Acta Geophysica
vol. 62, no. 1, Feb. 2014, pp. 241-258
DOI: 10.2478/s11600-013-0170-9

Sub-monthly Gravity Field Recovery from Simulated Multi-GRACE Mission Type

Basem ELSAKA

Institute of Geodesy and Geoinformation, Bonn University, Bonn, Germany
e-mail: elsaka@geod.uni-bonn.de

National Research Institute of Astronomy and Geophysics, NRIAG, Helwan, Cairo, Egypt

Abstract

Monthly solutions of the current GRACE mission are affected by the aliasing problem. In fact, sub-monthly temporal sampling may reduce the temporal aliasing errors but this will be done at the cost of reduced spatial sampling.

Reducing the effects of temporal aliasing can be achieved by setting two pairs of satellites in different orbital planes. In this paper, we investigate the so-called Multi-GRACE constellation to improve temporal and spatial resolution for the GRACE-type mission without deteriorating accuracy. We investigate two scenarios: the Multi-GRACE ΔM that improves the temporal sampling only and the Multi-GRACE $\Delta\Omega$ that improves the spatial sampling besides the temporal one in time span of only 12 days for the hydrological signal as a time-varying gravity field component.

Our findings indicate that the hydrological signal can be submonthly recovered and the aliasing errors can be reduced as well by increasing temporal resolution (sub-month) via the Multi-GRACE $\Delta\Omega$ constellations.

Key words: gravity field recovery, Multi-GRACE constellation, hydrological signal recovery, aliasing effects.

1. INTRODUCTION

The Gravity Recovery and Climate Experiment (GRACE) mission (Tapley *et al.* 2004) flies in a simple formation consisting of two identical satellites separated from each other by approximately 220 km in the along-track direction. GRACE has been providing a continuous sequence of monthly gravity field solutions and numerous model series, such as ITG-GRACE, EIGEN and GGM series (see, *e.g.*, Mayer-Gürr *et al.* 2011, Förste *et al.* 2008, Tapley *et al.* 2007, respectively), although errors in recovering the high-frequency temporal signals during the processing contaminate its monthly solutions. These error sources are due to the fact that the GRACE mission suffers from under-sampling (time-varying) geophysical signals that have a period less than twice the (orbital) sampling period (of one month), according to the Nyquist sampling theorem. This is considered as a temporal sampling problem on the one hand. Insufficient sub-satellite track pattern produced by GRACE mission is considered also as an error source of spectrum anisotropy in its gravity solutions, and hence is considered as spatial sampling problem on the other hand.

Consequently, considering a single gravity satellite mission (*e.g.*, GRACE mission), one finds that it cannot actually accomplish the global temporal and spatial resolutions together at an accurate level. This issue is analogous to the well known Heisenberg uncertainty principle of spatio-temporal sampling, which explains the relation between the spatial and temporal sampling. The Heisenberg theorem shows a reverse relation that the better the spatial sampling is, the worse the temporal sampling becomes and *vice versa*. This rule can be proved when considering the spatial and temporal scales as (Reubelt *et al.* 2009),

$$D_{\text{space}} = 2\pi/\beta\ , \quad D_{\text{time}} = \alpha\ [\text{rad}]\ , \tag{1}$$

where β stands for the satellite revolutions in α nodal days. The product between the spatial sampling D_{space} and the temporal sampling D_{time}, taking into account the revolution time T_{rev} from the third Kepler's law ($T_{\text{rev}} = \sqrt{GM/a^3}$), reads

$$D_{\text{space}} \cdot D_{\text{time}} = \frac{2\pi\alpha}{\beta \cdot 24 \cdot 60 \cdot 60}\,[\text{s}] = 2\pi T_{\text{rev}}\ [\text{s}]\ . \tag{2}$$

For the low Earth orbit satellites, this product is actually very small and can be considered as constant

$$D_{\text{space}} \cdot D_{\text{time}} = c\ . \tag{3}$$

This relation is regarded as Heisenberg-type rule of spatio-temporal sampling of a satellite (analogously see Dirac 1958). So, a satellite mission with an adequate temporal resolution (*i.e.*, short periodic time span) will provide a worse spatial sampling due to the lack of the coverage of the Earth with satellite tracks. Conversely, sufficient spatial sampling of a satellite mission provides insufficient temporal resolution because of the short periodic signals (*e.g.*, daily and weekly) that will alias into the monthly signals. This effect is known as "aliasing" in the sampling theorem.

The effect of temporal aliasing errors has been considered to be one of the largest error sources for the GRACE mission (Thompson *et al.* 2004, Han *et al.* 2004). They can be reduced based on three primary methods, as mentioned in Wiese *et al.* (2011). The first is to increase sampling more frequently, which can be performed using multiple pairs of satellites. The second is to improve the models of the high frequency signals that are taking place at short periods, such as the atmosphere, ocean, and ocean tides. The third is to consistently adjust the spherical harmonic coefficients of the gravity field by co-estimating the orbital parameters and additional empirical parameters.

This paper aims to recover continental hydrological signal as well as to reduce the affecting temporal aliasing errors via the first method, in which sampling frequency of one pair of GRACE-like satellites increases via another GRACE-like pair to constitute the so-called Multi-GRACE constellations. One has to mention that previous studies investigated the combination of sampling with multiple satellite pairs (*e.g.*, Bender *et al.* 2008, Elsaka and Ilk 2009, 2012, Elsaka 2010, Wiese *et al.* 2012). However, this paper does not aim to reduce the temporal aliasing via multiple pair having different inclinations, as given by Bender *et al.* (2008) and Wiese *et al.* (2012). Additionally, it explores the continental hydrological signal recovery with (*cf.*, Wiese 2011) and without the existence of other high-frequency mass variations (*e.g.*, non-tidal atmosphere, and ocean and ocean tides).

To summarize, a single satellite mission like GRACE mission is not able to detect the mass variations in both space and time without producing a complex effect (aliasing) in its gravity solution. The aliasing effects will indeed be the main problem that future missions will face, especially for the GRACE Follow-on mission, a joint US/German satellite mission, which is currently being developed and is presently scheduled to be launched in 2017 in order to continue the observation of the spatial and temporal variations of the Earth's gravity field. For this reason, an alternative mission configuration using multiple-satellite sensors is highly recommended.

