



Originally published as:

Müller, A. (2000): A new method to compensate for bias in magnetotellurics. - *Geophysical Journal International*, 142, 1, pp. 257—269.

DOI: <https://doi.org/10.1046/j.1365-246x.2000.00152.x>

A new method to compensate for bias in magnetotellurics

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Accepted 2000 March 7. Received 1999 December 12; in original form 1999 February 16

SUMMARY

Error estimates from statistical regression analysis are often obviously too small, leading to doubts about the given equations, the statistical method itself and finally, with resignation, to the conclusion that mathematical equations and reality never agree. However, for magnetotelluric data we have found an almost perfect fit between observed scattering and predicted confidence limits of regression coefficients after accounting for a systematic error—the bias.

Different methods to compensate for bias in magnetotelluric impedance estimation have been described using additional data from a reference station. However, sufficiently accurate reference data are often not available. A new method has been developed that enables bias compensation without additional data. For the new method we derive a linear relationship between the effect of bias and an expression depending on the data fit. From this we extrapolate the solution for the unbiased impedance. The new method assumes a special model of uncorrelated noise as well as an approximation for the structure of the impedance tensor. From each pair of components of the unrotated impedance tensor corresponding to the same output channel, one of the pair can be compensated if its magnitude is large compared to that of the other.

The method has been successfully applied in many cases. We claim that the solution is closer to the true impedance than any solution based on the selection of events. It gives a measure of the partitioning of noise between the electric and magnetic channels.

We applied the method to measurements from the North Anatolian Fault Zone (Turkey) and from the Merapi volcano (Central Java) in the period range 10–2500 s. Different instrumentation was used for the two sets of measurements, but in both cases we used fluxgate magnetometers to measure the magnetic variations.

Key words: data processing, magnetotellurics, statistical methods.

1 INTRODUCTION

In regression analysis we estimate the relationship between at least two quantities and describe this relationship by regression coefficients. Usually it is necessary to distinguish between input and output channels. The input channels are assumed to be ‘noise-free’ while for the output we expect to have some data deviating from the estimated relationship due to Gaussian noise added to the output data.

The coherence between measured and predicted output data (CMP) describes the data fit. In cases of uncorrelated noise, a CMP close to unity indicates an overall agreement between the data and prediction, and hence little noise and thus a small error in the measurements. Much smaller CMPs than unity indicate noise in the data. Standard regression analysis does not give us information on whether the noise is in the input or the output channels. However, the distinction between the two cases is important. If the noise is in the output channel, our estimation is correctly carried out. Confidence limits from the regression analysis then accurately describe the uncertainty of our derived regression coefficients. However, if the noise is mainly in the input channel, then the assumptions and therefore the subsequent estimates are wrong, in particular the estimated confidence limits. Our estimates are then biased.

Common stacking techniques use the fact that the amount of possible bias depends on the CMP. With a CMP close to unity, the decision on which channel will be called input or output makes no difference to the result, as we have little bias. Regression coefficients and CMPs based on different data subsets are calculated. Only the data subsets with a CMP above a certain threshold are used to contribute to the stacked solution.

In the technique described here all estimates from the data subsets are used. We observe the relationship between regression coefficients based on different data subsets and the CMPs. If there is a systematic behaviour, we have bias. If the results are independent of the CMP, then we have evidence that our assumptions are correct, and can continue with standard regression analysis.

If bias is present, we can go one step further if we can describe the systematic behaviour of the coefficients in relation to the CMPs. Then we are able to extrapolate a solution for a CMP equal to unity and thus we can find an optimal solution.

2 REGRESSION ANALYSIS IN MAGNETOTELLURICS

In magnetotellurics the impedance \mathbf{Z} is estimated from the linear equations

$$E_x = Z_{xx}B_x + Z_{xy}B_y, \quad (1a)$$

$$E_y = Z_{yx}B_x + Z_{yy}B_y, \quad (1b)$$

where \mathbf{E} and \mathbf{B} are measured horizontal electric and magnetic field variations transformed into the frequency domain. This special case of a regression problem with two input variables B_x and B_y and without a constant offset is called a bivariate problem. The elements of the impedance tensor \mathbf{Z} are complex and frequency-dependent. From the impedance \mathbf{Z} one can derive apparent resistivities and phases using

$$\frac{\rho_{ij}}{[\Omega\text{m}]} \approx \frac{T/[\text{s}]}{5} \frac{|Z_{ij}|^2}{[\text{mV}/(\text{nT km})]^2} \quad (2)$$

and

$$\varphi_{ij} = \arg(Z_{ij}), \quad (3)$$

with $i, j \in \{x, y\}$.

One critical point is the numerical estimation of the impedance tensor from measured data. While the solution from eq. (1) is straightforward for high-quality data, it is more problematic to estimate the true impedance \mathbf{Z} when the electric and magnetic fields depart from the relation represented by eq. (1). For our purposes, we call these influences ‘noise’. Sources of noise are as follows.

(i) Source effects. One condition for eq. (1) to hold is that the electromagnetic source is uniform with respect to the area examined. This assumption might fail because of manmade noise due to mechanical sources such as mains, trains, pipelines, electric machines, transmitters, etc., or may be attributed to the natural variations of ionospheric source configuration.

