitrary, and the value of $\beta$ definitely has to be further investigated. Presumably, it is dependent on the maturity of the fault. However, in our model there are no real deflections from
the fault plane, but only variations in the local fault mechanism. That means the geometry variations are projected on the fault plane. Käser and Gallović (2008) demonstrate that projecting geometry variations on the fault plane has very similar effects on the simulated ground motions as true deflections from the plane.

The discretization has to be fine enough to avoid the aliasing effect due to the discrete point sources; insufficient discretization causes artificial periodicities that are due to the discrete summation. For correct results, the time difference between the signals of adjacent fault patches has to be smaller than the inverse Nyquist frequency. In other words, a maximal frequency \( f_{\text{max}} \) requires a maximal discretization step \( \Delta x \), defined by

\[
\Delta x = \frac{1}{2} \left( \frac{1}{V_r} + \frac{1}{\beta_{\text{min}}} \right)^{-1} \frac{1}{f_{\text{max}}}
\]  

(Heimann, 2010), in which \( \beta_{\text{min}} \) is the minimal shear-wave velocity of the material in the fault region and \( V_r \) is the rupture velocity along the fault. Equation (1) is valid not only for our method, but for any deterministic wave propagation approach, which is based on the summation of discrete point sources. It should be stressed that the discretization is independent of the earthquake size, but mainly dependent on the considered frequency range. Indeed scientists are probably interested in higher frequencies in the case of smaller earthquakes; and, therefore, a finer discretization is needed.

The rupture velocity \( V_r \) is assumed to be related to the surrounding 5 velocity \( V_S \) with

\[
V_r(z) = \begin{cases} 
0.6 \times \gamma \times V_S(z) & : z < 5 \text{ km} \\
\gamma \times V_S(z) & : z > 8 \text{ km} 
\end{cases}
\]

and a linear transition between 5 and 8 km. \( \gamma \) varies around 0.8 for crustal earthquakes (Somerville et al., 1999) but may be different for subduction earthquakes. This definition of equation (2) approximately follows the approach of Graves and Pitarka (2010), who argued that the reduction in velocity for the upper 5 km represents the shallow weak zone in surface rupturing events (Pitarka et al., 2009).

The stress drop \( \Delta \sigma \) is assumed to be the main factor influencing the ground acceleration \( \ddot{u} \), hence should be proportional to the slip \( u \) divided by the squared rise time \( r \),

\[
\ddot{u} \propto \Delta \sigma \propto \frac{u}{r^2}.
\]

As a first-order approximation, we assume constant stress drop along the fault to constrain the relation between rise time and slip. In this case, the rise time of each fault patch \( \tau_i \) should be related to the slip \( u_i \) along this patch by

\[
\tau_i \propto \frac{u_i}{\Delta \sigma} = \sqrt{\frac{u_i}{\text{const.}}}
\]

(4)

For this reason, we make the patch rise time proportional to the square root of the patch slip.

Figure 1. Comparison between \( M_e \) (determined by the Global Centroid Moment Tensor project) and \( M_w \) (determined by GeoForschungsZentren [GFZ], German Research Centre for Geosciences) for 990 events. Different symbols represent the type of mechanism. The 1:1 line is also plotted, as well as dotted lines indicating \( M_e = M_w \pm 0.5 \). Modified from Di Giacomo et al. (2010), including data from the year 2008. The color version of this figure is available only in the electronic edition.

Adjusting the Radiated Seismic Energy

After the earthquake source is described kinematically, the energy radiated from this modeled source is adjusted to the observed energy. Here, we always consider the energy observed and radiated by the whole fault, not by single fault patches.

In physical terms, the moment magnitude is derived from the seismic moment \( M_0 \) and represents a measure of the geometric size of the earthquake. The energy magnitude is a measure of the kinetic energy radiated by seismic waves and is determined in practice by integrating the power spectrum of teleseismic broadband seismograms (Choy and Boatwright, 1995; Di Giacomo et al., 2010). Hence, it is more suitable for assessing the potential ground shaking. For large earthquakes, both magnitudes are available from teleseismic observations within several tens of minutes after the earthquake. Figure 1 suggests a linear relation between the two magnitudes. Following Bormann and Di Giacomo (2011), it may be described by the formula

\[
M_e = M_w + (\Theta + 4.7)/1.5,
\]

