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Statistical Stress Model Calibration

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SUMMARY

The estimation of orientation and magnitude of crustal stresses is crucial for the design phase of technological and safe underground usage. As usually only a few in-situ data are available, stress prediction is challenging. Geomechanical-numerical modelling is the only tool, which allows stress prediction that takes material properties and inhomogeneities into account. The model calibration is essential to find the best-fitting stress results. The presented workflow allows the statistically proved calibration to determine the best-fit model. Furthermore, statistic tests speed up the calibration process. The order in which model-independent data are used for model calibration pay attention to the interrelation between the stress components. Therefore, data of vertical stress are tested first to optimize material properties. Second, the orientation of the maximum horizontal stress is used to optimize orientation of applied boundary conditions. Finally, the magnitudes of minimum and maximum horizontal stress are varied to find the optimal strain, applied by the boundary conditions.

Introduction

The Earth's crust is used by the mankind for extraction of energy or minerals (hydrocarbon, geothermal energy etc.), for interim storage sites (gas and pressurized air), for way of transportation (tunnels) as well as for waste repositories (nuclear and chemicals). A safe and efficient underground usage is important, but also the (long-term) stability is crucial for all, the operator, the society and environment. Potential users of the underground need a good understanding of the local stress state greatest before the first well. This is important in terms of well stability or well configuration of several corresponding wells, in the case of reservoir stimulation by hydraulic fracturing. (e.g. Bell and McLellan, 1995; Peska and Zoback, 1995). An inadequate understanding of the spatial stress variability during the planning phase of a reservoir is a crucial point for the success (e.g. Brown, 2009; Duchane and Brown, 2002). However, stress prediction is also essential for any cavity like tunnels or nuclear waste repositories (e.g. Fuchs and Müller, 2001; Gunzburger and Magnenet, 2014; Heidbach et al., 2013).

The stress state is formally described with a second rank tensor with six independent components (e.g. Schmitt et al., 2012; Zang and Stephansson, 2010). Assuming that the vertical stress (S_V) is a principal stress component, the minimum and maximum horizontal stress (S_{Hmax} and S_{Hmin}) respectively, are also principal stresses. In this case the stress can be described with only four components, i.e. the orientation of S_{Hmax} as well as the three magnitudes of the principal stresses (S_V , S_{Hmax} and S_{Hmin}). Data of S_{Hmax} orientations are collected in the world stress map database (WSM - Zoback, 1992; Heidbach et al., 2010) since decades. For the magnitude data only a first attempt of database exists (Zang et al., 2012). The relation between the stress magnitudes defines the stress regime (faulting type) according Anderson's theory, which are thrust-, strike-slip- or normal faulting regime (Anderson, 1951).

As stress data are sparsely distributed for the most regions, geomechanical-numerical modeling is an important tool to predict the in-situ stresses still in the design phase of an underground project. The method allows incorporation of several petrological units and inhomogeneities like faults or detachments. Such models provide an estimate of the full stress tensor. Nevertheless, such models need calibration versus available in-situ stress data (e.g. Fischer and Henk, 2013; Henk, 2009). But uncertainties of calibration are usually unknown, as mostly just a few data are available (e.g. Fischer and Henk, 2013; Buchmann and Connolly, 2007; Heidbach et al., 2013). A major progress for reliability of models would be availability of a sufficient amount of data, so that a statistical calibration procedure could be applied. Such statistically calibrated models would improve the significance of numerical stress models. In the case that data are available for several or all stress components, the model have to be calibrated in a reasonable order. This is important as some components of the stress tensor are independent from boundary condition which will be applied in a later stage to fit other in-situ data.

Calibration of geomechanical-numerical stress models

Geomechanical-numerical models reproduce simplified the geometry of the chosen investigation volume. The model geometry is based on model-dependent data, which are seismic cross sections, well data etc. This geometry is combined with the internal forces (material properties like Young's modulus, Poisson's ratio) and external forces like body forces (gravity) as well as surface forces (boundary conditions). Models are calibrated versus model independent data which are S_V magnitudes from well logs, S_{Hmax} orientations from borehole breakouts, focal mechanisms etc., S_{Hmin} magnitudes from hydraulic fracturing (e.g. leak-off tests) as well as S_{Hmax} magnitudes from overcoring (or calculated data).

The here presented workflow allows the meaningful stepwise statistical calibration of the model on the in-situ data. The statistic test of one stress component compares each in-situ stress magnitude or orientation with the model outcome at the same position within the model. The difference between both is ΔS :

$$\Delta S = S_{Measured} - S_{Model} \quad (1)$$

Figure 1 Sketch of the calibration workflow. The geomechanical model is prepared based on the model geometry, the material properties, the variable displacement boundary conditions and the initial stress state. The numerically modelled total stress tensor is calibrated on model independent in-situ stress data. First the S_V magnitude, by testing the density, second the S_{Hmax} orientation testing the orientation of applied boundary conditions and third the S_{hmin} and S_{Hmax} magnitude is testing the offset of applied boundary conditions, until the model fits all the model independent data.

As a best fit is intended, the median of all ΔS ($\widetilde{\Delta S}$) from each stress component should be very small. Therefore the calibration procedure seek for a model where the median of ΔS is as small as possible ($\widetilde{\Delta S} = 0$).