(ii) Instrumental effects. Sensor noise, amplifier effects and the truncation errors of digitization.

For a more detailed description of electromagnetic noise see Szarka (1988) and Junge (1996).

To account for these factors, a large number of time windows (events) in narrow frequency bands are used to obtain an overdetermined estimation of the impedance \mathbf{Z} . For this step one needs to decide on how to determine the result from the overestimated equation, which means making an assumption about the character of noise in order to get a good estimation of the true impedance. In other words, we need a noise model.

3 THE LEAST-SQUARES SOLUTION

Classically one assumes noise-free data in the magnetic measurement and Gaussian noise in the Fourier coefficients of the electric field. Then the best solution is obtained from minimizing the squared residuals of the electric fields with the equation (Sims *et al.* 1971)

$$\hat{Z}_{xy} = \frac{(\overline{B_x B_x^*})(\overline{E_x B_y^*}) - (\overline{B_x B_y^*})(\overline{E_x B_x^*})}{(\overline{B_x B_x^*})(\overline{B_y B_y^*}) - (\overline{B_x B_y^*})(\overline{B_y B_x^*})}, \quad (4)$$

where an asterisk indicates a conjugate complex, an overbar indicates a stack over events and frequencies in one band, and a hat indicates an estimate, with similar equations for the other components (here and in the following we will use just the equations corresponding to the impedance component Z_{xy} in order to avoid confusion).

The (squared) coherence between two complex parameters A , B is defined by

$$\text{Coh}^2(A, B) = \frac{|\overline{AB^*}|^2}{(\overline{AA^*})(\overline{BB^*})}. \quad (5)$$

The squared CMP is given by

$$\text{Coh}^2(E_x, E_x^p) = \frac{|\overline{E_x E_x^{p*}}|^2}{(\overline{E_x E_x^*})(\overline{E_x^p E_x^{p*}})}, \quad (6)$$

where the predicted field is defined by

$$E_x^p = \hat{Z}_{xx} B_x + \hat{Z}_{xy} B_y \quad (7)$$

and \hat{Z}_{xx} , \hat{Z}_{xy} are the statistically estimated impedances.

If the impedances are determined by minimization of the electric residual according to eq. (4), one can write more concisely

$$\text{Coh}^2(E_x, E_x^p) = \frac{\hat{Z}_{xx}(\overline{B_x E_x^*}) + \hat{Z}_{xy}(\overline{B_y E_x^*})}{(\overline{E_x E_x^*})}. \quad (8)$$

The CMP gives a measure of the validity to the assumptions. While the CMP is preferably close to 1, the coherence between the magnetic channels $\text{Coh}^2(B_x, B_y)$ should be low so that one can associate a variation in the electric field with the magnetic channel variation that causes it.

These two terms can be found in the equation describing the statistical error for each component of the impedance Z (Schmucker 1978; Pedersen 1982):

$$(\Delta Z_{xy})^2 = k \frac{F(k, 2N - 4, \delta = 0.05)}{2N - 4} \frac{[1 - \text{Coh}^2(E_x, E_x^p)] \overline{E_x E_x^*}}{[1 - \text{Coh}^2(B_x, B_y)] \overline{B_y B_y^*}}, \quad (9)$$

where $\delta = 0.05$ is the significance level corresponding to a 0.95 or 95 per cent confidence limit, N is the number of complex Fourier coefficients used, $2N - 4$ is the degree of freedom, $F(m, n, \delta)$ is the factor of the F -distribution (see mathematical tables, e.g. Bronstein & Semendjajew 1981) and k is 4 for the confidence limit of $|Z_{xy}|^2$ (Schmucker 1978) and 1 for the confidence limit of $\Re(Z_{xy})$ or $\Im(Z_{xy})$ (Pedersen 1982).

In a corresponding way we can assume that all noise is in the magnetic channels. Writing eq. (1) with the admittance tensor A ,

$$B_x = A_{xx} E_x + A_{xy} E_y, \quad (10a)$$

$$B_y = A_{yx} E_x + A_{yy} E_y, \quad (10b)$$

gives an analogous system to eqs (4), (8) and (9),

$$\hat{A}_{yx} = \frac{(E_y E_y)(B_y E_x) - (E_y E_x)(B_y E_y)}{(E_x E_x)(E_y E_y) - |E_x E_y|^2} \quad (11)$$

(A_{yx} is the admittance component corresponding to Z_{xy} because it describes the relationship between E_x and B_y),

$$\text{Coh}^2(B_y, B_y^p) = \frac{\hat{A}_{yx}(\overline{E_x B_y^*}) + \hat{A}_{yy}(\overline{E_y B_y^*})}{(\overline{B_y B_y^*})}, \quad (12)$$

$$(\Delta A_{yx})^2 = k \frac{F(1, 2N - 4, \delta = 0.05)}{(2N - 4)} \frac{[1 - \text{Coh}^2(B_y, B_y^p)] \overline{B_y B_y^*}}{[1 - \text{Coh}^2(E_x, E_y)] \overline{E_x E_y^*}}. \quad (13)$$

One can calculate the impedance components from the admittance by inversion of the tensor

$$Z = A^{-1}, \quad (14)$$

or, more concisely,

$$\frac{1}{Z_{xy}} = \frac{(\overline{B_x E_x^*})(\overline{E_x E_y^*}) - (\overline{B_x E_y^*})(\overline{E_x E_x^*})}{(\overline{B_x E_x^*})(\overline{B_y E_y^*}) - (\overline{B_x E_y^*})(\overline{B_y E_x^*})} \quad (15)$$

(Sims *et al.* 1971).