The scope of this paper is firstly to increase the spatial and temporal sampling towards improving the sampling resolutions of the satellite observations through the Multi-GRACE constellations, and secondly, to investigate if sub-monthly sampling provides any improvements in the gravity field solution of a temporal signal such as in hydrology.

This paper is organized as follows: Section 2 reviews briefly a solution for solving the sampling problem through the Multi-GRACE constellation with two mission scenarios: Multi-GRACE ΔM and Multi-GRACE $\Delta\Omega$. The selection of the temporal resolution for the sub-monthly gravity field solution is outlined in Section 3. Then, the procedures of the full-scale simulation scenarios using GROOPS software, which has been developed by the Group of Bonn University, are described in Section 4. The results for the static and temporal gravity field concerning the hydrological recovery and its aliasing errors are discussed in Section 5. Finally, on this basis, a general conclusion relevant for the analysis of the observations for new gravity satellite missions is outlined in Section 6.

2. SELECTION OF MULTI-GRACE CONSTELLATION

To overcome the sampling problem for reducing the temporal aliasing errors, a straightforward solution has been implemented by considering two multiple satellites' orbits of GRACE-type, flying simultaneously. This can be accounted for by combining two or more one-dimensional (along-track) observations. In other words, one can sample the globe at essentially different locations at the same time using the observations of multiple GRACE-like formations (or Multi-GRACE constellation as referred in the sequel). Basically, two constellations are investigated within this paper regarding the two-satellite GRACE-like formation (see Fig. 1). One of these constellations is composed of two pairs of GRACE-like formation with only a temporal shift. In other words, the two pairs sample with the same spatial sampling but are different in time. This means that the two pairs will pass a certain latitude at two epochs. Since the minimum achievable ground track distance of one day is approximately 2600 km (achieved by a mean anomaly differ-

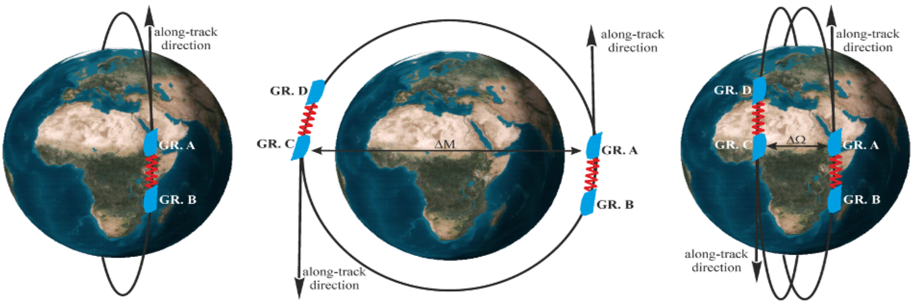

Fig. 1. Multi-GRACE constellation. Left: the two-satellite GRACE, middle: Multi-GRACE ΔM, and right: Multi-GRACE $\Delta\Omega$. The axis X stands for along-track directions.

ence of 180°), the constellation will have two formations of the same orbital elements which differ in the mean anomaly with $\Delta M = 180°$. The second constellation is additionally designed with a different spatial sampling. This can be achieved with two pairs of satellite formations having different right ascension of ascending node angles (RAAN) (*i.e.*, $\Delta\Omega$). Elsaka (2010) selected $\Delta\Omega$ fulfilling the condition that the sub-satellite tracks of the second formation shall be placed halfway between those of the first orbit to obtain a homogeneous spatial sampling (*i.e.*, $\Delta\Omega = 180°$). This means that the sub-satellite tracks can be tuned up to one half of the Earth's circumference. However, this constellation has as drawback a possible collision risk. A more recent study by Wiese (2011) showed that the best solutions are obtained when $\Delta\Omega$ is set as:

$$\Delta\Omega_{12} = \delta + \varepsilon , \tag{4}$$

with

$$\delta = \pi\left(1 + \omega_e \sqrt{\frac{a^3}{GM}}\right) , \tag{5}$$

and

$$\varepsilon = \frac{2\pi}{\beta}\left(\frac{1}{2} - \left(\frac{\beta\delta}{2\pi} - \left\lfloor \frac{\beta\delta}{2\pi} \right\rfloor\right)\right) , \tag{6}$$

where δ shifts the relative node between the two satellites pairs exactly 180° from each other plus the distance it takes for the Earth to rotate during one-half of a satellite revolution. As a correction, factor ε is added. The latter term allows the sub-satellite tracks of the second pair of satellites to fill in the gaps at the equator from those tracks of the first pair of satellites. Therefore, a RAAN angle $\Delta\Omega = 180.659°$ has been selected here, as shown in Table 1, in order to firstly obtain a more resulting homogeneous ascending and descending distribution of the satellite observations, and secondly, to be far away from any possibility of collision risk. The other Multi-GRACE ΔM constellation has no collision risk in space since it flies on the opposite side of the Earth (see Fig. 2).

It should be mentioned here that the Bender configuration (Bender *et al.* 2008) has not been taken into consideration within this paper. This constellation type has been introduced by considering one GRACE pair on a polar orbit (*e.g.*, $i = 90°$) and another GRACE pair on an orbit of significantly lower inclination ($i = 63°$). An advantage of this constellation is that a sufficient spatial sampling will be achieved at low latitudes due to the dense sub-satellite tracks of the satellites pair at relatively low latitudes (63°). Concurrently, this can be used to combine the data of two non-isotropic measurements of the two pairs to overall increased isotropic ones. Bender constella-