Order of data calibration

The stress component which have to be tested first (Fig. 1) is the vertical stress (S_V), as S_V is marginal influenced by later applied variation of (horizontal) boundary conditions. S_V by majority is driven by the overburden load of the sedimentary column as a function of density and gravity. As gravity changes only slightly on earth in general as well as into accessible depth, rock density is the solely variable for modellers to optimize the model versus S_V data. Each in-situ S_V data is compared to the same point in the model (Eq. 2). The best fit model is found when the median ΔS_V ($\widetilde{\Delta S_V}$) is nearby zero ($\widetilde{\Delta S_V} = 0$).

$$\Delta S_V = S_{V \text{ Measured}} - S_{V \text{ Model}} \quad (2)$$

After final definition of material properties, the stress orientation have to be calibrated in the second step (Fig. 1). A stress model will have either a more or less isotropic horizontal stress state ($S_{Hmax} = S_{hmin}$) or an anisotropic stress state ($S_{Hmax} > S_{hmin}$). In an anisotropic case, applied model boundary conditions are

similar ($\epsilon_x = \epsilon_y$). In such a case stress orientations will be affected by inhomogeneities within the model like topography, lateral changing density or near to faults. This can lead to large variation of orientations (e.g. Hergert et al., 2015). An anisotropic stress state will be reached when a different strain is applied to the model boundaries ($\epsilon_x \neq \epsilon_y$). As long as small differential stresses are applied, model inhomogeneities have an influence. Above a certain strain ratio, the stress pattern depends on the applied strain isotropy; the stress pattern will change only slightly for much larger applied strain ratios. Therefore orientations of applied boundary conditions have to be calibrated first, before calibration of horizontal stress magnitudes. Each orientation of applied boundary conditions are tested using S_{Hmax} orientation data (e.g. borehole breakouts or focal mechanisms from the WSM). The Equation to calculate the differences between the model and the in-situ data differs to the general equation, as stress orientations are circular data ($0^\circ=180^\circ$) and therefore $\Delta S_{Hmax\ Azi}$ have to range between -90° and $+90^\circ$ (Eq. 3) The best-fit can be found for boundary conditions which provide a tiny median $\Delta S_{Hmax\ Azi}$ ($\Delta S_{Hmax\ Azi} = 0$).

$$\begin{aligned} \Delta S_{Hmax\ Azi} &= S_{Hmax\ Azi\ Meas} - S_{Hmax\ Azi\ Model} \\ &\quad - 90(\text{sgn}(S_{Hmax\ Azi\ Meas} - S_{Hmax\ Azi\ Model} - 90)) \\ &\quad + \text{sgn}(S_{Hmax\ Azi\ Meas} - S_{Hmax\ Azi\ Model} + 90)) \end{aligned} \quad (3)$$

The applied strain to a rock mass is mainly stored elastically until failure. Up to the occurrence of failure, the stress magnitude is a linear function of rock properties and applied offset at the model boundaries. Therefore, the S_{Hmax} and S_{hmin} magnitudes will be tested and calibrated together in one calibration step (Fig. 1). Again differences between the model and the in-situ data stress data are calculated for each data point (Eqs. 4 and 5).

$$\Delta S_{hmin} = S_{hmin\ Measured} - S_{hmin\ Model} \quad (4)$$

$$\Delta S_{Hmax} = S_{Hmax\ Measured} - S_{Hmax\ Model} \quad (5)$$

Figure 2 Visualization of bivariate linear regression based on four models. (a and b): the median S_{hmin} and the median S_{Hmax} are plotted depending on the north-west to south-east extension (pull) and the south-west to north-east shortening (push). The isolines of the median ΔS_{hmin} and ΔS_{Hmax} are colour coded. (c) The isolines, where the median ΔS_{hmin} and ΔS_{Hmax} is zero are plotted alone. The intersection of both isolines indicated the push-pull values where the best-fit model can be found (Reiter and Heidbach, 2014).

The calculated $\widetilde{\Delta S_{hmin}}$ and $\widetilde{\Delta S_{Hmax}}$ of $\geq$ three models can be plotted separately in a push vs. pull diagram (Fig. 2a and b). The push is parallel to the S_{Hmax} orientation, where pull is perpendicular to that. To highlight the linear dependency between push and pull in an elastic model, colour coded isolines are plotted. Each model along the light blue line (Fig. 2a) would derive a model, which fits well to the in-situ S_{hmin} data. The same stands for the light blue line in Fig. 2b and S_{Hmax} data. As the determination of the *best-fit* model is intended, the intersection of both light blue lines from Fig. 2a and b would derive the best-fit. This is done with a bivariate linear regression based on the spatial distribution of the

$\widetilde{\Delta S}_{\text{hmin}}$ and $\widetilde{\Delta S}_{\text{Hmax}}$ (Fig. 2c). This method provides two linear equations, where the intersection can be calculated by equating both equations. The result is an x and y value of applied push and pull on the model boundaries, which derives the boundary conditions of the best-fit model.

Alberta stress model

This work-flow is first applied for a crustal scale model from the Alberta Basin. The model extend is $1200 \times 700 \times 80$ km. For the model calibration 321 S_{Hmax} orientation data, 981 S_{V} , 1720 S_{hmin} as well as 2(+11) S_{Hmax} magnitude data are available (Reiter and Heidbach, 2014). The correlation coefficient between the model and the data are in a range of $r = 0.935$ for S_{V} and $r = 0.835$ for S_{hmin} for data sets with sufficient amount of data.

Conclusion / Outlook

The calibration workflow allows calibration of geomechanical-numerical stress model using statistical methods, seeking for the best-fit model. The general attempt is the minimization of differences between in-situ stress data vs. the model outcome, using the whole model median as a measure for each data type. The calibration procedure has the following order: Test of S_{V} data to optimize rock density, S_{Hmax} orientations to find optimal orientation of boundary conditions and S_{hmin} and S_{Hmax} magnitudes in one step together to optimize boundary displacements. However, usually there are significant less data available. Further investigation are needed, how much data are necessary, to get a best-fit model with assessable uncertainties. Moreover, the question arises, under which circumstances model results can be transferred to other model regions with a similar tectonic setting, i.g. the Alpine Molasse Basin?

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