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Maximum Magnitude of Completeness in a Salt Mine

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Abstract In this study, we analyze acoustic emission (AE) data recorded at the Morsleben salt mine, Germany, to assess the catalog completeness, which plays an important role in any seismicity analysis. We introduce the new concept of a magnitude completeness interval consisting of a maximum magnitude of completeness ($M_{c(max)}$) in addition to the well-known minimum magnitude of completeness. This is required to describe the completeness of the catalog, both for the smallest events (for which the detection performance may be low) and for the largest ones (which may be missed because of sensors saturation). We suggest a method to compute the maximum magnitude of completeness and calculate it for a spatial grid based on (1) the prior estimation of saturation magnitude at each sensor, (2) the correction of the detection probability function at each sensor, including a drop in the detection performance when it saturates, and (3) the combination of detection probabilities of all sensors to obtain the network detection performance. The method is tested using about 130,000 AE events recorded in a period of five weeks, with sources confined within a small depth interval, and an example of the spatial distribution of $M_{c(max)}$ is derived. The comparison between the spatial distribution of $M_{c(max)}$ and of the maximum possible magnitude (M_{max}), which is here derived using a recently introduced Bayesian approach, indicates that M_{max} exceeds $M_{c(max)}$ in some parts of the mine. This suggests that some large and important events may be missed in the catalog, which could lead to a bias in the hazard evaluation.

Introduction

It is of major importance to evaluate the quality of seismic catalogs before using them to analyze the earthquake process and to assess seismic hazards. One of the basic parameters to assess such quality is the magnitude of completeness (M_c), defined as the lowest magnitude for which all earthquakes can be detected in a space–time volume (e.g., Rydelek and Sacks, 1989; Woessner and Wiemer, 2005). Its reliable estimation is essential for the statistical analysis of seismicity and hazard-related studies, such as the analysis of rate changes, static and dynamic triggering, probabilistic seismic-hazard assessment, and earthquake forecasting. M_c is usually heterogeneous in time and space, as it depends on factors such as network geometry and instrumentation, noise level, and attenuation of the seismic signals with distance. Especially in mining settings, due to the effects of local heterogeneities on the seismic wavefield, the M_c estimations may show strong spatial variations (e.g., Plenkens *et al.*, 2011; Maghsoudi *et al.*, 2013).

A number of statistical techniques are available to compute M_c (e.g., Mignan and Woessner, 2012). Most methods

are based on catalog data and define M_c as the lowest magnitude at which the observed frequency–magnitude distribution (FMD) deviates from the assumed Gutenberg–Richter (GR) relation. These methods include the maximum curvature technique (Wyss *et al.*, 1999; Wiemer and Wyss, 2000), entire magnitude range (Woessner and Wiemer, 2005), and goodness-of-fit test (Wiemer and Wyss, 2000). These techniques need to be applied for time intervals during which temporal variations of M_c can be neglected. This hampers their application to mining environments, where significant temporal changes in the fracturing processes and in the detection performance are common. For example, M_c may show strong variations depending on the radiation pattern of the events, which may vary due to changes in the fracturing processes in the mine. Moreover, multimodal FMDs and deviations from the GR distribution have been often observed in mines. Gibowicz and Kijko (1994) suggested some physical reasons to explain such multimodality. A reasonable hypothesis is that it results from strong structural heterogeneities, such as cavities or different geological bodies, which affect the small-scale distribution of the medium properties, local stress field, and stressing rates. Maghsoudi *et al.* (2014) suggested the multimodal FMDs can be explained by

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the superposition of two different FMDs before and after fracture initiations in mining environments. [Schorlemmer and Woessner \(2008\)](#) proposed an alternative method to infer M_c , namely the probability-based magnitude of completeness, which does not require assumptions about the FMD. Extended approaches of this method have been applied to catalogs of acoustic emission (AE) events in a gold mine in South Africa ([Plenkers et al., 2011](#)) and in a salt mine in Germany ([Maghsoudi et al., 2013](#)).

