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Validation of recent GOCE/GRACE geopotential models over Khartoum state – Sudan

Research Article

A. Abdalla[1]*, H. H. Fashir[2], A. Ali[3], D. Fairhead[4]

1 National School of Surveying, Faculty of Sciences, University of Otago, Dunedin, New Zealand
2 Engineering Division, Transitional Darfur Regional Authority, Khartoum, Sudan
3 Survey Department, Ministry of Planning and Physical Development, P.O Box 7265, Khartoum, Sudan
4 GETECH Group plc, Elmete Hall, Elmete Lane, Roundhay, Leeds LS8 2LJ, UK

Abstract:
This paper evaluates a number of latest releases of GOCE/GRACE global geopotential models (GGMs) using the GPS-levelling geometric geoid heights, terrestrial gravity data and existing local gravimetric models. We investigate each global model at every 5 degree of spherical harmonics. Our analysis shows that the satellite-only models derived by space-wise and time-wise approaches (SPW_R1, SPW_R2 TIM_R1 and TIM_R2), GOCO01S together with EGM08 (combined model) are very distinct and consistent to the local data, which guarantees one of them to be selected as the best of candidate models and then to be utilized in our further geoid studies. One of Satellite-only models will be employed for acquiring the long wavelength geoid component which is one of major steps in the geoid determination. EGM08 will be used to compensate and restore the missing gravity data points in the un-surveyed parts within the target area. We expect further improvements in geoid studies in Sudan due to the improved medium wavelength part of the gravity field from GOCE mission.

Keywords:
elevation • geoid heights • GPS-levelling data • global geopotential models • terrestrial gravity data

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1. Introduction

As many countries Sudan lacks adequate data for compilation of high resolution geoid, there are no enough GPS-levelling points yet, and moreover, most of the existing datasets is confidential and inaccessible. The available gravity dataset is only covering one-third of the country's area.

The attempts to determine geoid in Sudan have been started when Adam (1967) conducted a geoid study using astrogeodetic methods. However, the lack of data from the neighboring countries and the large gaps between measurements inside the country did not help to provide a proper definition to Sudan geoid. Salih (1983 and 1985) computed the geoid in Sudan using astrogeodetic and Satellite methods. The astrogeodetic observations were collected in 1937 while the satellite data were collected by Doppler receivers in 1975, 1978 and 1981. The results from the two methods were compared to each other and the comparison showed remarkable differences over the poor data areas in the eastern and western parts of Sudan, while the surveyed areas (from the north to the south) along the arc of 30° had better consistencies. Another study was conducted by Salih et al. (1990) to determine the shape of the geoid in Sudan using consecutive Doppler observations collected (from 1975 to 1986) by Transit Satellite. The geoid shape was described, however no information were reported regarding accuracy (see Fashir and Abdalla, 1991).

Historically, Sudan has two gravimetric geoid models; the first model (Geoid91) based on Geodetic Reference System 1980 (GRS80) was computed by Fashir (1991), the model was computed using the modified Stokes kernel (Vaníček et al., 1986). During the computation process of the first gravimetric geoid model for Sudan, the only existing reliable satellite only geopotential

*E-mail: ahmed.abdalla@otago.ac.nz, ahmed.abdalla@live.se

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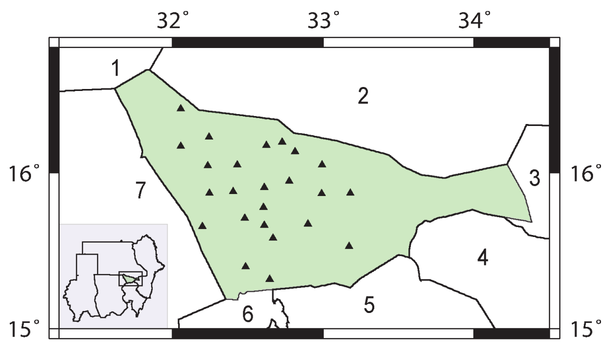

Figure 1. Location and boundary of Khartoum State (in green), GPS-levelling points and the surrounding states, 1) Northern, 2) Nile River, 3) Kassala, 4) Gadaref, 5) Gezira, 6) White Nile, 7) Northern Kordofan.

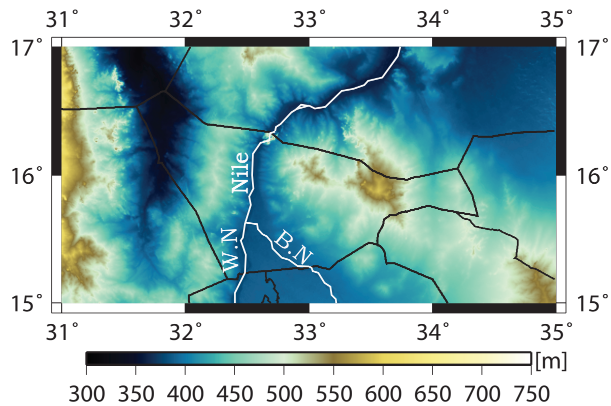

Figure 2. The digital terrain model of Khartoum state and the Nile river and its two major tributaries the Blue and white Niles (B.N and W.N).