Impedances for periods above 0.1 s are rarely estimated in this way for two reasons.

(1) The magnetic measurements are less affected by noise than the electric measurements, which may be more significantly disturbed by, for example, electrochemical effects at the buried electrodes.

(2) The electric field is polarized due to anisotropy (e.g. Eisel & Haak 1999) or a narrow lateral resistivity contrast (e.g. Müller 1997). This leads to a value of the electric coherence close to unity in a given coordinate system, and therefore, from eq. (13), to a large error.

Comparing eqs (4) and (15) resulting from minimizing the electric noise and the magnetic noise shows that the outcome may not be the same. Only for noise-free data, i.e. CMPs close to 1, is the result the same.

4 THE BIAS

Aside from other problems, one critical point in the method described above is the assumption of noise-free data in either the electric or the magnetic data. In the equations for the estimation of Z_{xy} (eqs 4 and 15), this assumption is implicit in the use of autopower spectra. Under the assumption of noise uncorrelated with the signal,

$$\overline{A^S B^{N*}} = 0,$$

where $A, B \in \{E_x, E_y, B_x, B_y\}$, A^S is the signal in component A and A^N is the noise in component A , one can write

$$\overline{AB^*} = \overline{(A^S + A^N)(B^{S*} + B^{N*})} = \begin{cases} \overline{A^S B^{S*}} & (A \neq B) \\ \overline{A^S A^{S*}} + \overline{A^N A^{N*}} & (A = B) \end{cases}. \quad (16)$$

While the estimates for the cross-power spectra are statistically distributed and yield a good approximation to the true value (as long as the noise is not correlated amongst different channels), the estimates of the autopowers are systematically too large, even for uncorrelated noise, because the noise is squared and therefore positive.

Many techniques have been applied to this problem.

(1) Coherency weights: from a number of events one preferentially weights those with a high CMP in order to increase the signal-to-noise ratio (Egbert & Livelybrooks 1996). In some cases events with a low CMP are rejected (e.g. $\text{CMP} > 0.8$; Spitzer 1987). Larsen *et al.* (1996) rejected events with very low as well as very high CMPs, the latter in order to avoid bias by correlated noise.

(2) Using a different approach Ritter *et al.* (1998a) chose effects with a high coherence between two magnetic stations.

(3) Remote reference: Goubau *et al.* (1978) and Gamble *et al.* (1979) used magnetic data from a second station in order to avoid autopowers. Combined with robust estimations, this technique is widely used and proves its effectiveness in a comparison of processing techniques (Jones *et al.* 1989).

(4) RMEV: Egbert (1997) used a so-called robust multivariate errors in variables algorithm in order to calculate correlated and uncorrelated noise levels iteratively by using data from multiple stations.

In the technique described the impedance at a single site is estimated for different time sections separately. The dependence of the different estimates on the coherences is then examined in order to extrapolate a value for the CMP equal to unity. Field data will demonstrate that it works well for the observed cases.

5 BIAS SUBJECT TO PARTICULAR ASSUMPTIONS

Following Pedersen (1982) we assume that just the autopowers are biased. From Pedersen (1982) we adopt the following notation:

$$\overline{AB^*} = \overline{(A^S + A^N)(B^{S*} + B^{N*})} = \begin{cases} \overline{A^S B^{S*}} =: (AB) = (AB)^0 & (A \neq B) \\ \overline{A^S A^{S*}} + \overline{A^N A^{N*}} =: (AB)^b = (AA)^0 + \Delta(AA) & (A = B) \end{cases}, \quad (17)$$

where $(AB)^0$ is the unbiased cross-power, $(AA)^0$ is the unbiased autopower, $(AA)^b$ is the biased autopower, $\Delta(AA)$ is the autopower of noise, Z_{xy}^0 is the unbiased impedance and Z_{xy}^b is the biased impedance. We insert this into the equation for the impedance estimation eq. (4),

$$Z_{xy}^b = \frac{[(B_x B_x)^0 + \Delta(B_x B_x)](E_x B_y) - (B_x B_y)(E_x B_x)}{[(B_x B_x)^0 + \Delta(B_x B_x)][(B_y B_y)^0 + \Delta(B_y B_y)] - |B_x B_y|^2}, \quad (18)$$

and replace unbiased components in the numerator by the unbiased impedance,

$$Z_{xy}^b = \frac{Z_{xy}^0 [(B_x B_x)^0 (B_y B_y)^0 - |B_x B_y|^2] + \Delta(B_x B_x)(E_x B_y)}{[(B_x B_x)^0 + \Delta(B_x B_x)][(B_y B_y)^0 + \Delta(B_y B_y)] - |B_x B_y|^2}. \quad (19)$$