The catalog completeness has been typically discussed in the past only for the smallest magnitudes within the catalog. In fact, this issue is always present, because seismic waveforms for small magnitude events have worse signal-to-noise ratios. Thus, below some magnitude threshold, events produce signals that are too weak to be correctly detected by a sufficient number of network stations; however, for specific acquisition systems, the catalog completeness may also be an issue for the largest observed events. Depending on the recording installation, sensors may saturate when the amplitude of the seismic signal reaching them is too high to be properly recorded. In such conditions, if most (or all) sensors are saturated, and the remaining information is not sufficient (e.g., to locate or estimate the magnitude of the seismic event), this may not be included in the catalog, resulting in a catalog incompleteness for large magnitude events. In this work, we discuss this latter case by considering an AE monitoring network in a salt mine and the corresponding catalog, which we analyzed previously ([Maghsoudi et al., 2013, 2014](#)). This data set is particularly well suited to test our hypothesis, because of the broad range of recordable magnitudes, the sensor characteristics, and the location procedure, which requires a minimum number of nonsaturated sensors. We propose here a method to estimate the maximum magnitude of completeness ($M_{c(max)}$), which is based on the modeling of sensor saturation and the detection performance at each single sensor.

When analyzing the seismicity in a certain region, another important goal is the assessment of the maximum possible earthquake magnitude M_{max} . In the context of probabilistic seismic-hazard assessment, this is defined as the upper truncation of the FMD, and its determination is crucial. It is important to ensure that M_{max} is not related to the problem of magnitude saturation, and the truncation in the FMD should be inferred, for example, using moment magnitude or a magnitude scale that shows no saturation.

The different methods proposed to estimate M_{max} can be divided into empirical and statistical types. A first, empirical approach is to define it as the largest observed magnitude in a given region, although there is no warranty that future earthquakes will not be larger. It has been further proposed that M_{max} can be estimated upon tectonic and fault parameters (e.g., [Wyss, 1979](#); [Singh et al., 1980](#)). Parameters such as the maximum dimensions of rupture, including rupture length, rupture area, and amount of slip are known to be empirically correlated with earthquake magnitude (e.g., [Stirling et al., 2013](#), and references therein). [Wheeler \(2009\)](#) discussed different empirical approaches to estimate M_{max} , depending on

the target region, local geological features, paleoseismic data, or using tectonic analogs.

M_{max} can also be evaluated from statistical procedures, such as extrapolating frequency–magnitude curves, or using the strain rate or seismic moment release rate (e.g., [Anderson et al., 1986](#); [Gibowicz and Kijko, 1994](#)). [Kijko \(2004\)](#) obtained point estimations of M_{max} for a frequency-size distribution that deviates moderately from the GR law. The main drawback of point estimators is that the calculation of uncertainties is mostly performed in approximation only ([Holschneider et al., 2011](#)). [Cornell \(1994\)](#) suggested using Bayesian analysis and allowed for a straightforward calculation of confidence level for M_{max} . The Bayesian analysis involves multiplication of a prior probability distribution by a likelihood function to form a posterior probability distribution. However, the choice of prior distribution has an inherently subjective component ([Vere-Jones and Ogata, 2003](#)). Finally, [Holschneider et al. \(2011\)](#) proposed an approach to calculate confidence intervals for M_{max} in a given earthquake catalog by assuming a doubly truncated GR distribution for the catalog magnitudes and showed the common approach to derive confidence intervals from the variance of a point estimator fails. Moreover, the Bayesian confidence intervals diverge unless particular prior assumptions are made. For example, using an upper-bound magnitude for M_{max} as *a priori* information, the Bayesian posterior distribution can be normalized, and thus its variance decreases with the number of observed events. Finally, the results of this study indicated that M_{max} can safely be derived with respect to a given time interval, as in that case the estimation of exact confidence intervals becomes straightforward ([Zöller et al., 2013](#)).