model was the Goddard Earth Model (GEM-T1) potential coefficients (Marsh et al., 1987) complete to degree and order 36.This implies that the smallest gravity field features represented in GEM-T1 model has a spatial resolution of 5 degree or approximately 555 km. The model was evaluated by Doppler data heights with a mean difference of 0.43m and RMS 1.93m. The second gravimetric model is known as KTH-SDG08, it was computed by (Abdalla, 2009; Abdalla and Fairhead, 2011). The computation of the KTH-SDG08 is based on the optimum least-squares modification of Stokes kernel which is widely known as the KTH method. EIGEN-GRACE02S satellite-only model (Reigber et al. 2005) was adopted for KTH-SDG08 final computation at spherical harmonic degree and order 120. Moreover, the gravity anomaly was evaluated for gross error detection. The KTH method is based on combining the terrestrial gravity and GGM data by using the modified Stokes formula in least-squares sense (Sjóberg 1984, 1991, 2003a and 2003b). The major distinction of this method is that we can employ terrestrial gravity data directly without corrections, in contrast to other method of gravimetric geoid determination. Alternatively, four corrections (the additive corrections) accounting for the effects of topography, atmosphere, ellipsoid and downward continuation of the gravity data are to be added directly to the geoid, for more details the readers are referred to Sjóberg (1997, 2000, 2003a and 2004). KTH-SDG08 was evaluated using 19 GPS-levelling points, showing a standard deviation (STD) of differences between the gravimetric and geometric geoid heights of about 0.3 m after applying 7-parameter model.

Since the current accuracy of the regional gravimetric geoid for Sudan is not yet satisfactory and in meantime more GPS-levelling become available in Khartoum area, we are planning to compile a first gravimetric geoid model for Khartoum State, after that we will compute a new improved geoid model for Sudan. In order to do that we conduct several investigations for newly released GGMs. We utilize GGMs from dedicated satellite gravity missions to conduct our study, there are three well-known satellite missions, CHAllenging Mini-Satellite Payload (CHAMP) (Reigber, et al. 1996), Gravity Recovery And Climate Experiment (GRACE) (Tapley et al. 1996) and Gravity field and steady-state Ocean Circulation Explorer (GOCE) (ESA 1999) were launched in 2000, 2002 and 2009 respectively. We will particularly investigate the performance of a significant number of recent global geopotential models from GRACE and GOCE missions. The investigation will be conducted through two evaluation procedures: firstly, we will evaluate the GGMs against the newly computed GPS-levelling of Khartoum State. A precise evaluation of several newly released models enables us to fairly select the most appropriate model that converges with the local data in terms of the geoid undulations. Secondly, the GGMs will be evaluated against the local gravity grid of Sudan.

2. Study area and input data

The first part of this study has been conducted over the region of Khartoum State, the candidate region lies between the parallels of 15 and 17 arc-degree of northern latitude and the meridians of 31.5 and 35 arc-degree of eastern longitude, occupying an area of about 86,500 km^2. It is bordered to the north and the east side on the River Nile State, to North and North-west on the Northern and River Nile States, as well as to the west and South-west on North Kordofan State and to the east and south-eastern on states of Kassala, Gedaref and Gezira (see Fig. ??).

The topography in Khartoum state is almost flat, with slight slopes towards the crossing rivers (Blue Nile, White Nile and The Nile) only interrupted by a few hills of rocky outcrops. In the western part, we find ripple topography due to the sand dunes. In contrast, topography is much solid in far eastern and northern parts. As a part of the Nile River and its major tributaries (Blue and White Niles) cross the study area, topography is permeated by the terraces and floors of the Nile valleys middle part and valleys and narrow ravines in the north-eastern part (see Fig. ??).

We used a number of 25 points of orthometric heights and similar of co-located ellipsoidal heights. The dataset has been collected for the sake of evaluating the vertical and horizontal control of the State (cf. Ali, submitted).

The levelling network campaign was conducted in 2010. The spirit levelling was used to determine orthometric height differences. Modern digital level instruments were employed to start from eight old benchmarks established during the British colony. Orthometric heights are referred to Alexandria tide gauge. The accuracy of levelling network is within a tolerance of $10\sqrt{k}\ mm$, where k is the length of levelling circuit in kilometers (cf. Ali, submitted).

Soon later in 2010, the GPS points were co-located with the levelling points to compute the ellipsoid heights statically using the differential method. At first, two receivers were used to measure the baseline length (based on two known coordinate points). Thereafter, more two receivers had been attached in measurement, considering the same configuration of the baseline receivers.

We compare the geoid heights derived from the GGMs against the geometrical geoid heights derived from the GPS-levelling data. The geometrical geoid height is obtained from the following formula:

$$N_{gps} = h - H \tag{1}$$

where h is the ellipsoidal height and H is the orthometric height

Each GGM consists of a set of fully-normalized spherical harmonic coefficients. The Global models of the Earth's gravitational potential provide information required for a variety of geodetic, geophysical and oceanographic investigations and applications. The process of selecting the best model in the determination of a gravimetric geoid is very important and sensitive at the same time. They can possibly affect the solution of the reference surface for the regional geoid computations especially when the accuracy is supposed to reach a centimeter level (Kiamehr and Sjöberg 2005 and Abdalla 2009). Overall, the dedicated satellite missions (CHAMP, GRACE and GOCE) for gravity mapping have certainly provided valuable information of the gravity field. The list of adopted GGMs in this article is illustrated in Table ?? which indicates the name, maximum degree, type and reference.