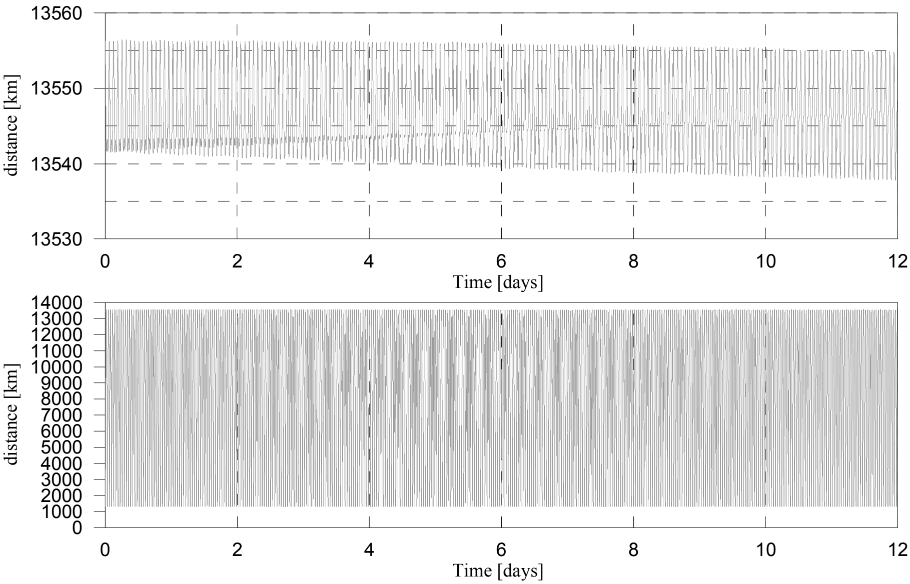

Fig. 2. Distances between Satellite GRACE A and GRACE C of Multi-GRACE ΔM (top) and Multi-GRACE $\Delta\Omega$ (bottom) constellations.

tion has been also investigated by Visser *et al.* (2010), who showed that the aliasing due to ocean tide model errors can be reduced with flying more tandems (*cf.*, BEN12 and BEN1).

However, a drawback occurs when implementing Bender-type constellation at the same orbital altitude. This drawback is represented in the different repeat periods (due to the different inclinations) of the sub-satellite tracks of the two satellite pairs forming the Bender-type, and hence, different sub-satellite track distances (plus different orbital altitudes as well) have to be selected for each pair to overcome this problem. Furthermore, different inclinations and altitudes make the precession rate of the nodal drift (Kaula 1966, P39) between the two satellite pairs to change:

$$\frac{d\Omega}{dt} = \frac{3nC_{20}R_e^2 \cos i}{2(1-e^2)^2 a^2}, \tag{7}$$

where a, i, and e are the Keplerian altitude, inclination and eccentricity, respectively, n is the mean motion of the satellite, and R_e is the Earth's radius.

Another drawback for the Bender-type constellation away from the repeat period, concerning the spatio-temporal resolution, is that the desire to obtain full Earth's coverage requires a month of sampling for each satellite pair to fill all data gaps at the pole (for the polar pair). This means that the constellation orbital sampling frequency is still less than the highest frequency of the temporal gravity variations. According to the Nyquist sampling theorem, these temporal variations arising from the under-sampling geophysical signals will alias strongly. This can humble this mission.

Strengthening the sampling frequency using shorter orbital repeat period of 13 days (at $i = 72°$ and $a = 299$ km), as presented by Wiese *et al.* (2012), can overcome this problem but this will be at the cost of the spatial sampling; as mentioned, there would be polar gaps in coverage should the polar pair of satellites fail (see Fig. 3, top).

In this paper, it was interesting to keep all satellite periods as same as possible in the satellite mission, and therefore the results of this constellation type (*i.e.*, Bender-type) have not been taken into consideration. Maintaining

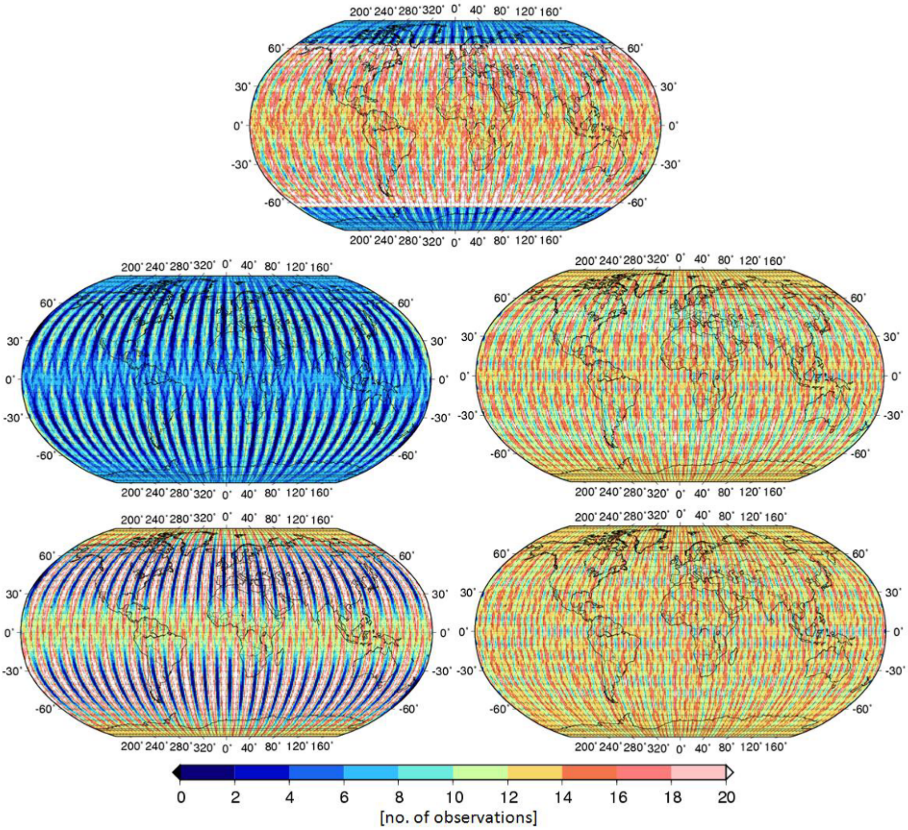

Fig. 3. Ascending and descending distribution of satellite observations of the Bender configuration (top), two GRACE 12-day satellites (middle-left), two GRACE 24-day satellites (middle-right), Multi-GRACE ΔM constellation (bottom-left), and Multi-GRACE $\Delta\Omega$ constellation (bottom-right). Colour version of this figure is available in electronic edition only.

satellite periods as same as possible makes the orbital sampling for each satellite pair to be identical, and hence, they can detect the temporal variations (in supporting way) at the same time.