In this study, we focus on an AE catalog recorded along 35 days in the Morsleben salt mine, Germany. We introduce the concept of the maximum magnitude of completeness and provide an approach to estimate it. In the present case, we show the definition of minimum magnitude of completeness, understood as the lowest magnitude above which all events can be detected in a space–time volume, cannot be used anymore in this strict sense, due to the saturation of sensors in the presence of large-amplitude signals. To investigate whether it is likely that some large events may be missed in some parts of the mine, we use the Bayesian approach proposed by [Holschneider et al. \(2011\)](#) to estimate M_{max} . Then, we check whether the estimated M_{max} values correspond to a range of magnitude where the catalog is complete. Wherever M_{max} exceeds the maximum magnitude of completeness, there is a significant potential to miss the largest events.

Data

This work is entirely based on catalog data. The catalog information was compiled from the data recorded by a microseismic network that was installed to monitor microcrack processes at the Morsleben salt mine, Allertal region, Germany. This site has been extensively monitored for AE

Figure 1. (a) Vertical section and (b) hypocenter depth distribution of the recorded earthquakes in the central part of the Morsleben salt mine. The depth interval 270 ± 2.5 m was selected for further analysis, where about 130,000 events were located (marked in black in [b]). The corresponding (c) epicenter and (d) cumulative (triangles) and noncumulative (circles) frequency–magnitude distribution (FMD) of these selected events (using an arbitrary magnitude scale M_{AE}). In (c), the small square indicates the cell that is discussed in Figures 2 and 3. The triangles in (a) and (c) mark the location of acoustic emission (AE) sensors.

studies (e.g., Spies and Eisenblätter, 2001; Becker *et al.*, 2010; Maghsoudi *et al.*, 2013, 2014).

The data set specifically used in this case was recorded within a period of five weeks (27 April–31 May 2010), with a total of 578,333 located AE events (Fig. 1a). This time period was chosen because the monitoring network was homogeneous during its whole duration and was composed of 31 working sensors, although 5 showed low detection capability and were thus removed from the analysis. The catalog information includes hypocentral locations and uncertainties, magnitudes, and the maximum recorded amplitudes at each sensor. Magnitudes of AE data (M_{AE}) were calculated from the recorded amplitudes and corresponding source–receiver distances. They are reported in decibels at a reference epicentral distance (Köhler *et al.*, 2009). M_{AE} values of this catalog were arbitrarily scaled by a factor of 0.05 to range between 0 and 5, which could correspond to M_w from -8 to -3 , according to empirical energy relations (Eisenblätter and Spies, 2000; Kai-

ser, 2011). The M_c using this M_{AE} scale, derived from the directional probabilistic magnitude of completeness (PMC) approach, was found to be $M_c \approx 1$ at the center of the network and increases up to $M_c \approx 4$ at further distances outside the network (Maghsoudi *et al.*, 2013, 2014). Figure 1a illustrates the distribution of epicenters in the central part of the Morsleben mine. Because the depth distribution is very heterogeneous, we focus on a 5 m thick depth interval (270 ± 2.5 m), in which 132,357 events were located (Fig. 1b). The higher AE activity observed at this depth interval is not a consequence of a bias introduced by the location procedure and is also confirmed when considering only best-located events. The epicenter distribution of the events in the considered depth interval is shown in Figure 1c, and the corresponding cumulative and noncumulative FMDs are shown in Figure 1d.

The AE signals detected by the piezoelectric sensors were recorded using a 16-bit analog-to-digital converter with a dynamic range of $\pm 5V$. If the dynamic range is low, the

large amplitude of the signal will saturate the dynamic range of the recording system. In the considered period, the installation was limited to 32,768 samples, the maximum count of the converter. Whenever the recorded digital amplitude reaches the given threshold, the sensor is considered saturated and its information is not further used for the location or the magnitude estimation of the AE event (J. Philipp, personal comm., 2014).