The combined global models are useful for assessing the local gravity data because they contain high degree spherical harmonics. The advantage of employing the combined models is that the long- to medium wavelength part from the unprecedented performance of the dedicated satellite missions is improved in the higher frequency part (Reigber et al. 1996).

The grid of gravity data used in this study were provided by GETECH-UK, it consists of a 23509 points of free-air gravity anomalies. The gravity database was compiled by GETECH in 1988 (Fairhead, 1988) from all available land based surveys and include academic data, GRAS data, Strojexport and oil company, the distribution of the gravity data is shown in Figure ??. The age of the data ranges from 1960's to 1980's.There is a mix of altitude measurement methods used from spirit levelling, benchmark, trigonometric point to barometric all tied to bench marks and trigonometric

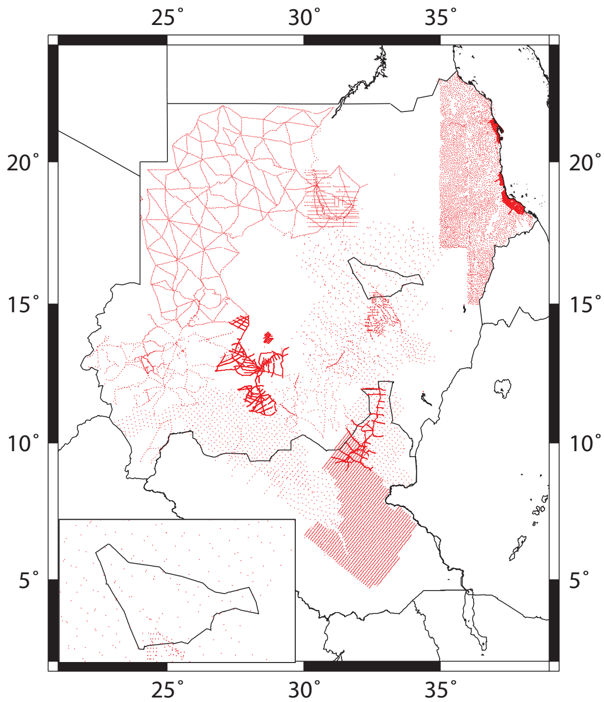

Figure 3. Gravity stations in Sudan (upper north). Inset shows the distribution of gravity stations in Khartoum state area.

points, least accurate was barometric at +/- 3m (~0.6 mGal error). The Getech free-air anomalies are compared with free-air anomalies derived from the GGMs (see Section 4.2).

The comparison process is conducted in terms of residual of differences between local and global anomalies.

$$\delta\Delta g = \Delta g^{t} - \Delta g^{ggm} \tag{2}$$

where Δg^{t} is the terrestrial anomaly and Δg^{ggm} is the free-air anomaly from the global model

3. Methodology

The Earth's disturbing potential T can be approximately described by means of spherical harmonics (Heiskanen and Moritz 1967) using the following formula:

$$T = W - U \tag{3}$$

where W is the geopotential and U is the normal potential at a certain point

The above equation can be written in terms of spherical harmonics to represent the employed dataset as follows

$$T = \frac{GM}{r}\sum_{n=2}^{l_{\max}}\left(\frac{a_e}{r}\right)^{n+1}\sum_{m=o}^{n}\bar{P}_{nm}(\cos\,\phi)\times \left[\bar{C}_{nm}\cos\,m\lambda + \bar{S}_{nm}\sin\,m\lambda\right] \tag{4}$$

Table 1. The tested GGMs over the study area.

Model		degree	Data	Reference
EGM08		2160	S(Grace),G,A*	Pavlis et al. 2008
EIGEN-GL04C		360	S(Grace, Lageos), G, A	Förste et al. 2006
GOCO	01S	224	S(Goce,Grace)	Pail et al, 2010b
	02S	250	S(Grace)	Goiginger et al, 2011
GO_CONS_GCF_2	DIR_R1	240	S(Goce)	Bruinsma et al. 2010
	DIR_R2	240		
	SPW_R1	210		Migliaccio et al. 2011
	SPW_R2	240		
	TIM_R1	224		Pail et al. 2010a
	TIM_R2	250		Pail et al, 2011
ITG-GRACE10		180	S(Grace)	Mayer-Gürr et al. 2010

*. S=Satellite Tracking Data, G = Gravity Data, A = Altimetry Data

where n and m are the degree and order of the spherical harmonic expansion, $\bar{P}_{nm}(\cos\phi)$ are the normalized associated Legendre polynomial functions, GM = 3986005×10^{-8} m^{3}s^{-2} is the Earth's gravitational constant (including the atmosphere) and a_e represents the equatorial radius of the Earth and $l_{\max}$ is the maximum degree of the geopodel, ϕ and λ are the geocentric latitude and longitude respectively. The fully normalized spherical harmonic coefficients $\bar{C}_{nm}$ and $\bar{S}_{nm}$ represent the zonal, tesseral and sectoral components of the potential.