Multiplying eq. (1a) for the unbiased impedances by B_y^S ,

$$(E_x B_y) = Z_{xx}^0 (B_x B_y) + Z_{xy}^0 (B_y B_y)^0, \quad (20)$$

substituting for $(E_x B_y)$ in eq. (19) and transforming the first term in the numerator gives

$$Z_{xy}^b = \frac{Z_{xy}^0 [(B_x B_x)^b (B_y B_y)^b - |B_x B_y|^2 - (B_x B_x)^b \Delta(B_y B_y) - (B_y B_y)^0 \Delta(B_x B_x)] + \Delta(B_x B_x) [Z_{xx}^0 (B_x B_y) + Z_{xy}^0 (B_y B_y)^0]}{(B_x B_x)^b (B_y B_y)^b - |B_x B_y|^2}, \quad (21)$$

and after some algebraic manipulation (autopowers are real; therefore multiplication is commutative)

$$Z_{xy}^b = Z_{xy}^0 \left(1 - \frac{(B_x B_x)^b \Delta(B_y B_y) - \left[\frac{Z_{xx}^0}{Z_{xy}^0} (B_x B_y) \right] \Delta(B_x B_x)}{(B_x B_x)^b (B_y B_y)^b - |B_x B_y|^2} \right). \quad (22)$$

To obtain a relation between biased and unbiased impedances, two further assumptions will be made.

Assumption 1

This assumption is justified by the typical structure of the magnetotelluric impedance tensor.

In the case of a uniform half-space, the diagonal elements of the matrix in eq. (2), $\rho_{xx} = \rho_{yy} = 0$, and the off-diagonal elements, $\rho_{xy} = \rho_{yx} = \rho$, are equal to the true resistivity of the half-space, while the phases $\varphi_{xy} = \varphi_{yx} + 180^\circ = 45^\circ$ and both φ_{xx} and φ_{yy} are undefined.

In more realistic cases, that is, those of a heterogeneous resistivity distribution, the diagonal elements are non-zero. However, in most cases the off-diagonal components of the impedance tensor are large compared with the diagonal components.

For the numerator in eq. (22) we assume

$$(B_x B_x)^b \Delta(B_y B_y) \gg \left| \frac{Z_{xx}^0}{Z_{xy}^0} (B_x B_y) \Delta(B_x B_x) \right| = \frac{|Z_{xx}^0|}{|Z_{xy}^0|} |(B_x B_y)| \Delta(B_x B_x) \quad (23)$$

under the conditions

$$|Z_{xx}^0| \ll |Z_{xy}^0|, \quad \Delta(B_x B_x) \approx \Delta(B_y B_y),$$

$$(B_x B_x)^b \approx (B_y B_y)^b \Rightarrow (B_x B_x)^b > |(B_x B_y)|.$$

If the assumption is inaccurate in an arbitrary coordinate system, one might want to fulfil this condition by rotating the impedance tensor \mathbf{Z} . However, this would have consequences for the equations because the noise between the channels would then be correlated after the rotation. For the impedances examined here a rotation was unnecessary.

With the approximation eq. (23) we can neglect the term in the squared brackets of eq. (22). Together with the biased magnetic coherence $\text{Coh}_b^2(B_x, B_y)$ (eq. 5), we can write

$$Z_{xy}^b \approx Z_{xy}^0 \left(1 - \frac{\Delta(B_y B_y)/(B_y B_y)^b}{1 - \text{Coh}_b^2(B_x, B_y)} \right). \quad (24)$$

In the limit (23) the bias effect is mainly due to noise in one of the magnetic components. In contrast to an approximation by Pedersen (1982), the relation between the biased and the unbiased impedances contains the observable biased coherence of the magnetic field variations instead of the unknown unbiased coherency.

Assumption 2

The amount of noise in the component B_y is still unknown. An upper limit is given by the deviation of the CMP from unity $[1 - \text{Coh}_b^2(B_y, B_y^p)]$. However, this term is a function of noise in B_y , as well as noise in the electric components.

We define a new quantity, the average noise distribution index (ANDI α):

$$\frac{\Delta(B_y B_y)}{(B_y B_y)^b} = \alpha [1 - \text{Coh}_b^2(B_y; E_x, E_y)]. \quad (25)$$

The assumption we use is

$$\alpha \approx \text{const} \quad (26)$$

for different events in each period band at one station. The assumption, empirically found, is certainly not true for all kinds of data. It will be tested for several cases below, before we argue why this assumption should be valid. To give a preliminary interpretation of the ANDI α we can use a first-order Taylor approximation derived by Pedersen (1982) for small autopowers of the noise:

$$1 - \text{Coh}_b^2(B_y, B_y^p) \approx \frac{\Delta(B_y B_y)}{(B_y B_y)^0} + \frac{\Delta(E_x E_x)}{(E_x E_x)^0}. \quad (27)$$

For little relative noise,

$$\Delta(B_y B_y) \ll (B_y B_y)^0; \Delta(E_x E_x) \ll (E_x E_x)^0; \text{Coh}_b^2(B_y, B_y^p) \approx \text{Coh}_b^2(B_y, B_y^p),$$

the ANDI α is given by

$$\alpha \approx \frac{\frac{\Delta(B_y B_y)}{B_y B_y}}{\frac{\Delta(B_y B_y)}{B_y B_y} + \frac{\Delta(E_x E_x)}{E_x E_x}}, \quad (28)$$

with $0 < \alpha < 1$.