Depending on the hypocentral location and magnitude of an event, several sensors can be saturated and are therefore useless for the location and magnitude estimation procedures. The automated AE location at Morsleben requires a minimum number of 10 phase detections (cumulative number of P and/or S picks) for a valid location (Spies and Eisenblätter, 2001; Becker *et al.*, 2010). Events with fewer picks are identified but not located, and they therefore are not included in the resulting catalog. If the number of nonsaturated sensors is too small, the available information becomes insufficient to perform location and magnitude estimations, and the event is excluded from the catalog. In this case, the seismic catalog is incomplete not only below the magnitude of completeness threshold M_c , but also above a magnitude threshold, which we will refer to as maximum magnitude of completeness ($M_{c(\max)}$).

Determination of the Maximum Magnitude of Completeness

This section describes our approach to estimate the $M_{c(\max)}$ in different parts of the mine in the considered depth layer. First, we discuss how to estimate the saturation magnitude for each sensor. Then, we describe how the information derived separately for each sensor can be joined to derive the $M_{c(\max)}$ of the catalog.

The region is first divided into cubic cells. The cell size should be chosen to be as small as possible to minimize source-sensor path differences for different sources within the cell. On the other hand, cells should contain a sufficient number of events. For our case, we scan the target region using nonoverlapping cubic cells of $5 \times 5 \times 5 \text{ m}^3$. The following analysis is only performed for those cells that include at least 200 events. A location precision of about 1 m is obtained at distances up to 50 m around the sensor network. This location precision limits the effective cell size down to 1 m.

For a specific cell and a given sensor, the recorded amplitudes depend on the scalar moment and the radiation pattern of the AE source. For each sensor, we can then plot event magnitude versus the logarithm of the recorded maximal amplitude. Because AE magnitudes are derived from the distribution of maximal recorded amplitudes and the source-receiver distances, they have been chosen as the dependent variable (y), whereas the logarithm of maximal amplitudes is the independent variable x . Figure 2 illustrates this plot for an example reference cell with 859 events (small square in Fig. 1c) and the best working 26 sensors. The five remaining

sensors showed very low detection capabilities and were removed from the analysis for all cells. A sensor with low detection capability (e.g., due to a bad coupling with the rock) can have a large magnitude of saturation. Its detection performance, however, is always very low and does not change the overall network detection performance. For this reason, it is adequate to neglect these sensors.

We find that the magnitude scales linearly with the logarithm of the maximum amplitude. In each graph, there is only minor scatter around the linear trend, resulting from marginal effects of variations in the radiation patterns or the slightly different distances between the sensor and the hypocenter. This indicates that either different events in the same cell have similar focal mechanisms or a variation of the focal mechanisms does not strongly affect the pattern of maximum amplitudes.

The average $M_{c(\max)}$ for each sensor and this specific cell can be estimated as follows. For each of the 26 different sensors, we use a linear fit between the event magnitude and the logarithm of its maximum amplitude recorded at the sensor (Fig. 2). Because of larger errors in the magnitude estimations compared with maximum recorded amplitudes, we performed a standard least-squares linear regression. Considering the highest recordable value of maximum amplitude (32,768), the average magnitude of saturation for a given sensor can be extrapolated from this linear regression. The linear regression is different for different sensors, because the far sensors record smaller amplitudes for the same event.

Maghsoudi *et al.* (2013) used the same data set to estimate M_c defined as the minimum magnitude reflecting the desired detection probability level of 0.999. The estimation for our reference cell was $M_c = 1.0$. The analysis was performed in different steps: first, the probability of detection was derived for each station, then detection probabilities for different stations were combined into a joint probability of detection for the whole network. In a last step, the spatial distributions of network detection probabilities were used to derive a map of completeness (for more details, see also Schorlemmer and Woessner, 2008). To remove artifacts due to sparse data and statistical fluctuation, Maghsoudi *et al.* (2013) proposed a magnitude-smoothing criterion in agreement with Schorlemmer and Woessner (2008), which assumed the detection probability at a given distance to the sensor cannot decrease as magnitude increases. The distance-smoothing algorithm from Schorlemmer and Woessner (2008), constraining the detection probability not to increase with the distance to the sensor, was not implemented for Morsleben, because such a constraint has been not been shown to hold for strongly heterogeneous mining environments (Maghsoudi *et al.*, 2013). In this study, we additionally find the magnitude of saturation acts as a threshold for the detection probability of a sensor, which should drop to zero for larger magnitudes.