Further drawback regarding the rationale for selecting Bender configuration is the relatively higher cost option than the mid-cost two polar pairs of satellite, as the Bender architecture requires two launch vehicles to force the inclined pair directed East-West. This was a further reason making the Multi-GRACE constellation of two polar pairs of satellites more optimal in this paper.

3. SELECTION OF "SUB-MONTHLY" TEMPORAL RESOLUTION

The accuracy of the estimated Earth's gravity field has benefited significantly from the current satellite missions. However, their duration is limited. This limitation is considered as the sampling problem which the gravity field recovery is still facing nowadays. The spatial and temporal sampling is an important factor for the gravity field analysis because it determines the obtainable accuracy of the potential coefficients. Therefore, adequate orbital parameters have to be selected in order to obtain sufficient sampling.

To examine the Multi-GRACE constellations, in this paper the orbital parameters have been chosen to yield polar and approximate circular orbits. The orbital altitude has been set to 400 km, as shown in Table 1. In this study, 24-day sampling for the GRACE formation, and 12-days sampling for the Multi-GRACE constellations have been selected. This is because at the chosen satellite's altitude of 400 km, each satellite pair will complete a full coverage of 360 revolutions after exactly 23.14 days. This means that in 24 days, each latitude compartment will be covered by satellite observations.

Table 1

Multi-GRACE constellations with their orbital parameters

Constellation / Orbital parameter	Multi-GRACE ΔM				Multi-GRACE $\Delta\Omega$			
	GRACE A	GRACE B	GRACE C	GRACE D	GRACE A	GRACE B	GRACE C	GRACE D
A [km]	6778.137	6778.137	6778.137	6778.137	6778.137	6778.137	6778.137	6778.137
E	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
i [°]	90	90	90	90	90	90	90	90
ω [°]	0	0	0	0	0	0	0	0
Ω [°]	0	0	0	0	0	0	183.659	183.659
M [°]	0	359.15	180	179.15	0	359.15	180	179.15

Figure 3 displays the ascending and descending distribution of the sub-satellite tracks of the two-satellite and four-satellite GRACE configurations in 24 and 12 days in addition to the Bender configuration (Fig. 3, top). The 24-day GRACE formation (Fig. 3, middle-right) has sufficient coverage of sub-satellite tracks while the 12-day formation (Fig. 3, middle-left) suffers from the deteriorating satellite coverage since its spatial sampling is approximately half of that of the 24-day sampling. The Multi-GRACE ΔM has twice number of satellite observations compared to the 12-day formation, which differs only in the temporal sampling (see Fig. 3, bottom-left). In contrast, the Multi-GRACE $\Delta\Omega$ (Fig. 3, bottom-right) has the temporal resolution the same as the GRACE 12-day formation but its spatial resolution is similar to the GRACE 24-day one. With the latter constellation, one can observe that sufficient spatial resolution is achievable with a temporal resolution of only 12 days. It has to be mentioned here that the temporal resolution of 12 days is a suitable sub-monthly resolution for the satellite-gravimetric mission of satellite altitude between 250 to 500 km.

4. SIMULATION SCENARIOS

In order to compare the performance of the four configuration scenarios, full-scale numerical simulations using the Gravity Recovery Object Oriented Programming System (GROOPS) software have been performed. GROOPS has been developed in the Astronomical, Physical, and Mathematical Geodesy group at Bonn University to estimate gravity field parameters from satellite measurements, and it is routinely used to compute, *e.g.*, the ITG series of GRACE solutions (see Mayer-Gürr 2006). The detailed procedures of these numerical simulations have already been described in Elsaka (2010).

For this study, all orbits were integrated at 5-second steps using ITG-GRACE03s (Mayer-Gürr *et al.* 2010) as the background mean gravity field model up to degree and order 180. In addition, temporal variations of gravity have been simulated using background models for ocean tidal forces as well as “non-tidal” atmospheric, “non-tidal” oceanic, and hydrologic mass variations. In the gravity analysis step, a second, differing set of force models has been used to simulate incomplete de-aliasing of the non-tidal atmospheric and oceanic masses.

Error-free inter-satellite measurements (relative distance, velocity, and acceleration) of each formation were then computed from the orbits. In processing scheme, the orbits (of 24 and 12 days) are first divided into short arcs of 35 minutes length. The spherical harmonics partials and the arc-wise boundary positions are then created using the integral equation approach (Mayer-Gürr 2006), so that one obtains a system of observation equations per arc. The solution of the accumulated normal equations then provides a set of stocks coefficients up to degree and order (d/o) 90, which can be represented in various representations, *e.g.*, degree variances and cumulative geoid errors.

It would be mentioned that the normal equation systems of GRACE 24-day and 12-day formations have been solved independently. Firstly, the normal equation of GRACE 24-day observations has been solved. Secondly, the corresponding normal equation for GRACE 12-day formation has been also solved independently. Thirdly, the latter normal equation has been combined with the other GRACE 12-day formations with a RAAN shift on the one hand and a mean anomaly shift on the other hand to form the Multi-GRACE $\Delta\Omega$ and Multi-GRACE ΔM solutions, respectively.

It is worthwhile to mention also that no noisy measurements have been used in this study in order to only investigate the recovery as well as the aliasing error of temporal gravity field without the contamination of observations errors. For instance, Elsaka (2010) showed that selecting relatively higher noise levels (*e.g.*, SST K-Band of micro levels) negatively affect the recovery of time-variable signals (*e.g.*, ocean tides). As a result, the recovery as well as the aliasing effects of the hydrological signal, whose accuracy is lower than the ocean tides, may underlie the measurement errors. Therefore, the error-free case has been only considered in this paper. Table 2 shows the simulations of temporal scenarios, as indicated in the following sections.