Applying Bruns formula, the relation between the geoid N and the disturbing potential T is defined as (Heiskanen & Moritz 1967):

$$N = \frac{T}{\gamma} \tag{5}$$

where γ is the normal gravity on the reference ellipsoid

Inserting equation ?? in ?? we get

$$N_{ggm} = \frac{GM}{r\gamma}\sum_{n=2}^{l_{\max}}\left(\frac{a_e}{r}\right)^{n+1}\sum_{m=o}^{n}\bar{P}_{nm}(\cos\phi)\left[\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right] \tag{6}$$

and for free-air gravity anomaly the following equation is used:

$$\Delta g_{ggm} = \frac{GM}{r^2}\sum_{n=2}^{l_{\max}}\left(\frac{a_e}{r}\right)^{n+2}(n-1)\sum_{m=o}^{n}\bar{P}_{nm}(\cos\phi)\left[\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right] \tag{7}$$

4. Numerical investigations

In this section we will show the results of the comparison of GGM- derived gravity field quantities against the GPS-levelling data, the local gravimetric models and gravity data. The differences between the reference data (terrestrial gravity data and GPS-levelling-based geoid heights) and the GGM-based data return to the reasons that the terrestrial data contain the full signal extension (frequencies) of the Earth's gravity field, while the GGMs do not have the same frequencies due to the limitations of the spherical harmonic expansion and therefore they become affected by the omission errors. Therefore any comparison that committed to a specific region will contain several of errors from different sources, for example commission error of the spherical harmonic coefficients and will be affected as well by the omission error due to the truncation of the selected region, and random errors from GPS, levelling and gravity data respectively.

4.1. Comparison with GPS-levelling data

The residual geoid undulation ΔN is computed as:

$$\Delta N = N_{gps} - N_{ggm} \tag{8}$$

The GGM-based geoid heights are compared with the GPS-levelling-based heights. All models (mainly satellite-only) show similar RMS behaviors when expanding the geoid solutions to different expansion degrees up. Satellite-only models have even shown better agreement that combined models EGM08 and EIGEN-GL04C. For the sake of an evenhanded comparison, we truncate our GGMs at a similar spherical harmonic degree to 210. The GPS-levelling geoid undulations correspond to 5 degree spectral interval. As shown in Fig. ??, the minimum RMS for all models (36 to 39 cm) is remarkably detected at spherical harmonic degree 150. The comparison results of all models are illustrated in Table ??.

Based on statistics illustrated in Table ??, the GOCE satellite-only models SPW_R1, SPW_ R2, TIM_R1, TIM_R2 and GOCO01S show good consistency with GPS-levelling data at the degree of spherical harmonic expansion 150, while they do not show better results beyond degree 150 as seen in Figure ??. Hence, one of these models will be used later for the determination of the gravimetric geoid model of Khartoum state.

Table 2. Differences between geometrical and GGM-based geoid heights derived from the candidate models at degree and order of 150, units:[m].

Model	Min[m]	Max[m]	Mean[m]	RMS [m]
EGM08	-0.91	0.78	-0.10	0.38
EIGEN-GL04C	-0.67	1.05	0.23	0.44
GOCO01S	-0.81	0.87	0.02	0.36
GOCO02S	-0.67	1.00	0.17	0.39
ITG-GRACE10	-0.53	1.14	0.30	0.47
DIR_R1	-0.53	1.11	0.26	0.44
DIR_R2	-0.65	1.01	0.18	0.40
SPW_R1	-0.83	0.83	0.00	0.36
SPW_R2	-0.82	0.85	0.00	0.36
TIM_R1	-0.80	0.87	0.03	0.36
TIM_R2	-0.84	0.83	0.00	0.36

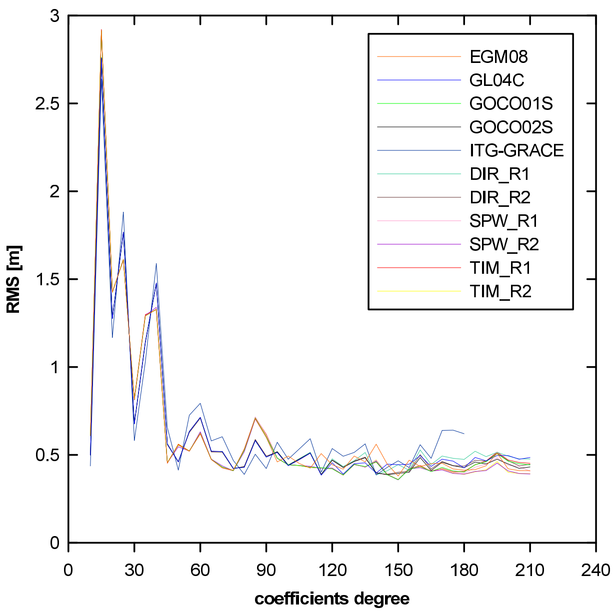

Figure 4. RMS of residuals of the differences between geometrical and GGM geoid heights, GGM heights were derived at $n = l_{max} = 10, \ldots, 210$.

