Hainzl, S., Christophersen, A. (2016): Comment on “Revisiting the 1894 Omori Aftershock Dataset with the Stretched Exponential Function” by A. Mignan. - *Seismological Research Letters*, *87*, 5, pp. 1130—1133.

DOI: http://doi.org/10.1785/0220160098

Comment on "Revisiting the 1894 Omori Aftershock Dataset with the Stretched Exponential Function" by A. Mignan

by S. Hainzl and A. Christophersen

ABSTRACT

Mignan (2016) claimed that a stretched exponential function describes Omori's original aftershock data of the 1891 Great Nobi earthquake better than the well-known Omori law, which is a power law. Besides his preference for the stretched exponential function based on general physical reasoning, he proposed that the Omori law does not hold when the proper visualization method is used; that is, the complementary cumulative density function (CCDF) in a log–log plot. However, his proposed plot is misleading, because it compares data of a finite observation interval with functions integrated over infinite periods. Using the same data as Mignan (2016), we find that an appropriate comparison leads to visually indistinguishable fits for this dataset. The Omori law is preferred based on maximum-likelihood values. Moreover, the extrapolation shows that it also fits the long-term data until the centennial anniversary of the Nobi event significantly better than the stretched exponential.

INTRODUCTION

The decay of aftershock rate $n(t)$ is most commonly described by the modified Omori law, which is a power law of the form $n(t) \sim (t + c)^{-p}$ (Utsu *et al.*, 1995; Mignan, 2015). The time-offset parameter c is generally much smaller than one day and may be affected by the reduced detection ability of the operating seismic network following a large earthquake (Kagan, 2004; Hainzl, 2016). The decay parameter p typically takes values in the 0.8–1.2 range (Utsu *et al.*, 1995). If $p \leq 1$, the modified Omori law has to be truncated at a finite triggering time for the expected number of aftershocks to be finite. A finite triggering time is consistent with physical models (Hainzl *et al.*, 2016). For $1 < p < 1.2$ and without truncation, the theoretical duration of the modified Omori law exceeds the duration of earthquake catalogs. For example, for $c = 0.0083$ days (about 12 min) and $p = 1.07$ (the median of New Zealand aftershock sequences) it would take 165 days for 50% of aftershocks to occur and more than 200,000 years for 80% of the aftershocks to occur (Harte, 2013). Such long durations cannot be verified with existing data. Furthermore, the challenge of distinguishing between aftershocks and independent earthquakes would be accentuated.

Mignan (2015) showed that aftershock sequences in three regional earthquake catalogs generally followed a stretched exponential decay better than a power law, suggesting that aftershocks follow a simple relaxation process. In his analysis, Mignan employed the complementary cumulative density function (CCDF). At time t, the CCDF is defined as the integral between t and infinity of the probability density function that is normalized in the interval 0 to infinity. Thus the presentation of the modified Omori law as CCDF requires $p > 1$, despite some best-fitting observations suggesting $p \leq 1$. When comparing data to the CCDF, it is important that the data are complete; otherwise the data may fall below the CCDF curve for high t. To visually assess whether the data are a good fit, the CCDF needs to be normalized between 0 and $t_{\max}$, in which $t_{\max}$ is the time of the latest observation.

Mignan (2016) applied the CCDF to the first 795 days of daily aftershock observations for the 1891 Great Nobi earthquake. The Nobi sequence is famous for still having its aftershock continue 100 years after the mainshock (Utsu *et al.*, 1995). Here, we fit different decay functions to the same data as Mignan (2016) and apply statistical methods to find the best-fitting model. Furthermore, we compare the results with the continued recordings of the subsequent 98 years.

DATA AND METHOD

Following Mignan (2016), we used the daily number of events recorded at the station of Gifu between end of October 1891 and December 1893 (column *sum* of tables IV and V of Omori, 1894). As in Mignan (2016), we ignored the likely incomplete recordings at the day of the Great Nobi earthquake which occurred at 6:38 a.m. (local time) on 28 October 1891. Thus, the analyzed dataset is composed of 795 days with daily numbers

 doi: 10.1785/0220160098

Table 1
Tested Models and Maximum-Likelihood Results

Model	Functional Form	Parameters (Times Units Days)	LL	AIC
Omori	$K_0(c_0 + t)^{-1}$	$K_0 = 506.9; c_0 = 0.47$	−1474.5	2953.0
Omori–Utsu	$K_1(c_1 + t)^{-p}$	$K_1 = 551.7; c_1 = 0.63; p = 1.02$	−1473.8	2953.6
Stretched exponential	$K_2 t^{\beta-1} \exp(-\lambda t^{\beta})$	$K_2 = 548.9; \beta = 0.21; \lambda = 0.39$	−1476.4	2958.8

LL, log likelihood; AIC, Akaike information criterion.

between 0 and 318 and a total of 3295 events in the whole period.