Table 2

Simulation scenarios

Temporal fields	Hydrological recovery		Hydrological aliasing	
	Nominal	Reduction	Nominal	Reduction
Atmosphere	ECMWF	ECMWF	ECMWF	ECMWF
Ocean	OMCT	OMCT	OMCT	OMCT
Ocean tide	EOT08a	EOT08a	EOT08a	EOT08a
Hydrology	GLDAS		GLDAS	LaD

5. RESULTS

First one has to mention that the maximum resolvable degree ($n_{\max}$) of the SH coefficients of all gravity field solutions have been determined in this paper up to degree and order (d/o) 90. This is according to the Columbo–Nyquist (Columbo 1984, paragraph 3.6a, p. 117) rule of thumb, which states that the maximum resolvable degree in the gravity field solution should be smaller than (or equal) half of the number of orbital revolutions of the pair of satellites to avoid aliasing, *i.e.*, $n_{\max} \leq \beta/2$, where β is the number of satellite revolutions (*i.e.*, sub-satellite tracks). During 24 days, GRACE produces

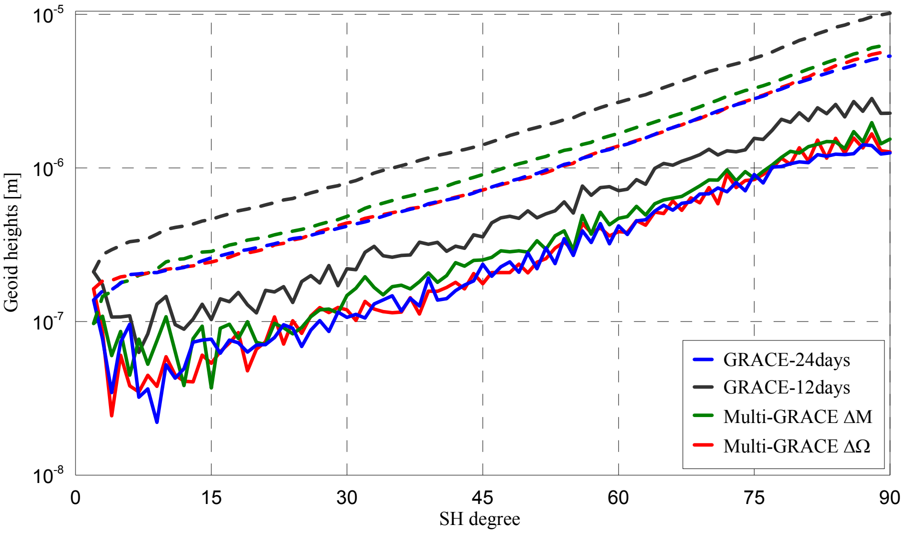

Fig. 4. Geoid DDV between true solution (ITG-GRACE03s) and the solutions obtained from the simulated formation flights at spherical harmonics d/o 90. Solid curves represent the degree variances and dashed ones for the cumulative geoid errors. Colour version of this figure is available in electronic edition only.

approximately 360 orbits and the 12-day produces 180 orbits. Therefore, the maximal gravity field solution for 12 days can be retrieved at degree $n_{\max} = 90$. Any spatial pattern with $n > 90$ will be under-sampled and will alias in the solution.

A valuable tool to examine the differences (*i.e.*, errors) between the true signal and the recovered signal in the spectral domain in a global sense is using the degree variances (see Figs. 4 and 6) based on:

$$\Delta\sigma_n^2 = R_E \sum_{m=0}^{n} \left(\Delta c_{nm}^2 + \Delta s_{nm}^2 \right) \, , , \tag{8}$$

where

$$\Delta c_{nm} = \left(c_{nm} \right)_{\text{estimated}} - \left(c_{nm} \right)_{\text{reference}} ,$$
$$\Delta s_{nm} = \left(s_{nm} \right)_{\text{estimated}} - \left(s_{nm} \right)_{\text{reference}} ,$$

as well as in terms of cumulative geoid errors as:

$$\Delta\sigma_{nm(\text{cumulative})}^2 = R_E \sum_{n=2}^{n_{\max}} \sum_{m=0}^{n} \left(\Delta c_{nm}^2 + \Delta s_{nm}^2 \right) , \tag{9}$$

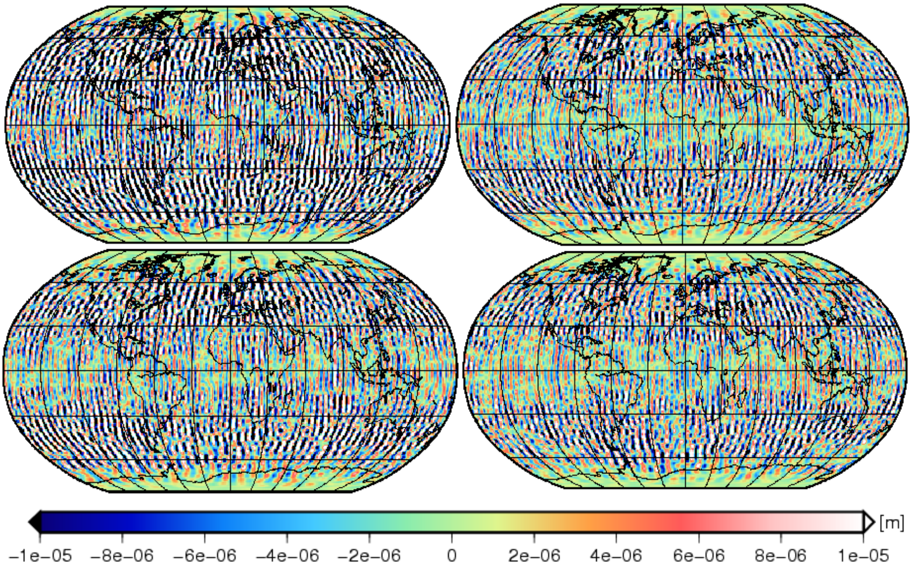

Fig. 5. Static gravity field solutions of GRACE 12-day (top-lift), GRACE 24-day (top-right), Multi-GRACE ΔM (bottom-left), and Multi-GRACE ΔΩ(bottom-right), $n_{\max} = 90$. Colour version of this figure is available in electronic edition only.