(5)

with the parameter \( \Theta = \log(E_e/M_0) \); that is, the logarithmic ratio between the radiated seismic energy \( E_e \) and the seismic moment \( M_0 \). However, a strict linear relation assumes that \( \Theta \) is constant, while in reality the ratio \( \Theta \) depends on highly variable parameters such as stress drop. That is why \( M_e \) and
The observed energy $E_{\text{obs}}$ is related to the energy magnitude $M_e$ by

$$E_{\text{obs}} = 10^{1.5M_e + 4.4}$$

(Bormann et al., 2002). Using some basic formulas given by Haskell (1964) and following the derivation in Venkataraman and Kanamori (2004), the energy radiated from the modeled source $E_{\text{model}}$ can be calculated via

$$E_{\text{model}} = \left(1 + \frac{2 V_S^5}{3 V_p^5}\right) \frac{1}{10\pi\rho V_S^5} \int_0^\infty \dot{M}^2(t) dt.$$  \hspace{1cm} (7)

Here, $V_p$ and $V_S$ denote the $P$ and $S$ velocities, $\rho$ is the density, $t$ is the time since the earthquake origin, and $\dot{M}(t)$ is the moment release rate, which is usually called the source time function. Except for $\dot{M}(t)$, all parameters are determined by the Earth model. Generally, $E_{\text{model}}$ does not equal $E_{\text{obs}}$, thus, we have to adjust the source time function. When using Brune's source time function, it has the functional form

$$\dot{M}(t) = M_0 \frac{t}{\tau^3} e^{\frac{-t}{\tau}} H(t)$$

(Brune, 1970), with $H(t)$ as the Heaviside function and $\tau$ as the rise time (i.e., the time when the slip rate function reaches its maximum). For a fault consisting of one single patch, the following relation holds for the integral in equation (7):

$$\int_0^\infty \dot{M}^2(t) dt \propto \frac{M_0^2}{\tau}. \hspace{1cm} (9)$$

Because we assume that the seismic moment is also fixed by the observation, the only parameter to adjust the energy is the rise time. Finally, we scale the rise time by multiplying it with the third root of the ratio of modeled and observed energy.

$$\tau^* = \tau \left(\frac{E_{\text{model}}}{E_{\text{obs}}}\right)^{\frac{1}{3}}.$$ \hspace{1cm} (10)

and recalculate the source time function with the modified rise time $\tau^*$.

For a fault consisting of $N_S$ patches, the radiated energy can be estimated using

$$E_{\text{model}} = \int_0^\infty \left\{ \sum_i \sqrt{\frac{1 + \frac{2 V_S^5}{3 V_p^5}}{10\pi\rho V_S^5}} \right\} \frac{1}{\tau_i^*} \left(\frac{M_i}{\tau_i^*} e^{\frac{-\infty - t_i}{\tau_i^*}} H(t - t_i)\right)^2 dt,$$ \hspace{1cm} (11)

in which $M_i$ and $\tau_i$ are the seismic moment and the rise time of the $i$th patch, respectively. The scaled rise time $\tau_i^*$ is still determined in the same way as for a single patch, but it does not lead to an exact agreement between $E_{\text{obs}}$ and $E_{\text{model}}$. Therefore, the procedure has to be applied iteratively. Adjusting the source time function in this way results in seismic energy radiation, which is in agreement with the observed energy magnitude $M_e$.

An alternative approach would be to scale the rupture velocity instead of the rise time or to scale both simultaneously. As the rupture velocity also determines the duration of the seismic moment release, it directly influences the moment rate and can therefore be used to achieve a certain seismic energy given a fixed seismic moment. However, the rupture velocity also influences the signal length, while scaling the rise time keeps the signal length approximately constant. Furthermore, the mean rupture velocity is easier to determine from the data (e.g., from the signal duration of teleseismic records) than the rise time. These are the reasons why, in this study, we fix the rupture velocity and constrain the rise time by the energy magnitude.