4.2. Comparison with local gravity grid

The point-value free-air anomaly grid synthesized from GGM in a similar expansion range as in Section 4.1 was compared against the terrestrial gravity data. The comparison in Figure ?? shows that EGM08 has better agreement among the candidate models with RMS of 11.34, 9.97 and 9.19 mGal at degrees 210, 360 and 2190 (see Table ??). We use GRAFIM software (Janák and □prlák, 2006) to compute gravity anomalies and geoidal heights from EGM08 up to degree 2160.

Most of the Satellite-only models have similar performance trends with all expansions from 10 and up to 210, except ITG-GRACE and GOCO02S as shown in Figure ??. The satellite-only models show competitive agreements against EGM08 and EIGEN-GL04C at the degree of 210, From Table ?? we can clearly see that EGM08 has better agreement with the local gravity and we can see how satellite-only models DIR-R1, DIR_R2, TIM_R1 and TIM_R2 show good agreements with respect to the terrestrial gravity data.

Table 3. Comparison between terrestrial gravity and EGM08 gravity data at degrees 2160 and 360, and EIGEN-GL04C at degree 360, units:[mGal].

Stat [mGal]	EGM08 (2160)	EGM08(360)	GL04C(360)
Min	-222.46	-78.70	-60.12
Max	73.58	74.84	67.85
Mean	-0.86	-0.90	-0.83
RMS	9.16	10	10.64

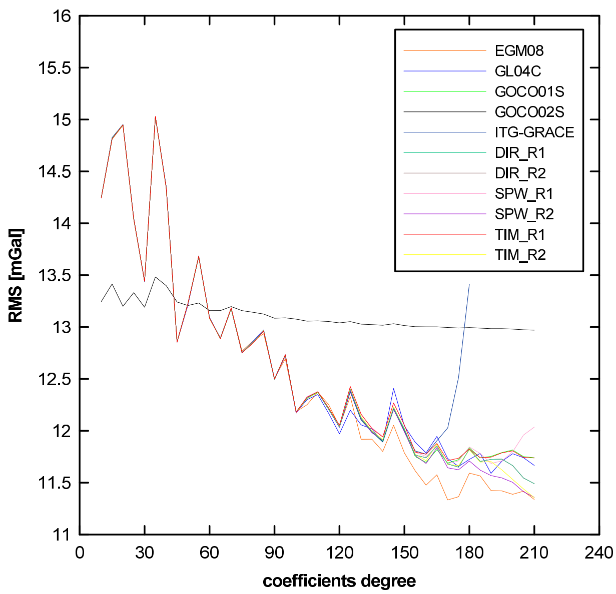

Figure 5. RMS of differences between local and global gravity anomalies derived at $n = l_{max} = 10, \ldots, 210$.

On the other hand, a pure comparison between the combined models (EGM08 and EIGEN-GL04C) has been conducted to the degree and order 360 (see Figure ??). The discrepancies between the two models are clearly starting from degree 95 to 360. EGM08 has always better agreement than EIGEN-GL04C. Histograms of the differences between the terrestrial free air gravity and free air gravity derived from both EGM08 and EIGEN-GL04C are shown in Figures ??**a** and ??**b**.

In contrast to the other candidate models, the behavior of RMS with respect to the higher degrees has shown an insignificant improvement in DIR_R2 and TIM_R2 in a similar trend, RMS has been hardly improved from 13.4 mGal to 12.9 mGal in both models. Furthermore, in ITG-GRACE model, RMS considerably increases between 165 and 180. We also see that the RMS is slightly increasing in higher degrees of TIM_R1 and SPW_R1.

Table 4. Comparisons between terrestrial gravity and GGM gravity data derived from different global models at degree and order 210, unit:[mGal].

Stat [mGal]	EGM08	EIGEN-GL04C	DIR_R2	DIR_R1	TIM_R1	TIM_R1
Min	-65.59	-64.27	-71.24	-71.78	-72.44	-76.77
max	87.14	88.52	85.84	86.28	89.10	89.03
mean	-0.92	-0.87	-0.93	-0.89	-0.89	-0.90
RMS	11.34	11.67	11.43	11.49	11.36	11.36

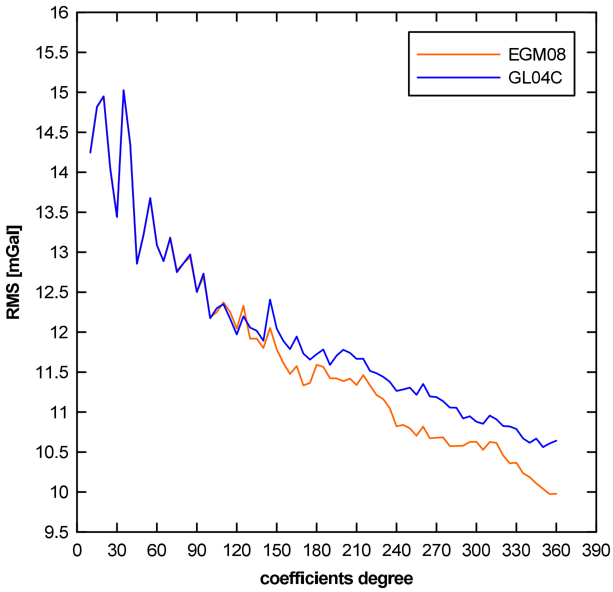

Figure 6. RMS of residual gravity anomalies of EGM08 and EIGEN-GL04C against GPS-levelling data at different expansion degrees of spherical harmonics truncated at the degree and order $n = l_{max} = 360$.