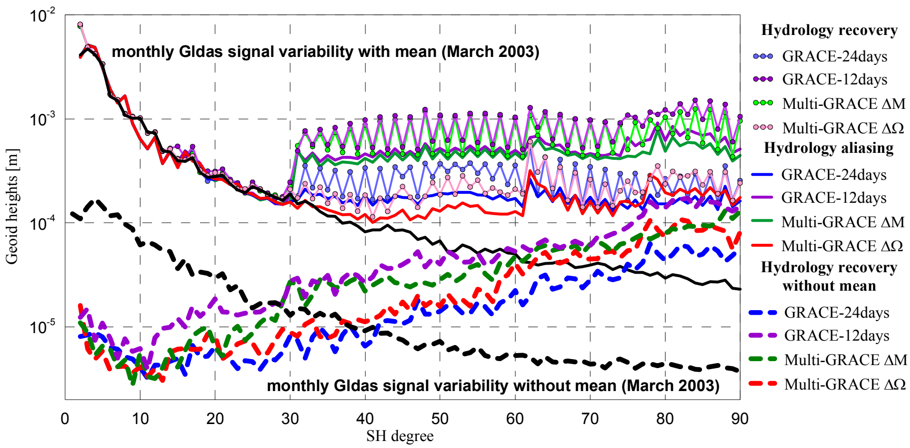

Fig. 6. The estimated temporal gravity solutions concerning the hydrological aliasing. Colour version of this figure is available in electronic edition only.

where R_E represents the Earth's radius. Spatially, the results obtained in this paper are plotted on $1° \times 1°$ grid on an Earth's map in terms of geoid errors, as shown in Figs. 5 and 7.

5.1 Static scenario

All satellite orbits have been integrated every 5 seconds as mentioned in the procedures of the simulation scenarios (Section 4). The ITG-GRACE03s (a product of the GRACE mission) has been used within the integration step as a reference gravity field model up to a SH d/o $n = 90$. In addition, the GLDAS hydrological model (Rodell *et al.* 2004) has been added as a background model. Further time variable and force models related to the ocean tides, atmospheric-oceanic masses, atmospheric air drag, solar radiation pressure, and Earth albedo were not included. The GLDAS model used in the orbit integration step has been used again during the gravity field estimation process in order to only retrieve the static gravitational signal for each mission scenario.

The static gravity field solutions are given spectrally in Fig. 4 in terms of degree variances of the geoid heights and in terms of accumulated geoid errors as well. Spatially, the results are plotted in Fig. 5 in terms of geoids heights on an Earth's man. It is clearly shown that the spatial sampling plays a very important role for improving the gravity solutions when comparing the error spectrum of the GRACE 24-day solution with the GRACE 12-day solution (Fig. 4). As the spatial sampling becomes poorer (in case of a 12-day scenario), a less accurate gravity solution is obtained. This is already shown in Fig. 5, in which the GRACE 24-day displays less striping pattern having root mean square (RMS) of geoid errors of 5.9 μm with respect to the GRACE 12-day with RMS of geoid errors of 11.3 μm.

Now one considers the Multi-GRACE ΔM constellation, which has same spatial sampling as the GRACE 12-day formation but with a temporal sampling twice that of the 12-day formation (see Fig. 3). Correspondingly, the recovered static ΔM solution with RMS of 7.4 μm surpasses the 12-day one at all wavelength ranges of the gravity spectrum (see Fig. 4). This indicates that the ΔM solution is more accurate than the 12-day solution.

The Multi-GRACE $\Delta\Omega$ constellation has the advantage of an adequate spatial coverage similar to the GRACE 24-day formation but with an improved temporal resolution of only 12 days. This constellation provides an accurate solution with RMS of geoid errors of 5.8 μm approximately same as the GRACE 24-day solution. This can be seen obviously from their error spectrum and cumulative geoid errors in Fig. 4.

These sets of experiments emphasize that a static gravity solution is now from a 12-day mission instead of 24-day single GRACE mission (for maximum d/o of 90).

5.2 Temporal scenario

Basically, the GRACE mission has been designed to recover the long wavelength part of the gravitational field variations (SH $n_{\max} \leq 30$), which are estimated monthly. This study explores for the first time how hydrological signals can be recovered in a sub-monthly period of only 12 days via the simulated observations of Multi-GRACE constellations without applying any of smoothing techniques (see, *e.g.*, Swenson and Wahr 2006, Kusche 2007) on the one hand. Secondly, this study investigates if these constellations are able to reduce the aliasing errors of the hydrological variations.

As the Global Land Data Assimilation System (GLDAS) global hydrological model was used in the orbit integration step, another hydrological model (Land Dynamics, LaD) has been assumed in the gravity estimation step to be representative of the aliasing errors, which were derived from model differences.

One month (March 2004) has been selected for the recovery (as well as for the study of aliasing errors) of the hydrological signal from the simulated observations of the Multi-GRACE constellations. The corresponding results of these two scenarios are given in terms of degree variances in the spectral domain as shown in Fig. 6 and are plotted spatially on an Earth's map up to SH degree and order 90 (Fig. 7). Table 3 indicates the geoidal statistical values from degree 2 to 90.

Figure 6 shows mainly the degree variances per degree in terms of geoid heights for three hydrological scenarios. The first scenario, as given by the dashed curves, explains the hydrological variability after subtracting the mean monthly hydrological signal from the gravity solutions. It is already well known that the spectra of the temporal signal (hydrological here) decrease with increasing the spherical harmonics degree, while those error spectra of the GRACE and Multi-GRACE mean monthly solutions increase with the increasing SH degree. The intersection of both spectra types indicates that, *e.g.*, GRACE 24-day can recover the hydrological variations up to SH degree and order ≤ 41, while GRACE12-day can recover them up to only SH degree and order ≤ 29.

In the second scenario, the monthly mean signal of GLDAS has not been removed during the calculations. This amplitude spectrum (containing its mean signal) is an order of magnitude worse than that one which does not contain the mean hydrological signal. The GRACE 12-day formation yields the largest variances as seen from Table 2. This result was expected since the formation suffers from poor spatial resolution. Despite the poor spatial coverage of the Multi-GRACE ΔM constellation, it has provided a slightly refined solution w.r.t. the GRACE 12-day formation. This is due to the double number of satellite orbits of the Multi-GRACE ΔM constellation than the GRACE-type 12-days FF. The two-satellite GRACE 24-day formation as well as the Multi-GRACE $\Delta\Omega$ constellation provides approximately the most accurate gravity solutions of lowest geoid variances. This emphasizes that