Convolution and Superposition

In the last step, the seismograms are computed. As a simple approach, the source time functions of all subpatches of the kinematic source could be convolved with their corresponding Green’s functions. However, regarding the high number of fault subpatches needed to avoid spatial aliasing, the huge number of convolutions leads to a very high computation time. To avoid this, the fault subpatches are now binned in larger fault patches that have the same size as the grid of the Green’s functions database. The moment rate function is calculated for each subpatch–receiver pair, and the functions of all subpatches binned in one fault patch are summed with the time shift according to their rupture time and relative position regarding the receiver. However, the time-shift regarding the direct $S$ phase is applied to the source time function, so there remains a small error for the other phases. It is important to note that the shape of the resulting moment rate function of the large patch is dependent on the receiver position. This dependence makes the important difference between a simple coarse discretization and the refined discretization used here. The resulting moment rate function of each fault patch is convolved with its corresponding Green’s function. Finally, the contributions of all patches are superposed, which results in the time series of ground velocity at the receiver. To obtain ground acceleration and ground displacement, the derivatives and integrals of the seismograms are calculated in the time domain, respectively.
For an overview, we have summarized all input parameters of the simulations in Table 1. All the examples were calculated on a normal desktop computer.

Application

In the following subsections, we present applications of our methodology for the cases of the 2008 Wenchuan and the 2003 Tokachi-Oki earthquakes. Maps of the two earthquakes and the strong-motion and Global Positioning System (GPS) stations used are shown in Figure 2. We also aim to validate the rise-time scaling approach by comparing simulations of the Wenchuan earthquake with varied rupture velocity.

2008 Wenchuan Earthquake

On 12 May 2008, the $M_w$ 7.9 Wenchuan earthquake occurred in the region of Sichuan in China. The rupture extended about 300 km along the Longmen Shan thrust belt and was one of the most devastating intraplate thrust earthquakes. Just before the earthquake, the installation of the National Strong Motion Observation Network System of China was completed (Li et al., 2008), which ensured many near-field observations of the event. This enables the comparison of the observed strong-motion records at 95 stations with the corresponding time series obtained by our simulation.

For the simulations, Green’s functions were calculated on a grid of approximately 1 km for the layered velocity model given in Table 2, which is similar to the IASP91 reference Earth model (Kennett, 1991) but accounts for slightly smaller $S$ velocities in the upper crust as suggested by Y. Xu, et al. (2010). We use the source geometry and slip model obtained by C. Xu, et al. (2010) from joint inversion of GPS and Interferometric Synthetic Aperture Radar coseismic displacement. Their model consists of five fault segments, which are further discretized to patches of 1.5 km size. The dip of the segments at depth is smaller than the dip close to the surface. High slip mainly occurs close to the surface. The moment magnitude of 7.9 provided by the Global Centroid Moment Tensor solution (see Data and Resources) corresponds to a seismic moment of $0.9 \times 10^{21}$ N·m. We rediscrize to 100 m (see Table S1 available in the electronic supplement to this article), leading to a maximum frequency of around 7.8 Hz. In this example, the energy magnitude of 7.9 (U.S. Geological Survey [USGS]; see Data

<table>
<thead>
<tr>
<th>Input Parameters Used in the Simulations</th>
</tr>
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<tbody>
<tr>
<td>Earth model (layered half-space)</td>
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<tr>
<td>Source model</td>
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Summary of all input parameters used in the simulations.
and Resources) equals the moment magnitude and leads to a mean subpatch rise time of 0.022 s. According to our definition, 36% of the energy is radiated over this period. For a 90% energy release, which is an alternative definition of the rise time, our value must be multiplied by 2.4. The rupture velocity is related to the $S$ velocity by equation (2), with $\gamma = 0.8$.

It is important to note that we use a rupture model inferred only from the coseismic static deformation. Consequently, our model provides forward simulations constrained mainly by geodetic information and does not incorporate detailed information of the kinematic rupture process. Thus, we cannot expect to match every single detail of the waveform. Instead, we aim to reproduce major characteristics of the strong ground motion. We want to show that the radiated seismic energy is a good mean to calibrate the high-frequency content and that deterministic wave propagation based on a sufficiently discretized source model is suitable for the simulation of strong ground motions.

Figure 3 shows modeled and observed waveforms at four selected stations for the 2008 Wenchuan earthquake. (a) Unfiltered seismograms of east–west (E), north–south (N), and vertical (Z) components (from top to bottom). The station name and its distance to the surface projection of the fault are given above each set of seismograms. (b) Same for seismograms low-pass filtered at 1 Hz. The color version of this figure is available only in the electronic edition.