The ANDI α describes the ratio of relative noise in the component B_y to the total relative noise. With this parameter we achieve a generalization of the assumptions typically used:

- (1) noise-free magnetic data, which lead to a minimization of the noise in the electric field components corresponding to $\alpha = 0$;
- (2) noise-free electric data, which lead to a minimization of the noise in the magnetic field components corresponding to $\alpha = 1$.

Olsen (1998) introduced an analogue variable η relating noise in the output and input channels for the solution of a univariate problem, the determination of the so-called C -response. He assumed η to be constant over the measurement time of 90 months, but additionally for different sites and frequencies (in the period range 3–720 hr). From that he determined a value for α by minimizing the misfit of the resulting C to a 1-D resistivity distribution at each station. He stated himself that the frequency-independence of his variable is questionable, but argued that the assumption is better than assuming one measured component as error-free.

The linear equation

Inserting definition (25) into eq. (24) yields

$$Z_{xy}^b \approx Z_{xy}^0 - Z_{xy}^0 \alpha_{xy} \left(\frac{1 - \text{Coh}_b^2(B_y, B_y^p)}{1 - \text{Coh}_b^2(B_x, B_y)} \right). \quad (29)$$

By defining a misfit factor q_y^b (depending on B_y),

$$q_y^b := \left(\frac{1 - \text{Coh}_b^2(B_y, B_y^p)}{1 - \text{Coh}_b^2(B_x, B_y)} \right), \quad (30)$$

we can write

$$Z_{xy}^b \approx Z_{xy}^0 - Z_{xy}^0 \alpha_{xy} q_y^b. \quad (31)$$

Eq. (31) is a linear equation for Z_{xy}^b in terms of q_y^b . Regression analysis enables the intercept Z_{xy}^0 and the gradient $\alpha_{xy} Z_{xy}^0$ to be determined. It will be shown empirically that the assumptions are valid in many cases.

6 APPLICATION TO MAGNETOTELLURIC DATA FROM THE NORTH ANATOLIAN FAULT ZONE (TURKEY)

As a first application, we use a long-term magnetotelluric time-series measured from May 1993 to August 1995 at the North Anatolian Fault Zone (Turkey). The measurements were carried out in the framework of the Joint Turkish–German Earthquake Research Project (Berckhemer *et al.* 1991) in order to identify resistivity changes connected to tectonic activity. We used a newly developed data logger HESLOG 08/20 (Erkul *et al.* 1994) and fluxgate magnetometers MAG-03MC (Bartington) for the magnetic measurements. The analogue signal from the fluxgate magnetometer was neither compensated nor high-pass filtered. This unusual configuration was used in order to avoid additional time-dependent signals from analogue electronic parts during the long-term measurements. With the measurement of the large offset in B_x and B_z , we had diagnostic control of the system because changes inside the instrument of about 1 per cent would have been easily detected by an anomalous change in the absolute values. A disadvantage of this method is the inefficient use of the dynamic range of the preamplifier as well as the AD converter. Thus we expected the magnetic measurements to be noisier than usual.

6.1 Estimation of the effect of bias

We divided these time-series into 105 independent data sets and estimated the impedances for each set. Each set contained at least 4096 values for every channel with a sampling period of 60 s. Each section was processed using a program from Egbert & Booker (1986). While the impedances estimated for periods above 2500 s seem to be stable, the impedances below 2500 s show an excessive scattering. This could have been incorrectly interpreted as a subterranean resistivity change.

In Fig. 1, the real and imaginary parts of the largest component Z_{xy} are shown, sorted by the amplitude of Z_{xy} . Additional terms from statistical error-analysis (eq. 9) are plotted. Error bars are calculated for a 95 per cent confidence limit. A significant number of impedances deviate from the mean value, although all of them have been estimated for data from the same site.