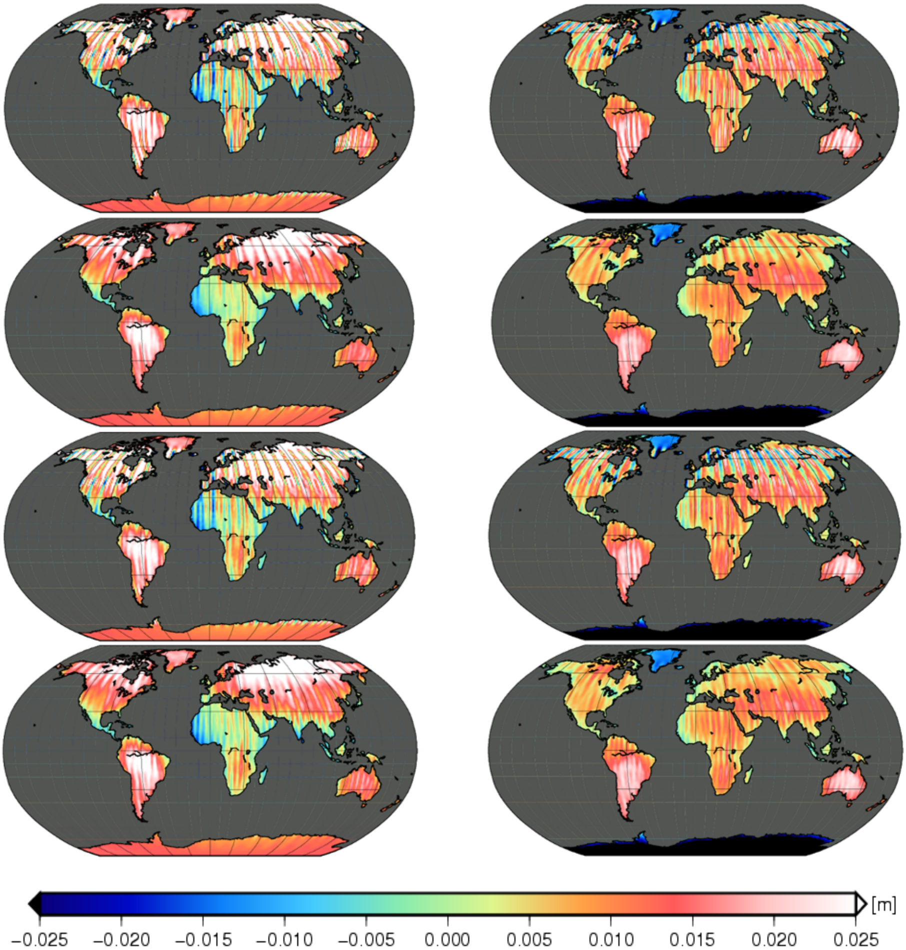

Fig. 7. Hydrological recovery indicated in geoid heights: from top to bottom: GRACE 12-day, GRACE 24-day, Multi-GRACE ΔM, and Multi-GRACE $\Delta\Omega$. The left and right columns represent the hydrological recovery case without and with aliasing errors, respectively, SH $n_{\max} = 90$. Colour version of this figure is available in electronic edition only.

a sub-monthly temporal recovery (in 12 days) of the hydrological signal is now achievable from satellite observations in a shorter Nyquist period.

Regarding the third scenario (the hydrological aliasing), one can see that the Multi-GRACE $\Delta\Omega$ constellation has reduced model differences (representing aliasing errors) w.r.t. the other configurations (see Table 2). Figure 7 shows that in hydrology the largest variations are visible over the continents, particularly those recognizable amplitudes in the large tropical river basins like the Amazon in South America, the Great Lakes in USA, the Congo and

Table 3

Geoid statistical values of the different gravity solutions as determined by Multi-GRACE constellations due to the hydrological signal (recovery and aliasing) up to SH degree $n = 90$

Constellation \ Geoid values [mm]	Hydrological recovery				Hydrological aliasing			
	RMS	Avg.	Min.	Max.	RMS	Avg.	Min.	Max.
GRACE 12-day	14.01	11.22	–68.9	67.5	10.65	7.85	–46.8	30.9
GRACE 24-day	11.51	9.62	–25.3	43.5	9.51	6.93	–46.5	24.0
Multi-GRACE Δ*M*	13.28	10.62	–52.6	69.2	10.34	7.61	–46.8	30.3
Multi-GRACE ΔΩ	11.65	9.80	–27.7	38.4	9.47	6.92	–46.2	23.0

the Niger in Africa, and the Ganges in India. The improvements are provided by the Multi-GRACE ΔΩ constellation.

6. CONCLUSION

This study has investigated the trade-off between temporal and spatial resolution for GRACE-like satellite missions without deteriorating accuracy via the so-called Multi-GRACE constellations. Two basic constellations, Multi-GRACE Δ*M* and ΔΩ, have been investigated within this paper. One of them (Multi-GRACE Δ*M*) improved the temporal resolution without sacrificing in the spatial resolution and the other improved the spatial sampling without losing the temporal resolution (Multi-GRACE ΔΩ).

It was found that in an idealized case, both Multi-GRACE Δ*M* and ΔΩ determine the static gravity field better than the GRACE12-day formation. Additionally, the ΔΩ constellation determines the geoid heights with same accuracy as the GRACE24-day with higher temporal resolution. The sub-monthly gravity determination from the Multi-GRACE constellations has been found to be temporal resolution of 12 days.

It has been also found that the ability to recover the temporal hydrological signal is now achievable in only 12 days via the Multi-GRACE ΔΩ, with the same accuracy as in 24 days. Moreover, the ΔΩ configuration reduces more aliasing errors w.r.t. the two-satellite GRACE 24-day formation.

One concludes from this that the Multi-GRACE ΔΩ leads to refined recovery of the static and time variable (*e.g.*, continental hydrology) gravity field with great potential and to a reduction of the aliasing effects. Additionally, the results from this paper suggest that a future mission is highly required to merge additional information (*e.g.*, two along-track components) in order to improve the static and temporal gravity solutions. Definitely, this can help to recover the time-varying gravity field signals in short periods and hence to reduce their high frequency effects causing the aliasing problem.

References

Bender, P.L., D.N. Wiese, and R.S. Nerem (2008), A possible dual-GRACE mission with 90 degree and 63 degree inclination orbits. **In:** *Proc. 3rd Int. Symp. on Formation Flying, Missions and Technologies, ESA/ESTEC, 23-25 April, Noordwijk, The Netherlands*, 1-6.