From Table ??, comparison and analysis show that EGM08 in its full expansion (to degree and order 2160) is the most suitable model that to be used for gravity prediction. The model will be used to recover the gravity, in other words EGM08 with full expansion will be employed in filling the gaps of un-surveyed areas of the entire country.

4.3. Comparison with existing regional gravimetric geoid models

Further evaluation for our potential model SPW-R1 has been conducted over the target area. The potential SPW-R1 satellite-only model is validated against the existing gravimetric models. In addition, the combined models EGM08 and EIGEN-GL04C have been evaluated on the same pattern. This evaluation will give a clear view about the agreement between global and regional geoid models and it also gives a direct approval to use the potential candidate model in further studies

In the comparison versus the geoid91 (Fig ??), the three GGMs have almost the same quality with an RMS of (73 cm) for SPW-R1 and (64 cm) EIGEN-GL04C, and the highest RMS is (84 cm) for the

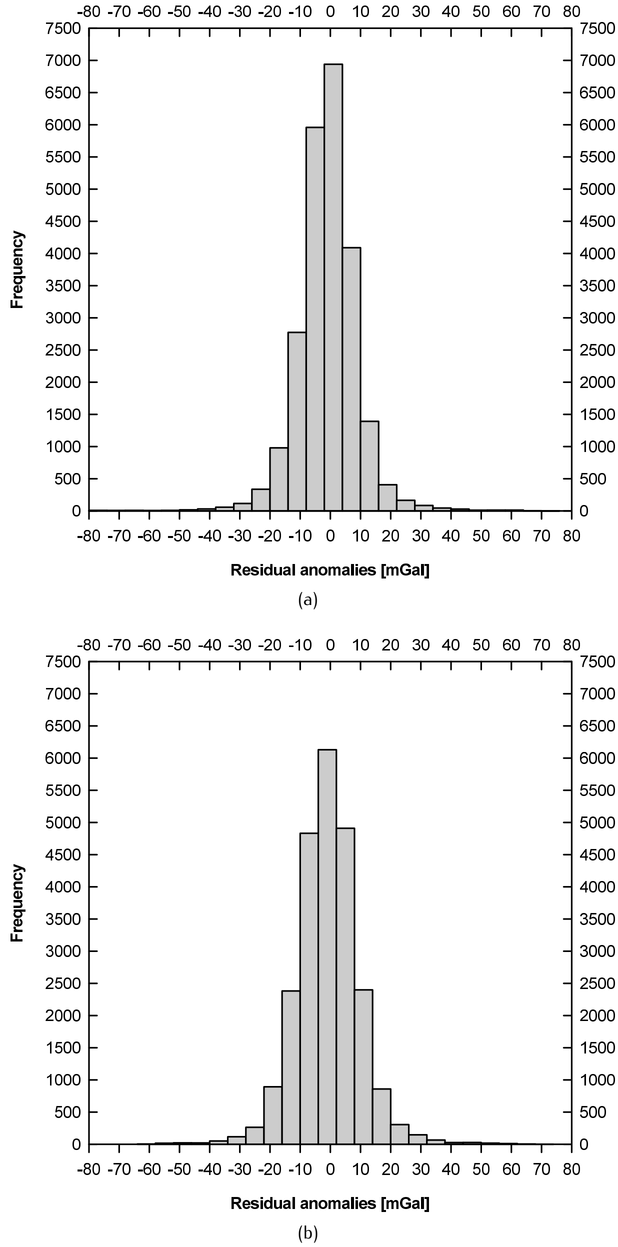

Figure 7. Histograms of differences between terrestrial and GGM gravity anomalies at degree and order 360, a) EGM08, b) EIGEN-GL04C.

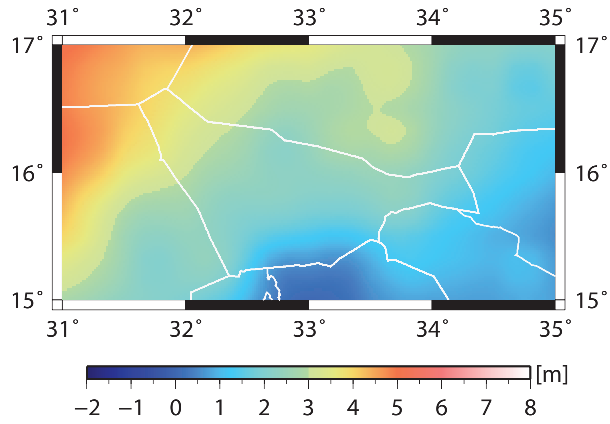

Figure 8. The Gravimetric Geoid91 over the target area.