![Figure 3. Comparison of observed and modeled velocity waveforms at four selected stations for the 2008 Wenchuan earthquake. (a) Unfiltered seismograms of east–west (E), north–south (N), and vertical (Z) components (from top to bottom). The station name and its distance to the surface projection of the fault are given above each set of seismograms. (b) Same for seismograms low-pass filtered at 1 Hz. The color version of this figure is available only in the electronic edition.](image)

<table>
<thead>
<tr>
<th>Depth (km)</th>
<th>$V_p$ (km/s)</th>
<th>$V_s$ (km/s)</th>
<th>Density (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0–5.1</td>
<td>5.50</td>
<td>3.20</td>
<td>2.6</td>
</tr>
<tr>
<td>5.1–18.0</td>
<td>6.00</td>
<td>3.46</td>
<td>2.7</td>
</tr>
<tr>
<td>18.0–34.5</td>
<td>7.60</td>
<td>3.87</td>
<td>2.8</td>
</tr>
<tr>
<td>34.5–40.0</td>
<td>7.50</td>
<td>3.43</td>
<td>3.0</td>
</tr>
<tr>
<td>&gt; 40.0</td>
<td>8.04</td>
<td>4.47</td>
<td>3.4</td>
</tr>
</tbody>
</table>

Crustal velocity model used for the calculation of the Green’s functions for the 2008 Wenchuan earthquake. Here, a constant $Q_S$ factor of 600 was used.
the peak values of the individual components. The maps of PGA show certain similarities in terms of directivity, but also the simulation overestimated the observed values in several areas. In addition, residuals are plotted to explicitly show differences at individual stations. For most of the stations, PGA is matched satisfactorily well. Large discrepancies occur mainly for stations close to the fault but predominantly on the footwall side. For these locations, PGA is significantly overestimated. This might be attributed to overly strong forward and, respectively, backward directivity effects that are caused by the unperturbed rupture velocity (Kurahashi and Irikura, 2010). To consider relative differences of PGA compared with the amplitude of observed PGA, one should also consider the logarithmic difference. Logarithmic difference of PGA is also large at the hanging-wall site, west of the fault plane, whereas logarithmic differences are less pronounced north and south of the northernmost and the southernmost edge of the fault, respectively. The reader should remember that the regional near-surface structure is not included in the simulation. Hence, the northernmost and the southernmost edge of the fault, respectively. The reader should remember that the regional near-surface structure is not included in the simulation. Hence, the local characteristics of the Earth model may largely influence energy radiation from the source at shallow depth and the site specific. The reader should remember that the regional near-surface structure is not included in the simulation. Hence, the northernmost and the southernmost edge of the fault, respectively. The reader should remember that the regional near-surface structure is not included in the simulation. 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and Atkinson (2008) match the observation reasonably well for a uniform $V_{S30}$ value of 320 m/s. They tend to slightly underestimate the observations at larger distances. However, the level of the GMPE estimates is strongly dependent on the chosen value for $V_{S30}$. The underestimation becomes significant if higher $V_{S30}$ values are used. The variability of our simulation is comparable to the one standard deviation given by the GMPE for which we show the within-event variability (Atik et al., 2010) of the GMPE only, while excluding between-event variability.

2003 Tokachi-Oki Earthquake

On 25 September 2003, the Tokachi-Oki earthquake occurred off the Japanese coast, southeast of Hokkaido. The large thrust event along the upper boundary of the subducting Pacific slab had a seismic moment of $1.7 \times 10^{21}$ N·m (Yagi, 2004) corresponding to $M_w$ 8.1.

Thanks to a dense network of strong-motion records provided by KiK-net of the National Research Institute for Earth Science and Disaster Prevention (Aoi et al., 2004), we are able to compare simulations with 145 strong-motion stations. Throughout this section, we use KiK-net borehole data for the comparison. Moreover, 1 Hz GPS data are provided by the Japanese Geographical Survey Institute, which established the permanent GPS Earth Observation Network System (GEONET) that covers all the Japanese islands. The GPS measurements also permit comparison of the results of our simulations with the recorded displacements.

For the simulation of the 2003 Tokachi-Oki earthquake, we calculated Green’s functions in a layered earth on a grid of approximately 1 km. The 1D Earth model is adopted from Yagi (2004) and given in Table 3. Again, no station-specific near-surface information was included in the simulation. The source geometry, the slip distribution, and a seismic moment of $2.2 \times 10^{21}$ N·m are obtained from the inversion of static GPS displacements by Mingpeji Jin (personal comm., 2011); see Table S2. Also in this case, the slip model was constrained only by static displacement, and no kinematic information about the rupture process is included. The hypocenter is situated at latitude 41.78° N and longitude 144.08° E, resulting in a down-dip propagating rupture. We use a rupture velocity of $V_{rup} = 0.7 \times V_S$, and the assumed energy magnitude is $M_e$ 8.0 (Di Giacomo et al., 2008). Using a discretization of 200 m, this leads to a rise time of 0.05 s.