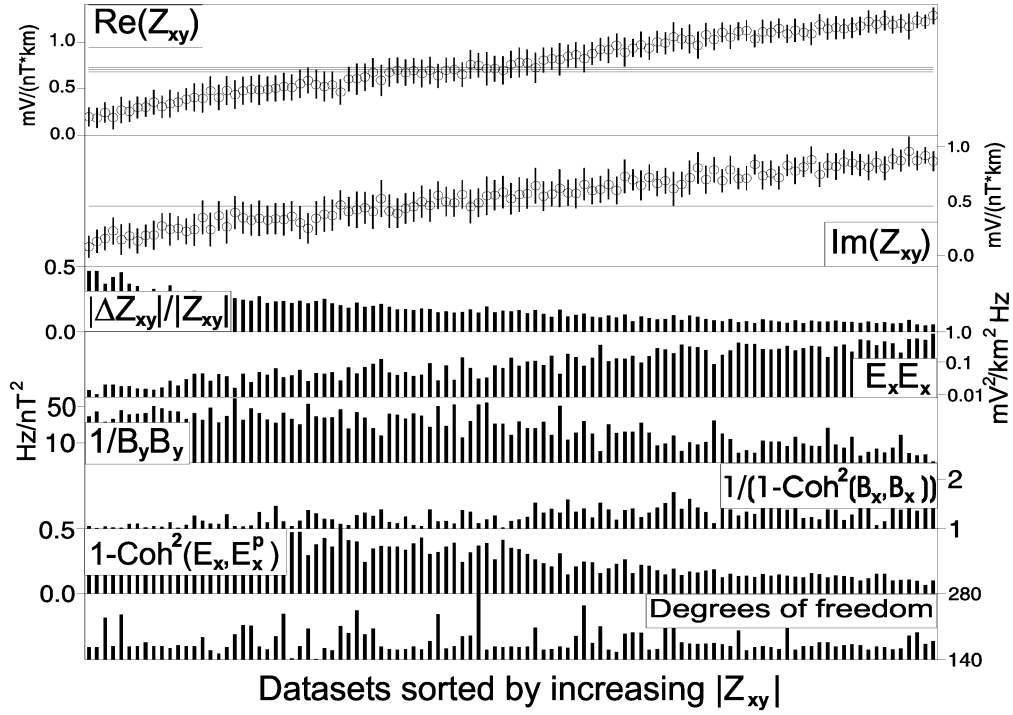


Figure 1. Comparison of terms in eq. (9) for the confidence limits of Z with the 105 estimated impedance components Z_{xy} for the period range 400–510 s. The sections are sorted by the amplitude of Z_{xy} for comparison.

Systematic correlation can be seen between the impedances, the autopowers $E_x E_x$, $B_y B_y$ and the CMP. A systematic error can be seen in the estimation of the impedances, which should, of course, be independent of the energy of the field variations.

The correlation can be explained by biased impedance estimates. Low energy in the electric and magnetic field variations leads to a low signal-to-noise ratio and consequently to a small CMP. However, the impedances should not be affected by this if the noise is in the electric channels. This indicates that at least a part of the noise comes from the magnetic measurements, or in other words, the ANDI α is larger than zero.

Fig. 2 shows Z_{xy} from Fig. 1 plotted against the misfit factor from the linear equation (31). It appears that the linear relation is a useful approximation for a wide range of coherences. The whole variety of estimates can easily be reduced to a constant

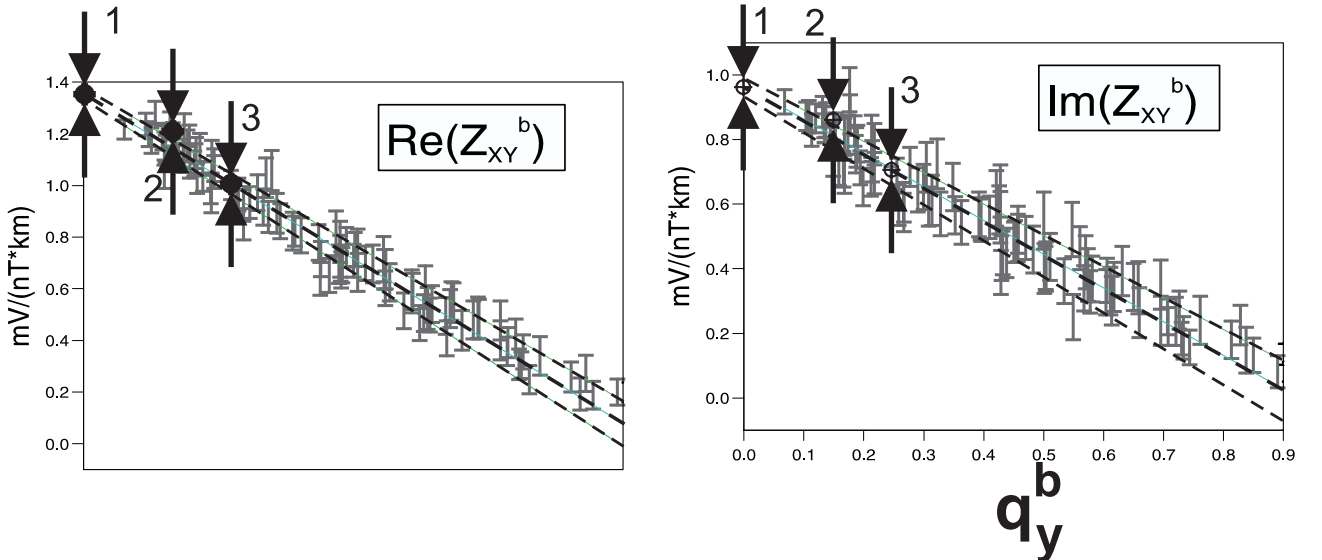


Figure 2. Real and imaginary parts of Z_{xy} (confidence limits) from 105 data sets plotted against the misfit factor q_y^b for the period range 400–510 s. Even impedance estimates with large q_y^b , and hence small CMPs, follow the linear relationship according to eq. (31). The arrows indicate (1) the extrapolated impedance Z_{xy}^0 , (2) impedance from stacked spectra with a CMP above 80 per cent, and (3) impedance from all stacked spectra. For the real part, none of the single events reaches the value of the extrapolated impedance Z_{xy}^0 .

unbiased impedance and the effect of bias described by the ANDI α . Both quantities can be simply determined by regression analysis, weighting the errors of each value. For the regression analysis we assume the impedances to be normally distributed with respect to the regression line and the misfit factor q_y^b to be noise-free. While q_y^b contains empirical variables and therefore noise, we argue that, because it contains the term $(1-\text{CMP})$, the relative error of q_y^b is small even when we have highly contaminated data, leading to a small CMP. It is also small for a low noise level. This is confirmed by the finding that regression analysis under the assumption of noise-free impedances led to larger standard deviations. We found no statistical significant difference between these two different assumptions.