For each sensor, the probability of detecting a given magnitude is here defined as the ratio of the number of events

Figure 2. The results of the linear regression procedure and estimations of saturation magnitude (M_{AE} , marked by horizontal dashed lines) for the reference cell (shown in Fig. 1c) at 26 sensors. This magnitude corresponds to the maximum recordable amplitude (32,768 samples, vertical dashed lines).

Figure 3. (Top) Number of nonsaturated sensors as a function of the earthquake magnitude in the reference cell (shown in Fig. 1c). The vertical dashed line refers to the inferred maximum magnitude of completeness ($M_{c(\max)}$) if the minimum number of required sensors is set to 10 (bottom). The resulting probability (solid line) that the network will detect earthquakes of magnitude M_{AE} in the reference cell, which drops to 0 for both too small ($M_{AE} < 1.0$) or too large ($M_{AE} > 3.6$) magnitudes. The dashed line is the detection probability function calculated for the same cell by Maghsoudi *et al.* (2013).

in the cell with a phase pick at the sensor and the number of all recorded events in the cell. This approach is different than in Maghsoudi *et al.* (2013), in which the detection probability of a sensor was derived from the information of all events in a given direction, whereas now we only rely on events in a given cell. The former approach remains more general and preferable to estimate M_c in the whole mine, because it is less dependent on the density of events. Now the analysis can only be performed for those cells in which a sufficient number of events are available. Figure 3 (bottom) shows the detection probability for the target cell in the network as derived in Maghsoudi *et al.* (2013, dashed line) and from the

available events in the cell (solid line). Despite some differences for $M_{AE} < 1$, both approaches agree that $M_c = 1.0$ (both solid and dashed lines are overlapped for magnitudes in the M_{AE} 1.0–3.5 range). According to the former magnitude smoothing, the detection probability for $M_c > 1.0$ cannot be smaller than the chosen probability level of 0.999. However, we have estimated the saturation magnitude for each sensor and know that the detection probability of a given sensor will drop to zero for magnitudes above the threshold. If we include this information when computing probabilities for all sensors and combine them to estimate the detection probability of the network (e.g., still reflecting the overall probability level of 0.999), we find that, above a given magnitude, too few sensors are not saturated. At Morsleben, a minimum of 10 picks are required to include an event in the catalog (see Data and Resources). Those events which have too few picks will be ignored (in our case, the number is limited because the events have a large magnitude that saturates many sensors), and the catalog will become incomplete. Figure 3 (top) shows the number of nonsaturated sensors for the reference cell. If the minimum number of required unsaturated sensors is set to 10, the maximum magnitude of completeness will be equal to M_{AE} 3.6. In contrast to the previously assumed smoothing approaches (Maghsoudi *et al.*, 2013), the detection probability of the network thus drops to zero for $M_{AE} > 3.6$ (Fig. 3, bottom, solid line).

To calculate the statistical uncertainty of the estimated $M_{c(\max)}$, we performed linear regressions with 1000 bootstrap samples and estimated $M_{c(\max)}$ for each sample. Doing so in a given cell for each sensor, we obtained a distribution of $M_{c(\max)}$ estimations, instead of a single value. The corresponding standard deviations are found to be < 0.15 in the analyzed cells. The decreasing detection probability at large magnitudes is modeled as the reverse cumulative distribution of $M_{c(\max)}$ estimations. This produces a smoother drop of the detection probability around the mean $M_{c(\max)}$ value.