Figure 6 shows modeled and observed waveforms at four selected stations at different distances from the fault. For these examples, similar characteristics in terms of amplitude and signal duration are found for simulations and observations at the near-fault stations (TKCH09 and TKCH04) and also at greater distances from the fault (station IKHR01). At IKHR01, the simulation appears to have stronger high-frequency ground motion at the beginning of the shaking than did the observation. The same is true for AKTH09, for which the simulation shaking as a whole was too strong compared with the observation, especially on the vertical component. See Table S3. From the refined kinematic model, we compare observed and simulated peak ground accelerations. On Hokkaido, the general level of PGA values is well matched. The simulation tends to
underpredict values at the coast, especially at the southeastern part of the island. The PGA values at the western part of the island are slightly overpredicted. The logarithmic difference shows that PGA is overestimated at larger distances from the fault, in the northern part of Hokkaido and for stations on Honshu south of 40° latitude. On average, the simulation matches the observation well, as shown by the model bias in Figure 8. For response spectra, PGA, and PGV, the model bias is close to zero. However, the standard error is still up to 1.0 natural log units, resulting from a large difference between observations and simulations for some of the stations. Here, we also compare the observations with the GMPE of Zhao et al. (2006). This comparison should be regarded with care, as the regression was not made for borehole stations, but for surface stations. Regardless, we show its results under the assumption that borehole stations behave similarly to hard-rock stations with high $V_{S30}$. In this case, the general ground-motion level is met by the prediction for stiff rock ($V_{S30} = 1500$ m/s). The GMPE tends to overpredict the observations at small periods, whereas the predicted ground motion at large periods is too weak. The prediction works well for periods between 1 and 4 s. Figure 8b shows that the observed PGA seems to decrease more rapidly with increasing distance to the fault than predicted by Zhao et al. (2006). Our simulation matches this decrease with distance better than the GMPE, even though the simulated attenuation is still weaker than the observed. Again, the variability of our simulation is comparable to the one standard deviation of within-event variability of the GMPE.

As the observations of the GEONET 1 Hz GPS data are available for this earthquake, we compare the results of our simulations with the recorded displacement time series. Figure 9 shows the comparison of the simulated and observed displacements after low-pass filtering at 0.05 Hz suppressing noise at higher frequencies in the GPS data. We are able to approximately reproduce the maximal displacement amplitude. Because the slip distribution was inverted from the permanent displacement, at most stations and components the permanent displacement is reproduced, but the alignment with the waveforms is also fairly good. This shows that our approach is able to simulate the low-frequency displacement data up to the permanent displacement.

Validation of Rise-Time Scaling

One major advantage of our method is that there is no parameter that has to be tuned manually to achieve
ground-motion simulations with approximately correct high-frequency content. To demonstrate this, we conduct a simulation in which the rupture velocity is changed in the case of the Wenchuan earthquake, while we still apply the rise-time scaling to the energy.

We perform two additional simulations with rupture velocities decreased to $0.75 \times V_S$ and increased to $0.87 \times V_S$. For the main part of the fault, this leads to rupture velocities of around 2.6 and 3.0 km/s, respectively.

As a lower rupture velocity results in a larger overall fault rise time and because the signal is extended, without energy-related rise-time scaling we expect the ground-motion amplitudes to be smaller than in the previous cases. However, after fixing the rupture times of each fault patch, we again adjust the rise time, such that the radiated seismic energy is reproduced. Figure 10a shows that thereby we are able to match the observed energy level almost equally well for a smaller rupture velocity. The model bias for the medium spectral periods changes due to the change in rupture velocity. Compared with the previous simulation, less energy is radiated for periods ranging between 0.5 and 10 s. However, the small periods (i.e., the high-frequency ground motion) are almost unchanged. The same is true for the increased rupture velocity (Fig. 10b). Again, the changes in rupture velocity cause changes in the medium-frequency range. However, the high-frequency content is fixed due to the adjusting of the rise time and is very similar to that of the previous simulation. This demonstrates the energy scaling works as expected. Changes in the kinematic rupture model influence the simulation outcome; however, with approximately correct input parameters, the observation can be satisfactorily reproduced.