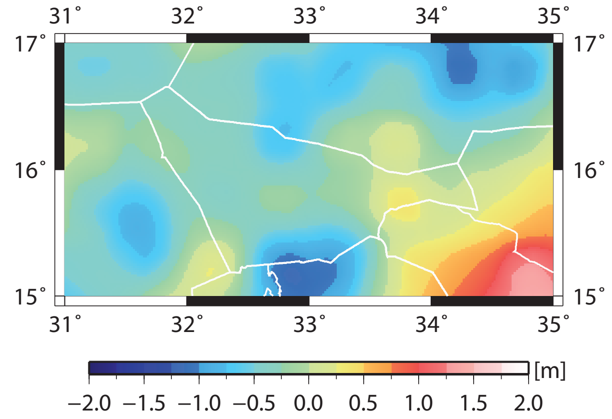

Figure 10. Differences between Geoid91 and EIGEN-GL04C.

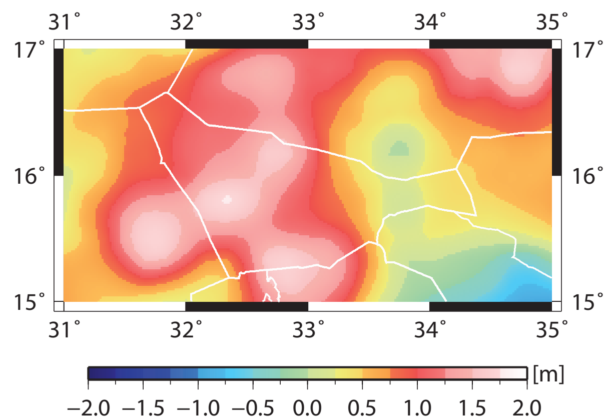

Figure 9. Differences between Geoid91 and EGM08.

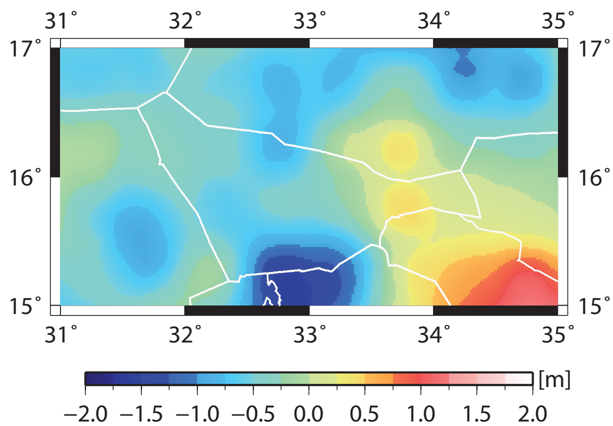

Figure 11. Differences between Geoid91 and SPW_R1.

EGM08. The comparisons and numerical statistics are illustrated in Figures ??, ??, ?? and Table ??, respectively.

On the other hand, the results of the candidate models evaluation against the second regional geoid model KTH-SDG08 (see Fig. ??) show significant improvements for all models, the qualities in terms of RMS have increased up to 49 cm for SPW-R1, 37 cm for EGM08 and 60 cm for EIGEN-GL04C.

The EGM08 is as expected consistent with the local gravity and the GPS-leveling data as well as the KTH-SDG08 regional geoid models, and it always has a better quality which means that the model is efficient enough to contribute in further precise gravimetric computations and other related studies that involve with GGMs and gravity modelling. The comparisons and numerical statistics are illustrated in Figures ??, ??, ?? and Table ??, respectively.

Table 5. Geoid1991 model against EGM08, EIGEN-GL04C and SPW_R1 geoid heights, units: [m].

Stat [m]	EGM08	EIGEN-GL04C	SPW_R1
Min	-1.70	-1.54	-1.63
Max	1.65	1.75	1.68
Mean	-0.44	-0.28	-0.44
RMS	0.93	0.64	0.73

Table 6. KTH-SDG08 gravimetric heights against EGM08, EIGEN-GL04C and SPW_R1 geoid heights, units: [m]

Stat [m]	EGM08	EIGEN-GL04C	SPW_R1
Min	-0.78	-0.77	-0.88
Max	1.22	1.77	1.27
Mean	0.30	0.46	0.30
RMS	0.37	0.60	0.49

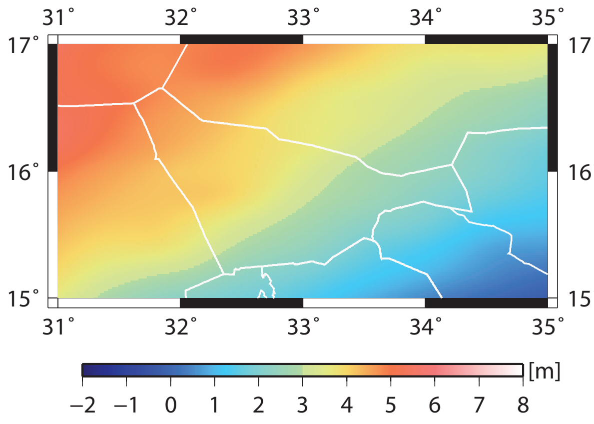

Figure 12. The KTH-SDG08 gravimetric heights over the test area.