A consequence of our findings is that the catalog could be significantly incomplete for small magnitudes far from the network (related to larger M_c values), whereas sensor saturation problems could make it incomplete for large magnitudes in the center of the network. To test this hypothesis, we plot the spatial event density for different magnitude intervals (Fig. 4). The normalized number of events (i.e., the number divided by the maximum observed number) is shown here, because we are mostly interested in discussing the spatial pattern of the event density. The concentration of AE activity within a north-northwest–south-southeast elongated region roughly corresponds to the geometry of the salt body in the mine. Figure 4 (left) shows that most small magnitude events ($M_{AE} \approx 1.0 \pm 0.1$) are recorded well within the network, whereas their number decreases with distance to the network, where the catalog may be incomplete due to M_c . In contrast, larger magnitude events ($M_{AE} \approx 2.5 \pm 0.1$; Fig. 4, right) are mostly located outside the network (toward the north-northwest–south-southeast), which could suggest that some large events were not detected in the network region due to

Figure 4. Density of events for different magnitude values. Small magnitude events ($M_{AE} \approx 1.0 \pm 0.1$, left) are recorded in the center of the network, whereas larger magnitude events ($M_{AE} \approx 2.5 \pm 0.1$, right) are mostly located toward the north-northwest and south-southeast. The color version of this figure is available only in the electronic edition.

$M_{c(max)}$. This spatial pattern is in agreement with our hypothesis. Our result is additionally supported by [Maghsoudi et al. \(2013\)](#), who showed the magnitude of completeness is increasing from the center to the borders of the network.

In our approach, $M_{c(max)}$ is defined on the basis of a minimum number of triggered stations, independently of whether only the P or both P - and S -phases are identified. Thus, we consider a minimum of 10 stations to ensure that at least 10 phases were detected for each event.

To assess the quality of the linear regression, we calculate the squared correlation coefficient R^2 (which has a possible range from 0 to 1) for each cell and sensor pair. Values close to 0 indicate a poor fit, whereas values close to 1 indicate a very good fit to the data. Figure 5c shows the spatial map of the median of R^2 estimations obtained by linear regression fitting for all 26 sensors in each cell. A poor fit may indicate a poor resolution of the saturation magnitude for some sensors and, therefore, an uncertain estimation of the maximum magnitude of completeness. In the following, $M_{c(max)}$ is only further discussed for those cells in which the distribution of amplitudes and magnitudes can be reasonably well fit (with a median R^2 of at least 0.7).

Figure 5 shows the spatial distribution of $M_{c(max)}$ estimations at the chosen depth, using a minimum of 10 sensors. The AE activity clusters in two regions, toward the north and the south. The $M_{c(max)}$ estimations are slightly larger in the northern region than in the southern region. We tested the spatial robustness of the computation by using cells which overlapped each other by 50%, obtaining compatible results, with smoother transitions between the $M_{c(max)}$ estimates. In those cells presenting good fits between magnitude and log (maximum amplitude), such as in the western part of the northern region and the central part of the southern region (Fig. 5c), the standard deviation of $M_{c(max)}$ is lowest. Results of $M_{c(max)}$ estimation for a second layer closer to the recording sensors present smaller values, which show that even smaller events saturate the closer located sensors.

Estimation of the Maximum Possible Magnitude and Comparison with $M_{c(max)}$

We follow here the Bayesian approach proposed by [Holschneider et al. \(2011\)](#) to calculate the maximum possible magnitude M_{max} . The occurrence of events with such large magnitudes is considered to be rare. The proposed model assumes the validity of the doubly truncated GR distribution for earthquake magnitudes and focuses on the confidence intervals for the maximum possible magnitude. [Holschneider et al. \(2011\)](#) showed the estimation of M_{max} in an infinite time horizon is only feasible if appropriate prior assumptions are made; otherwise, confidence intervals of M_{max} can diverge. We apply the method proposed by [Holschneider et al. \(2011\)](#), relying on two main assumptions: (1) earthquake catalog magnitudes follow a doubly truncated GR distribution and (2) the M_{max} in this particular case cannot exceed the value of 8.