Discussion

Although it is often stated that deterministic wave propagation approaches are not suitable for modeling of high frequencies (Graves and Pitarka, 2010), our simulations show that the outcomes are nevertheless reasonable. It is usually thought that the source and wave propagation phenomena are not sufficiently known at high frequencies and that stochastic simulations are the logical response to the stochastic character of natural high-frequency ground motion. However, here we test the assumption that high-frequency ground motion follows the same rules as low-frequency ground motion, and we use the same modeling approach for both frequency ranges. The inclusion of stochastic features in the slip distribution and rupture geometry ensures the natural variability of the seismic source. Indeed, we do not account for scattering effects at high frequencies during the wave propagation, but project all variabilities to the fault. As it is not yet possible to distinguish between effects coming from the propagation path or from the rupture process, the inclusion of variability in one feature seems sufficient to us. We did not permit variations of the rupture velocity along the fault. A detailed investigation of the influence of variable slip, fault geometry, rupture velocity, and rise time will be the subject of further investigation.

The approach has general advantages compared to simpler methods such as the application of attenuation laws:

- Our approach is in principle applicable to every region of the world where we know the mean underground velocity model.
- There is no limitation concerning the distance range.
The results of the simulations show that our approach performs reasonably well, keeping in mind that it is neither designed for a specific tectonic setting nor uses the regional velocity model, but only a layered Earth model. However, the usage of a layered earth makes the model design very easy and fast, as only a few parameters are needed to start the simulation; this is a major advantage of the method. Another possible explanation for deviations of the simulations from observations is a different rupture velocity or a seismic moment. For the Tokachi-Oki example, Honda et al. (2004) report that the seismic moments obtained from data inversions span a wide range (from $0.8 \times 10^{21}$ to $3.5 \times 10^{21}$ N-m) and that the rupture velocity obtained from different inversions ranges from 2.6 (Nozu and Irikura, 2008) to 4.5 km/s (Yagi, 2004). We did not yet include station-specific site characterizations. Alternatively, one could think of using empirical Green's functions instead of numerical ones (Nozu and Irikura, 2008; Kurahashi and Irikura, 2010), as this would be an approach to include path and site effects without detailed knowledge about the local and regional earth structures.

Regarding the overprediction for several stations in the Wenchuan case, a possible explanation would be an inappropriate modeling of shallow parts of the source. In reality, the rupture is most likely decelerating toward the surface because stress drop is decreasing with decreasing depth. Hence, the shallow part of the fault radiates less kinetic energy. As we neglect this behavior, our approach overestimates the kinetic energy radiated from parts of the fault close to the surface. Kagawa et al. (2004) found from kinematic slip inversion that stress drop at shallow asperities is lower than the stress drop from deep asperities and that the slip velocity is smaller, which would result in a smaller patch rise time. They confirmed this by numerical simulations of ground motions. In addition, Brune and Anoosheshpoor (1998) observe longer rise times and slower particle velocities in a rubber foam experiment when introducing a weak layer. According to their arguments, a significant reduction of the radiated seismic energy at high frequencies could originate from a low value for effective accelerating stress in the shallow weak zone and the velocity-strengthening frictional characteristics of the weak zone. In our simulation, we would have to account for that reduction in rise time by, for example, relating the rise time directly to the seismic moment, which is itself related to the $S$-wave velocity and therefore smaller in layers of low velocity. Consequently, an introduction of low velocities at the surface causes much less energy radiated from the shallow parts of the fault and mainly affects the near-field ground motion. On the other hand, it causes strong amplifications at all stations with site characteristics of soil or alluvium, such that a simulation including this effect would show even stronger overestimations. Overall, in a simulation with implemented soft sedimentary layers, the effect of site amplification and rise-time reduction would oppose each other, and it is difficult to say which contribution dominates. The method is limited by the usage of the simple layered half-space. In future investigations, the rise-time scaling and the refined source

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**Figure 8.** (a) Model bias for Fourier spectra using 145 sites for the Tokachi-Oki earthquake. The continuous line denotes the model bias, the gray shades show ± one standard deviation. Stars show the model bias for the spectral response estimate with the GMPE of Zhao et al. (2006) using a $V_{S30}$ value of 1500 m/s. On the right, the bias related to the PGA value is shown. (b) Comparison of recorded (small circles) and simulated (large circles) PGA plotted as functions of closest distance to the rupture for the Tokachi-Oki earthquake. The estimated PGA from Zhao et al. (2006) is shown as a broken line for $V_{S30} = 1500$ m/s, with the hashed area showing one standard deviation and as a continuous line for $V_{S30} = 320$ m/s with the gray shade showing one standard deviation. Here, only within-event variability is used for the empirical ground-motion prediction. The color version of this figure is available only in the electronic edition.