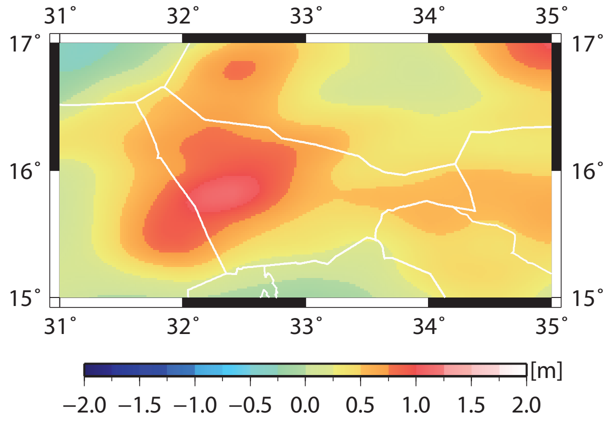

Figure 15. Differences between KTH-SDG08 and SPW_R1.

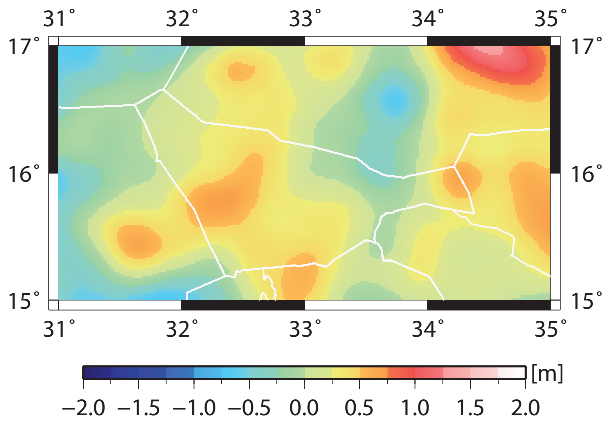

Figure 13. Differences between EGM08 and KTH-SDG08.

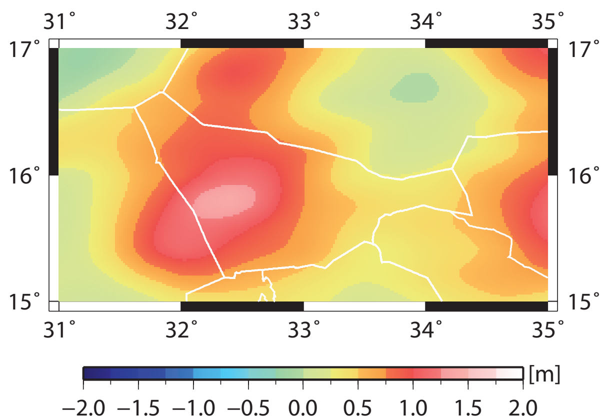

Figure 14. Differences between KTH-SDG08 and EIGEN-GL04C.

5. Summary

We examined the performance of the recent GOCE/GRACE geopotential models in order to select the most suitable model with regard to the gravimetric geoid undulations and terrestrial free-air gravity anomaly data. The successful candidate model will be used to obtain the long wavelength component of the geoid undulations as a partial contribution towards the determination of a precise geoid model for Khartoum State. A set of eleven GGMs including nine satellite-only models and two well known combined models (EGM08 and EIGEN-GL04C) has been validated against the local datasets, GPS-levelling data, terrestrial gravity data and the existing local gravimetric geoid models.

Three types of comparisons were conducted. Firstly, a comparison between the GPS-based geoid undulations and point-value geoid undulations derived from GGM. Secondly, a comparison between the regional free-air anomalies and point-value free-air anomalies synthesized from GGMs. Lastly, an affirmative comparison versus the existing gravimetric geoid models (Geoid91 and KTH-SDG08). The affirmative comparison is employed mainly for approving the likely selected candidate model.

In the first comparison, the geoid height is computed over the GPS-levelling data, all models upper limits were truncated to degree and order 210, based on performances, the satellite-only models showed similar accuracies at spherical harmonic degree of 150, precisions of around 36 to 39 cm with respect to the standard deviation were achieved. Among the similar precision of the models qualities the SPW-R1 satellite-only model has been selected as potential candidates, nevertheless, other models e.g. SPW_R2, TIM_R1, TIM_R2 and GOCO01S are still qualified.

In the second comparison, the combined models showed best performance with regard to the gravity anomalies; the EGM08 combined model showed an accuracy of 9.93 mGal versus the other models, EIGEN-GL04C is the closest one to the EGM08 with 10.6 mGal accuracy.

For the sake of the confirmation to our selected models, the third comparison was taking place. The potential candidate models in addition to EIGEN-GL04C were evaluated against the existing gravimetric geoid models. The recent KTH-SDG08 geoid models showed better agreement against trilogy models of comparison EGM08, EIGEN-GL04C and SPW-R.

The EGM08 model was truncated into 210 in the first and second comparisons. It is truncated again to 360 to be compared with EIGEN-GL04C as plotted in Fig. ?? and statistics are illustrated in Table ??. EGM08 was also used in its full expansion 2160 to compute geoid heights over the GPS-levelling data and GGM-based gravity anomalies for comparison against the terrestrial gravity anomalies. Overall, and from the models performance, it is expected to obtain a similar quality of long wave-length contribution from the other candidate models. Further inspection and more comparisons can be carried out in upcoming research works.

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