A detailed mathematical formulation and explanation of this method can be found in [Holschneider et al. \(2011\)](#) and [Zöller et al. \(2013\)](#). In this work, we estimate M_{max} for all cells in which a reliable estimate of $M_{c(max)}$ was possible and also require a minimum number of 500 events per cell. This limit number was chosen based on our tests showing that the M_{max} estimations are robust for subsets of 500 events in locations with more events. The output of the Bayesian approach is a full probability distribution of M_{max} , including uncertainties. For practical purposes, however, it is desirable to extract a single number from this distribution. To accomplish this, we fix a confidence level or, equivalently, a probability of the error one is willing to accept. If this probability approaches zero, we get $M_{max} = 8$ by definition. We decided to use the median value (50% error probability), which indicates the location of the probability distribution. For this moderate value, the results will be essentially unaffected by the truncation effect. Figure 5d shows the spatial distribution of differences between M_{max} and $M_{c(max)}$. In the majority of the cells,

Figure 5. (a) Maximum observed magnitude in each cell for the chosen depth; (b) estimated $M_{c(\max)}$; and (c) median coefficient of determination R^2 of the magnitude–log(maximum amplitude) fits (in which a value of 1 indicates a perfect linear relation), using events located by at least 10 sensors. In (a), (b), and (c), cells shown in gray have too few events (< 200) to make a meaningful analysis. (d) Difference between the estimated maximum possible magnitude ($M_{\max}$, calculated using a Bayesian approach with a moderate confidence level of 0.5) and $M_{c(\max)}$; it is shown only for those cells with > 500 events and $R^2 > 0.7$. The color version of this figure is available only in the electronic edition.

$M_{\max}$ is smaller than $M_{c(\max)}$, indicating that the catalog should be complete for large magnitudes. However, for a relevant number of cells, this condition is not met, and a large significant earthquake could take place and be missed from the catalog because of the saturation of the recording system.

The applied Bayesian method assumes that the FMD follows the GR distribution in each cell. In some cases, if the data show strong deviations from the linear GR trend, the uncertainties in the Bayesian posterior of $M_{\max}$ will increase. If the GR model does not represent the observed FMD, the method over- or underestimates $M_{\max}$ (in terms of confidence intervals). In such cases, it could be possible to use different FMD models, for example, to account for bimodal distributions; however, such models would require a larger number of parameters and are thus not considered in this study. We

instead point out that a large difference between $M_{\max}$ estimations at neighboring cells may result from different FMDs at those cells, as we verified for some examples.

Discussion

In this study, with the knowledge of the maximum recordable amplitude at each sensor of the network monitoring the Morsleben salt mine, we calculated $M_{c(\max)}$ for regions with a sufficient number of recorded events. Using this information, we found that the detectability of earthquakes does not necessarily increase but can also drop with increasing earthquake magnitude as a consequence of saturation problems.

Our approach to estimate saturation magnitudes $M_{c(\max)}$ is based on the assumption that, for any small source volume (cu-

Figure 6. Generalized calculation of $M_{c(max)}$ for cell-station pairs with linear and nonlinear relations between magnitude and log(maximum amplitude). Polynomial, multiple linear, and single linear regressions are performed for stations 1, 9, and 17, respectively. The corresponding detection probabilities for each sensor are shown in the third (bottom) row.

bic cells of 5 m edge length in our case), the event magnitude scales linearly with the logarithm of the maximum recorded amplitude. However, in a few cells and for specific sensors, we observed deviations from this expected behavior. Three illustrative cases are discussed in Figure 6. A first anomalous pattern (Fig. 6, sensor 1) is a curved distribution instead of

the expected linear trend. A second case, which has been observed for different cells, is the presence of two branches in the plot (Fig. 6, sensor 9). Different explanations are possible for these observations. One hypothesis is that events within a cell have different focal mechanisms. If we assume two dominant focal mechanisms with very different radiation patterns, for ex-

ample, this could explain the double branch plot and the fact that events with different magnitudes can produce comparable amplitudes with the same source–receiver geometry. A very likely alternative hypothesis involves the quality of the determined event parameters. If some of the events have large location errors or bad magnitude estimations, this can affect the observed distribution and would explain both observed anomalous patterns. This hypothesis is also supported by the fact that these anomalous features disappear when only the best-located events are plotted (for such a test, best-located events were chosen as those detected by a larger number of sensors; e.g., 15). Using only the best-located events may provide an option to infer reliable $M_{c(max)}$ estimations.