We applied the maximum-likelihood method assuming nonstationary Poisson processes with mean values decaying according to the alternative decay laws. In particular, we tested the Omori law in its original and modified (Omori–Utsu) version (Utsu *et al.*, 1995) and the stretched exponential function. The functional forms of the tested models together with their estimated parameters are provided in Table 1. In particular, we calculated for each day i the expected number of events x_i by the integration of these functions from the start to the end time of this day. The probability that the observed number n_i results from a Poisson process with mean rate x_i is given by

$$p_i = x_i^{n_i} \exp(-x_i)/n_i!.$$

Thus, the log-likelihood (LL) value of the whole dataset becomes

$$\mathrm{LL} = \sum_{i=1}^{795} \ln(p_i) = \sum_{i=1}^{795} [n_i \ln(x_i) - x_i - \ln(n_i!)].$$

The parameter values listed in Table 1 are the result of a maximization of this LL-value by means of the Davidon–Fletcher–Powell optimization algorithm. The Omori–Utsu law yields the highest LL-value with a p-value of 1.02. However, the number N of free parameters is 3 instead of 2 in the case of the Omori function, which only yields a slightly worse LL-value. According to the Akaike information criterion (AIC), $\mathrm{AIC} = 2(N - \mathrm{LL})$ is the lowest for the Omori function, which is thus preferable. We verified that our result is not strongly biased by potential incompleteness in the first days of the sequence. Ignoring the first 5, 10, or 20 days for the LL-fit always leads to the highest values for the modified Omori decay function and a preference for the Omori function, based on the AIC-value.

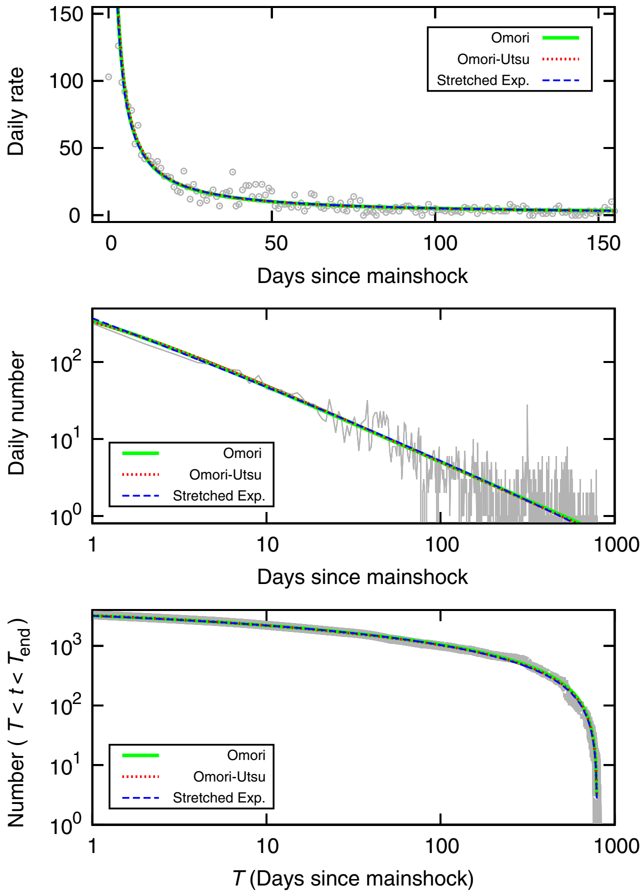
Figure 1 shows the alternative decay functions with optimized parameters in comparison to the observed data. All three models yield almost indistinguishable visual fits. Also in the case in which the cumulative number of events following a time T is plotted as function of T, all model curves overlap and fit the observations very well. This is in contrast to the CCDF plot of Mignan (2016), which suggests that the Omori law fails to reproduce the observations. However, the mistake of the previous representation is that aftershocks are recorded only in a finite time interval, although the aftershock process is ongoing, while the CCDF integrates the model expectation until infinite times, for which we have no information. If the integration of the models is constrained to the observational time interval, all models fit the observations very well, as shown in the bottom plot of Figure 1. This indicates that the two-year dataset is too short to visually distinguish the data fits.