- It reproduces directivity effects (at least as long as the fault geometry and the hypocenter are correctly determined from data inversion).
- It produces broadband ground motion, that is, the outcome is the seismic ground motion in a broad frequency band including PGA but also with approximately correct duration and phasing.

Previously, the disadvantage of the pure deterministic wave propagation was the need for dense discretization to model deterministically up to high frequencies, as an insufficient discretization step leads to artificial periodicities. However, we partially overcome this drawback using Green's functions for finite rupture sources instead of point sources.
model can be combined with more complicated Earth models, such that their performance can be tested in more realistic environments.

The limited knowledge of the Earth model also plays a role when considering simulations at large distances. At long distances, the factors determining the wave propagation have increasing influence on the ground motion and probably, out of those $Q$, is least known. On the other hand, the waves also travel increasingly larger distances through the depth layers, the parameters of which are globally more constant and, hence, better known on average, as for example the upper mantle. Because the aim of this study was merely to demonstrate the performance of the scaling of the rise time by the observed energy magnitude and the possibility of a deterministic wave propagation at all frequencies, we considered a simple $Q$ model as sufficient.

The scaling of the rise time to the observed energy magnitude provides means to control the rise time without adjusting it manually for each simulation. We showed that the simulations may be realized with changes in parameters (such as rupture velocity) without any interaction with the algorithm. Hence, the proposed rise-time scaling can be used to adjust the energy content radiated at high frequencies using only parameters that can be observed or are obtained by inversions.

The ability of the method to reproduce specific ground motions is limited by its simplicity. If all gross parameters are completely fixed for an earthquake, an existing model bias may not be corrected. A specific target spectrum and waveform may only be achieved by adapting the input parameters. In the near-fault region, the source parameters and slip distribution significantly influence the results while the velocity model is the major cause of model bias in the far field. The rupture velocity may significantly influence the ground motion at all distances.

Conclusions

In this article, we focus on two aspects of ground-motion modeling. First, we employ a combined stochastic source description and deterministic wave propagation approach up to high frequencies. Second, we propose the scaling of the rise time of the source time function to ensure that the modeled...
Figure 10. (a) Same as Figure 5 but for a rupture velocity of $0.75 \times V_S$ instead of $0.8 \times V_S$. The continuous line denotes the model bias regarding the observations, whereas the broken line shows the model bias to the reference simulation shown in Figure 5. Light gray shows standard deviation regarding the observations. Dark gray shows standard deviation regarding the reference simulation shown in Figure 5. (b) Same as (a) but for an increased rupture velocity of $0.87 \times V_S$. The color version of this figure is available only in the electronic edition.

The ground motions simulated for the 2008 Wenchuan and the 2003 Tokachi-Oki earthquakes appear realistic, with appropriate frequency content and peak amplitudes. Many regions of the world, detailed information on the local 3D Earth model is not available, and our model using the layered Earth model is a reasonable alternative. Finally, the energy-constrained rise-time scaling may as well be applied in the context of other approaches, such as 3D models or in combination with empirical Green’s functions.

Data and Resources

Wenchuan data were obtained from the China Earthquake Administration. Strong ground motion data for the Tokachi-Oki earthquake was obtained from the National Research Institute for Earth Science and Disaster Prevention and downloaded at http://www.kik.bosai.go.jp (last accessed October 2012). Processed high-rate GPS data were kindly provided by Kristine Larson. Global Centroid Moment Tensors were obtained from www.globalcmt.org (last accessed February 2014). The energy magnitude for the 2008 Wenchuan earthquake was obtained from http://neic.usgs.gov/neis/eq_depot/2008/eq_080512_ryan/neic_ryan_e.html (last accessed February 2014). Predicted ground motions from Zhao et al. (2006) and Boore and Atkinson (2008) were calculated using scripts by James Bronder and Yoshifumi Yamamoto, respectively (scripts are available at http://www.stanford.edu/~bakerjw/attenuation.html, last accessed June 2014).