Columbo, O.L. (1984), The global mapping of gravity with two satellites, Netherlands Geodetic Commission, Publications on Geodesy, Vol. 7, No. 3.

Dirac, P.A.M. (1958), *The Principles of Quantum Mechanics*, 4th ed., The International Series of Monographs on Physics, Vol. 27, Oxford University Press, Oxford.

Elsaka, B. (2010), Simulated satellite formation flights for detecting the temporal variations of the Earth's gravity field, Ph.D. Thesis, University of Bonn, Germany.

Elsaka, B., and K.-H. Ilk (2009), Simulated multiple formation flights for future gravity field recovery, *Geophys. Res. Abstr.* **11**, EGU General Assembly 2009, Abstr. no. EGU2009-529.

Elsaka, B., J. Kusche, and K.-H. Ilk (2012), Recovery of the Earth's gravity field from formation-flying satellites: Temporal aliasing issues, *Adv. Space Res.* **50**, 11, 1534-1552, DOI: 10.1016/j.asr.2012.07.016.

Förste, C., F. Flechtner, R. Schmidt, R. Stubenvoll, M. Rothacher, J. Kusche, H. Neumayer, R. Biancale, J.-M. Lemoine, F. Barthelmes, S. Bruinsma, R. König, and U. Meyer (2008), EIGEN-GL05C – A new global combined high-resolution GRACE-based gravity field model of the GFZ-GRGS cooperation, *Geophys. Res. Abstr.* **10**, EGU General Assembly 2008, Abstr. no. EGU2008-A-03426.

Han, S., C. Jekeli, and C. Shum (2004), Time-variable aliasing effects of ocean tides, atmosphere, and continental water mass on monthly mean GRACE gravity field, *J. Geophys. Res.* **109**, B4, 403, DOI: 10.1029/2003JB002, 501.

Kaula, W.M. (1966), *Theory of Satellite Geodesy: Applications of Satellites to Geodesy*, Blaisdell Publishing Company, Waltham, 124 pp.

Kusche, J. (2007), Approximate decorrelation and non-isotropic smoothing of time-variable GRACE-type gravity field models, *J. Geodesy* **81**, 11, 733-749, DOI: 10.1007/s00190-007-0143-3.

Mayer-Gürr, T. (2006), Gravitationsfeldbestimmung aus der Analyse kurzer Bahnbögen am Beispiel der Satellitenmissionen CHAMP und GRACE, Ph.D. Thesis, University of Bonn, Germany (in German).

Mayer-Gürr, T., A. Eicker, E. Kurtenbach, and K.-H. Ilk (2010), ITG-GRACE: Global static and temporal gravity field models from GRACE data. **In:** F.M. Flechtner, T. Gruber, A. Güntner, M. Mandea, M. Rothacher, T. Schöne, and J. Wickert (eds.), *System Earth via Geodetic-Geophysical*

Space Techniques, Advanced Technologies in Earth Sciences, Springer, Berlin Heidelberg, 159-168, DOI: 10.1007/978-3-642-10228-8_13.

Mayer-Gürr, T., E. Kurtenbach, A. Eicker, and J. Kusche (2011), The ITG-Grace 2010 gravity field model, Institute of Geodesy and Geoinformation, Bonn University, Bonn, Germany, http://www.igg.uni-bonn.de/apmg/ index.php.

Reubelt, T., N. Sneeuw, and M.A. Sharifi (2009), Future mission design options for spatio-temporal geopotential recovery. **In:** *Proc. IAG Int. Symposium on Gravity, Geoid and Earth Observation, 23-27 June 2008, Crete, Greece*.

Rodell, M., P.R. Houser, U. Jambor, J. Gottschalck, K. Mitchell, C.-J. Meng, K. Arsenault, B. Cosgrove, J. Radakovich, M. Bosilovich, J.K. Entin, J.P. Walker, D. Lohmann, and D. Toll (2004), The global land data assimilation system, *Bull. Am. Meteor. Soc*. **85**, 3, 381-394, DOI: 10.1175/BAMS-85-3-381.

Swenson, S, and J. Wahr (2006), Post-processing removal of correlated errors in GRACE data, *Geophys. Res. Lett*. **33**, 8, L08402, DOI: 10.1029/2005 GL025285.

Tapley, B., S. Bettadpur, M. Watkins, and C. Reigber (2004), The gravity recovery and climate experiment: Mission overview and early results, *Geophys. Res. Lett*. **31**, 9, DOI: 10.1029/2004GL019920.

Tapley, B., J. Ries, S. Bettadpur, D. Chambers, M. Cheng, F. Condi, and S. Poole (2007), The GGM03 mean earth gravity model from GRACE. **In:** *American Geophysical Union, Fall Meeting 2007*, Abstr. no. G42A-03.

Thompson, P.F., S.V. Bettadpur, and B.D. Tapley (2004), Impact of short period, non-tidal, temporal mass variability on GRACE gravity estimates, *Geophys. Res. Lett*. **31**, 6, 619, DOI: 10.1029/2003GL019285.

Visser, P.N.A.M., N. Sneeuw, T. Reubelt, M. Losch, and T. Van Dam (2010), Spaceborne gravimetric satellite constellations and ocean tides: aliasing effects, *Geophys. J. Int*. **181**, 2, 789-805, DOI: 10.1111/j.1365-246X.2010.04557.x.

Wiese, D.N. (2011) Optimizing two pairs of GRACE-like satellites for recovering temporal gravity variations. Ph.D. Thesis, Univ. Colorado, Boulder, USA.

Wiese, D.N., P. Visser, and R.S. Nerem (2011), Estimating low resolution gravity fields at short time intervals to reduce temporal aliasing errors, *Adv. Space Res*. **48**, 6, 1094-1107, DOI: 10.1016/j.asr.2011.05.027.

Wiese, D.N., R.S. Nerem, and F.G. Lemoine (2012), Design considerations for a dedicated gravity recovery satellite mission consisting of two pairs of satellites, *J. Geodesy* **86**, 2, 81-98, DOI: 10.1007/s00190-011-0493-8.

Received 21 January 2013
Received in revised form 3 June 2013
Accepted 13 June 2013