Fig. 2 also compares the results obtained by the extrapolation to those obtained by stacking methods. The stacked results are within the error bounds of the estimated single results. The level depends on which events are considered. The extrapolated impedance is larger than each single estimate and can therefore never be reached by any stacking algorithm.

6.2 Compensation for bias

Z^0 yields an optimal estimation for the mean magnetotelluric impedance. To detect conductivity changes with time, we require the unbiased impedance for each data subset. Taking just the well-estimated impedances with a CMP above, for example, 0.8 would lead to large gaps in the time-series.

After determining the unbiased impedance and the ANDI α , we are now able to compensate each impedance value for the bias

$$Z_{xyi}^c = \frac{\hat{Z}_{xyi}^b}{1 - \hat{\alpha}_{xy} q_y^b}, \quad (32)$$

where Z_{xyi}^c is the compensated impedance for data subset i .

To reject very poorly determined impedance estimates we use a CMP limit of 0.33. Fig. 3 shows the result of the compensation (3) compared with two other estimation methods assuming (1) noise-free magnetic data and (2) noise-free electric data.

The impedances estimated by the compensation scheme (3) show a smaller scattering and more reliable error bars than the estimates from minimizing the electrical variations (1). In order to examine conductivity changes one needs to ensure that a real effect will not be unintentionally compensated. This is ensured because all 98 values have been compensated in the same way and by only one free parameter, the empirical ANDI α .

Fig. 4 shows the estimated ANDI α for different period bands. The ANDI and therefore the relative amount of noise in the magnetic channel B_y increase towards shorter periods. In the period range 200–700 s they seem to reach values above unity but not above the 95 per cent confidence limit, hence this is not statistically significant. However, α slightly greater than unity is possible and is not in contradiction to the definition given in eq. (25), just to the approximation given by eq. (28). The biased CMP determined for least-squares residuals is maximal and therefore overestimated compared with the true CMP. For this reason

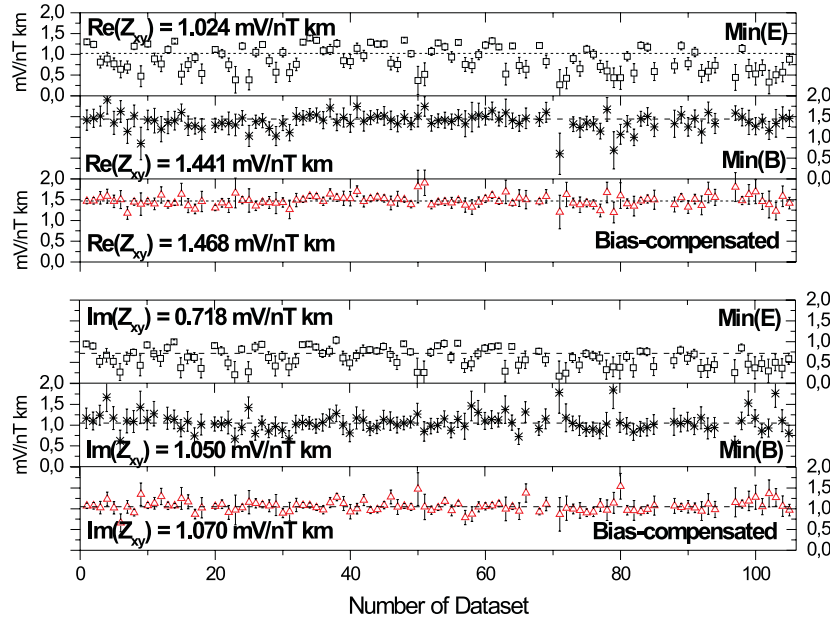


Figure 3. Real and imaginary parts of Z_{xy} impedances with a CMP above 0.33 (98 out of 105) for the period range 400–510 s estimated by three different methods: (1) minimization of noise in the electric components (eq. 4) [Min(E)]; (2) minimization of noise in the magnetic components (eq. 15) [Min(B)]; (3) compensation according to eq. (32) (Bias-compensated).