In addition to Mignan (2016) who limited his analysis to the Omori (1894) aftershock data of the first 2 years, we compare the resulting decay functions with the 100-years-aftershock data of the Great Nobi earthquake presented by Utsu *et al.* (1995). We do not have access to the data used in figure 1 of Utsu *et al.* (1995), which refer to the extended record of station Gifu until 1991. However, in Figure 2 we plotted the fitted functions from Table 1 on top of the long-term data. Both power-law functions fit the decay curve (Fig. 2a) well for all times, whereas the stretched exponential starts falling below the data from about 1000 days. Comparing the percentages of recorded events as a function of time (Fig. 2b), again the power-law functions follow the data well for all times, whereas the stretched exponential has a different shape than the data.

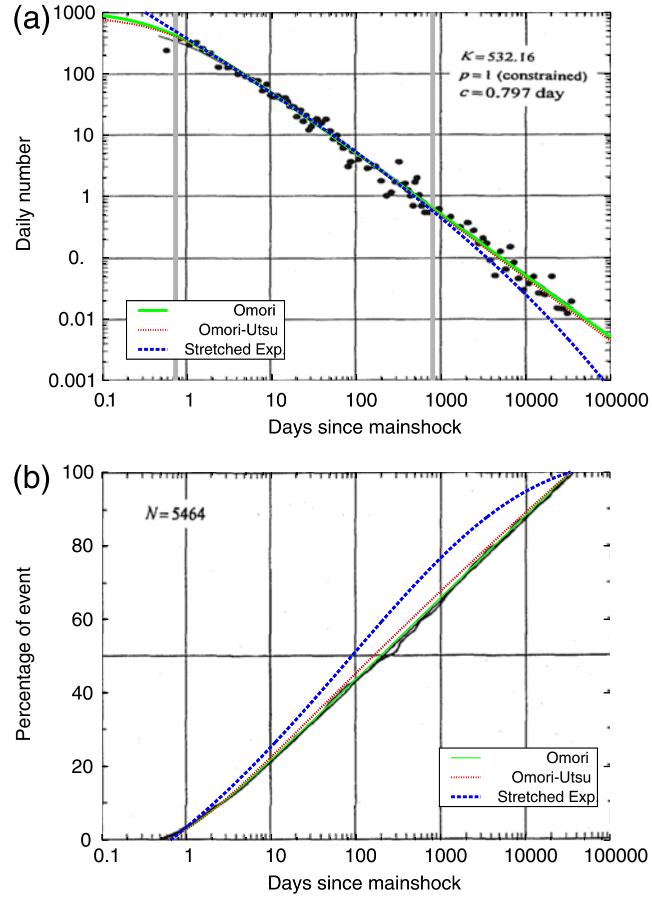
Often aftershock decay is more complicated than for the Great Nobi earthquake, in that large aftershocks trigger secondary sequences. For that purpose, the epidemic-type aftershock sequence (ETAS) model has been developed (Ogata, 1988). The ETAS model is used for aftershock modeling in California (Helmstetter *et al.*, 2006), Italy (Console *et al.*, 2003), Japan (Zhuang *et al.*, 2004), and New Zealand (Harte, 2013; Rhoades, 2013). The power-law decay of aftershocks can make it difficult to estimate the duration of an aftershock sequence, which is important for seismic-hazard modeling. However, the introduction of a finite duration of the Omori–Utsu-type triggering has been recently shown to lead to improved data fits of the ETAS model (Hainzl *et al.*, 2016). As a consequence, the ETAS parameters are more closely aligned with physical models for aftershock decay. Work on trials using the stretched exponential as alternative decay law in the ETAS framework is still ongoing.

CONCLUSION

Based on a misleading CCDF plot and some physical reasoning, Mignan (2016) suggested that the stretched exponential func-



▲ **Figure 1.** The aftershock data (gray) of Omori (1894) visualized using three different plotting methods, similar to figure 2 of Mignan (2016). The curves represent the maximum-likelihood fits of the Omori (solid), Omori–Utsu (dotted), and the stretched exponential function (dashed), which are almost indistinguishable. In contrast to Mignan's misleading bottom plot of the complementary cumulative density function, we show the comparison between the observed and expected number of events in the remaining time of the observation interval as the function of the starting time of this interval. The color version of this figure is available only in the electronic edition.