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References


Introduction of Small-Scale Variabilities in the Kinematic Description

For completeness, we introduce here the procedure to generate a rediscretized fractal slip model. The generation of the slip model starts with the smoothing of the original slip distribution. First a 2D Fourier transform is performed
to obtain the slip spectrum. In the wavenumber domain, a 2D filter is applied, which has the form

\[
f_1(k) = \begin{cases} 
1 & : \sqrt{k^2_x + k^2_y} < k_{co} - \Delta k_{co} \\
0 & : \sqrt{k^2_x + k^2_y} > k_{co} + \Delta k_{co} \\
\frac{1}{2} \left[ 1 + \sin \left( \frac{\pi(k-k_{co})}{2\Delta k_{co}} \right) \right] & : \text{elsewhere}
\end{cases}
\]

(A1)

in which \(k_{co}\) denotes the combination wavenumber (i.e., the wavenumber at which the original distribution is combined with the random distribution). To make sure that the main asperities are sufficiently visible, it is set to \(k_{co} = 1/(4d_{xold})\), with \(d_{xold}\) being the discretization of the original slip distribution. The resulting slip field is smoothed, such that the edges of the former discretization step are no longer visible. Unfortunately, this filter also introduces periodicity in the resulting slip distribution. The effect is negligible if the original slip distribution has low values at its borders. However, in case of surface ruptures, there is often significant amount of slip at the upper edge of the slip distribution. The filter hence introduces high slip values at the lower edge. We correct this with a taper at the lower limit of the smoothed distribution if the upper fault edge is in a depth lower than 2 km.

Second, a random distribution is created. A random field of numbers with the same dimension as the original distribution is transformed to the Fourier domain. The spectral amplitude is then modified such that the \(k\)-square model is obtained:

\[
u(k) = \begin{cases} 
\frac{1}{k^\alpha} & : \sqrt{k^2_x + k^2_y} < k_c \\
\frac{1}{\sqrt{(k^2_x + k^2_y)}} & : \sqrt{k^2_x + k^2_y} > k_c
\end{cases},
\]

(A2)

with \(u(k)\) being the spectral slip amplitude at wavenumber \(k\) and \(k_c\) as the corner wavenumber. According to Causse et al. (2010), the along-strike corner wavenumber \(k_{cx}\) (in kilometers) is linked to the moment magnitude \(M_w\):

\[k_{cx} = 10^{1.82 - 0.5M_w}.\]

(A3)

Here we neglect that they found slightly different results for the along-dip corner wavenumber and assume \(k_c = k_{cx}\). In the case of the slip distribution, the fractal dimension \(\alpha\) equals 2.0; that is, above the corner wavenumber we obtain a field with a random phase and an amplitude which decreases with the squared wavenumber.

Next, we apply a similar 2D filter as for the original distribution but as a high pass:

\[
f_2(k) = \begin{cases} 
0 & : \sqrt{k^2_x + k^2_y} < k_{co} - \Delta k_{co} \\
1 & : \sqrt{k^2_x + k^2_y} > k_{co} + \Delta k_{co} \\
\frac{1}{2} \left[ 1 + \sin \left( \frac{\pi(k-k_{co})}{2\Delta k_{co}} \right) \right] & : \text{elsewhere}
\end{cases}
\]

(A4)

We add a constant to make sure that all slip along the fault is positive and then apply a taper, such that the slip decreases to zero toward the edges of the fault. The taper forms a plateau at the center of the distribution and decays to 0 at the borders in the form of a quarter-cosine function as described by

\[
t(x) = \begin{cases} 
\cos \left( \frac{\pi(|x-L/2|-\lambda L/2)}{\lambda L/2} \right) & : |x - L/2| \geq \lambda L/2 \\
1 & : |x - L/2| \leq \lambda L/2
\end{cases}
\]

(A5)

This is similar to a Tukey window, but with a quarter-cosine function instead of the half-cosine function usually used. The value for \(\lambda\) used in our algorithm is 0.5. The resulting random slip distribution has a \(k^2\)-decay of amplitude in the wavenumber domain, a random phase, and decays to zero at the border of the distribution.

The smoothed original distribution is then combined with the random distribution to obtain a final distribution. This final distribution contains the essential information from the inverted slip model in the low-wavenumber part and random components in the high-wavenumber part. Its Fourier amplitude spectrum still shows a \(k^2\)-decay at high wavenumbers.

GFZ German Research Centre for Geosciences
Telegrafenberg
D-14473 Potsdam, Germany
katrin.kieling@gfz-potsdam.de
rongjiang.wang@gfz-potsdam.de
sebastian.hainzl@gfz-potsdam.de

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