Although not used toward the determination of $M_{c(max)}$ in this study, we propose alternative approaches to perform a polynomial or a multiple regression fitting on the anomalous observed features (Fig. 6). It is clear that performing the linear regression for sensors 1 and 9 would provide wrong estimations of $M_{c(max)}$. We can infer the detection probability for these sensors upon a polynomial regression for sensor 1, a multiple regression for sensor 9, and a simple linear regression for sensor 17 (second row). Figure 6 (third row) shows the smoothed probability of detection following proposed regression fits for these sensors. The detection probability drops to zero when the $M_{c(max)}$ value is reached. In the case of a multiple regression (sensor 9), the probability first decreases to about 0.5 at a first magnitude threshold, as events from the first cluster of the scatter plot would be missed above this magnitude (M_{AE} 1.82). The probability drops finally to zero at a second estimated magnitude threshold (M_{AE} 3.9), above which also the events of the remaining cluster would be missed.

These more complex models for sensors 1 and 9 clearly provide an improved fitting and more consistent residuals. In other situations, it might be less straightforward to choose whether it is worth using a more complex model instead of a simple linear one. In such cases, statistical criteria such as the Akaike information criterion (AIC; Akaike, 1974) may be helpful to decide which is the most robust model to fit to the data set. In general, AIC provides a measure of the model fit by accounting for the number of free model parameters. It allows the comparison of the data fits of models with different complexities. We have largely avoided these fitting problems by ignoring spatial locations with low goodness-of-fit values. However, implementing such alternative, more complex fits for sensor–cell pairs with low goodness-of-fit values might further improve the spatial extent and resolution of the $M_{c(max)}$ estimations.

Finally, in the Morsleben mine, saturated seismograms are directly eliminated from the hypocentral location procedures, whereas in other networks, such traces might still be used for location determinations (e.g., Yang and Ben-Zion, 2010; Ringler *et al.*, 2012). In such cases, magnitudes may be biased and underestimated if saturated traces are used for magnitude estimation. Assuming that these situations are not very frequent, such events could perhaps be detected (and their magnitudes reprocessed) by checking, event per

event, the fit of amplitude–magnitude tuples to the average regression lines at multiple sensors.

Conclusion

The analysis of a large catalog of AE data from the Morsleben mine allowed us to show that the known problem of catalog incompleteness may not only affect the smallest events, but also the largest ones. The standard completeness approach (which is entirely described by the magnitude of completeness) through the assumption that the catalog is complete above this magnitude threshold, is here modified to assess a completeness magnitude interval, the boundaries of which we refer to as minimum and maximum magnitude of completeness.

Our method to infer the maximum magnitude of completeness ($M_{c(max)}$) is based upon the prior calculation of the magnitude for which each sensor will saturate. This enables correction of its function of the detection probability by accounting for the drop in the detection performance above the saturation magnitude. The detection probability functions of different sensors are then merged to reconstruct the detection performance of the whole network, in a fashion consistent with the PMC method introduced by Schorlemmer and Woessner (2008) and later refined by Plenkens *et al.* (2011) and Maghsoudi *et al.* (2013).

The inference of a maximum magnitude of completeness is important to assess whether large events may be missed by the monitoring routine and indicate that hazard-related studies may be biased. To verify if this is the case, we suggested estimating the spatial distribution of the maximum possible magnitudes (M_{max}) and comparing it with the spatial distribution of $M_{c(max)}$. For this approach, our M_{max} estimations were derived from a Bayesian approach, according to Holschneider *et al.* (2011). The application of this method to our data set indicates that M_{max} exceeds $M_{c(max)}$ in different places. In these locations, some large and important events may be missed in the catalog, underlining the importance of determining the maximum magnitude of completeness for the given monitoring system. Although our approach thus far has only applied to a specific monitoring system in a mining environment, it also can be applied to other microseismic monitoring networks and potentially for local or regional networks, where large earthquakes may occur close to one or few stations and saturate them.

Data and Resources

The acoustic emission catalog used in this study was provided by Bundesanstalt für Geowissenschaften und Rohstoffe (BGR), Germany.

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