▲ **Figure 2.** The fitted functions superposed on the extended data until 1991 published in figure 1 of Utsu *et al.* (1995): (a) daily rate and (b) the cumulative percentage of the total number of observed aftershocks as a function of time. Note that the functions were only fitted between calendar days [1, 795] after the mainshock (indicated by vertical bars). The color version of this figure is available only in the electronic edition.

tion is a better representation for the original dataset of Omori than the Omori law. An alternative aftershock decay function could have important consequences for seismic-hazard estimation and thus needs careful testing. In this comment, we discuss neither the physical considerations nor the general applicability of the stretched exponential function to aftershock sequences (Mignan, 2015), but we show by quantitative fitting and appropriate visual representation that this conclusion cannot be drawn based on the empirical data of the Nobi sequence. According to maximum-likelihood estimates, the Omori law is preferable for this dataset. The two power-law functions also visually fit the long-term data for Nobi significantly better than the stretched exponential. However, aftershock sequences are, in general, more complex and better fitted by the ETAS model instead of single decay functions.

DATA AND RESOURCES

All data used in this article came from the published sources listed in the references.

ACKNOWLEDGMENTS

The New Zealand Earthquake Commission Time Varying Hazard Programme provided financial support through Project 14/TV682. We are grateful to our colleagues David Rhoades and Torsten Dahm for critical reading and valuable comments.

REFERENCES

Console, R., M. Murru, and A. M. Lombardi (2003). Refining earthquake clustering models, *J. Geophys. Res.* **108,** no. B10, doi: 10.1029/2002JB002130.

Hainzl, S. (2016). Rate-dependent incompleteness of earthquake catalogs, *Seismol. Res. Lett.* **87,** 337–344.

Hainzl, S., A. Christophersen, D. Rhoades, and D. Harte (2016). Statistical estimation of the duration of aftershock sequences, *Geophys. J. Int.* **205,** 1180–1189.

Harte, D. S. (2013). Bias in fitting the ETAS model: A case study based on New Zealand seismicity, *Geophys. J. Int.* **192,** 390–412.

Helmstetter, A., Y. Y. Kagan, and D. D. Jackson (2006). Comparison of short-term and time-independent earthquake forecast models for southern California, *Bull. Seismol. Soc. Am.* **96,** 90–106.

Kagan, Y. Y. (2004). Short-term properties of earthquake catalogs and models of earthquake source, *Bull. Seismol. Soc. Am.* **94,** no. 4, 1207–1228.

Mignan, A. (2015). Modeling aftershocks as a stretched exponential relaxation, *Geophys. Res. Lett.* **42,** no. 22, 9726–9732, doi: 10.1002/2015GL066232.

Mignan, A. (2016). Revisiting the 1894 Omori aftershock dataset with the stretched exponential function, *Seismol. Res. Lett.* **87,** no. 3, 685–689.

Ogata, Y. (1988). Statistical models for earthquake occurrence and residual analysis for point processes, *J. Am. Stat. Assoc.* **83,** 9–27.

Omori, F. (1894). On after-shocks of earthquakes, *J. Coll. Sci. Imp. Univ. Tokyo* **7,** 111–200.

Rhoades, D. A. (2013). Mixture models for improved earthquake forecasting with short-to-medium time horizons, *Bull. Seismol. Soc. Am.* **103,** 2203–2215.

Utsu, T., Y. Ogata, and R. S. Matsu'ura (1995). The centenary of the Omori formula for a decay of aftershock activity, *J. Phys. Earth* **43,** 1–33.

Zhuang, J. C., Y. Ogata, and D. Vere-Jones (2004). Analyzing earthquake clustering features by using stochastic reconstruction, *J. Geophys. Res.* **109,** no. B05301, doi: 10.1029/2003JB002879.

S. Hainzl
GFZ German Research Centre for Geosciences
Telegrafenberg
14473 Potsdam, Germany
hainzl@gfz-potsdam.de

A. Christophersen
GNS Science
1 Fairway Drive
Avalon 5010, New Zealand

Published Online 13 July 2016