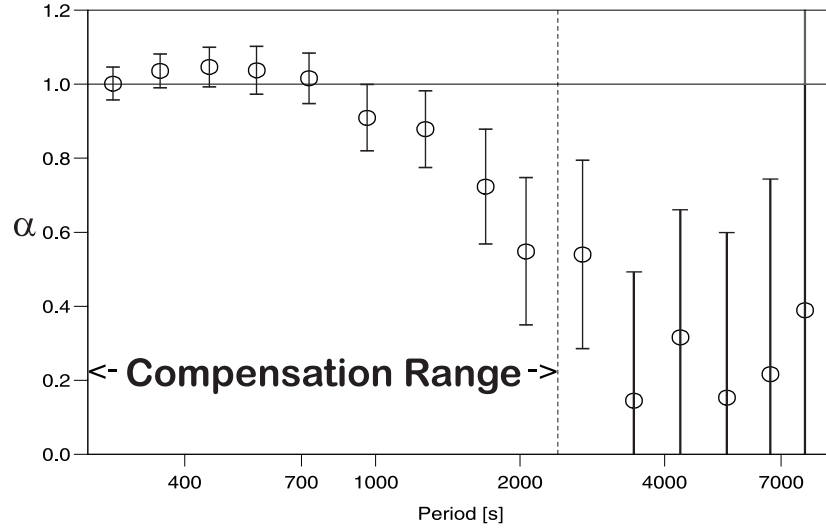


Figure 4. The ANDI α versus the period. The amount of noise in the magnetic component increases towards shorter periods. For long periods the ANDI α cannot be determined properly. $\alpha > 1$ is possible—see discussion in the main text.

the relative noise in one component might be larger than the deviation of the biased CMP from unity, which leads to $\alpha > 1$. For periods above 2500 s, the ANDI α can no longer be properly determined because it is small and therefore the bias effect is small.

Table 1 shows the half-intervals of 68 and 95 per cent of the impedance values. Even the estimates from the minimization of the magnetic variations (2) have larger confidence limits than the compensated estimates, although the ANDI α is close to unity (Fig. 4) and thus indicates that most of the noise is in the magnetic channels. Why did the estimates from eq. (15) work so badly? The reason here is a strongly polarized electric field that leads—in almost any coordinate system selected—to a coherence $\text{coh}(E_x, E_y)$ close to 1 and therefore, by eq. (13), to a large error. In cases of strong noise in the electric channels the impedances are poorly determined. Thus, the errors become small from eq. (13) because the coherence breaks down. This contradiction occurs because the assumption of noise-free electric fields fails in these cases.

Because of the strongly polarized electric field, the Z_{yx} -component was very small and therefore could not be compensated by this method. Z_{yx} and the effect of bias were small and within the statistical errors.

6.3 Confidence limits for the compensated impedances

While the confidence limits for the estimated quantities Z^0 and $(Z^0\alpha)$ are well known from regression analysis theory, the error estimation for the single corrected impedances Z^c need to be explained. In eq. (32) we see three quantities containing errors: the biased impedance Z^b , the ANDI α and the coherence term. Because the regression analysis to estimate the ANDI α is carried out for an error-free misfit factor q_y^b , the error in q_y^b cannot be taken into account. Its relative error is assumed to be negligible compared to the error in the ANDI α for low CMPs. For high CMPs its absolute error (relevant here for the error estimation) is small compared to the relative error of the biased impedance Z^b . Using a first-order Taylor approximation, we write

$$\frac{\partial Z_{xyi}^c}{\partial \alpha_{xy}} = \frac{-q_y^b}{1 - \alpha q_y^b} Z_{xyi}^c, \quad (33)$$

and for the total error,

$$\left| \frac{\Delta Z_{xyi}^c}{Z_{xyi}^c} \right|^2 \approx \left[\frac{\Delta \alpha_{xy} q_y^b}{1 - \alpha_{xy} q_y^b} \right]^2 + \left| \frac{\Delta Z_{xyi}^b}{Z_{xyi}^b} \right|^2. \quad (34)$$

Table 1. Empirical half-interval for 68 and 95 per cent of the impedance components Z_{xyi} determined by three different methods.

	Min(E)		Min(B)		Bias compensated	
	$\text{Re}(Z_{xy})$ (nT km ⁻¹)	$\text{Im}(Z_{xy})$ (nT km ⁻¹)	$\text{Re}(Z_{xy})$ (nT km ⁻¹)	$\text{Im}(Z_{xy})$ (nT km ⁻¹)	$\text{Re}(Z_{xy})$ (nT km ⁻¹)	$\text{Im}(Z_{xy})$ (nT km ⁻¹)
$\Delta Z_{68 \text{ per cent}}$	0.341	0.253	0.143	0.141	0.106	0.100
$\Delta Z_{95 \text{ per cent}}$	0.475	0.354	0.371	0.546	0.250	0.251

For a small q_y^b we can write

$$\frac{\Delta Z_{xy}^c}{Z_{xy}^c} \approx \frac{\Delta Z_{xy}^b}{Z_{xy}^b}. \tag{35}$$

This makes sense. The relative error of the biased impedance is at least the bivariate error of Z_{xy}^b . It cannot be reduced by bias compensation because the biased impedance is multiplied by a correction factor in eq. (32). In contrast, when the misfit factor q_y^b and the ANDI α are both close to unity, the error in α is amplified by the denominator in eq. (34).

In order to compare the estimated error of the corrected impedances with the variance of the 105 estimates we define a quantity ξ ,

$$\xi = \frac{2\sigma(Z^c)}{\Delta_{95 \text{ per cent}}(Z^c)}, \tag{36}$$

where σ is the empirical variance of the 105 impedances Z^c . The double 68 per cent variance corresponds approximately to a 95 per cent confidence limit in the case of the assumed normal distribution. $\Delta_{95 \text{ per cent}} Z$ is the 95 per cent confidence limit for the Z^c according to eq. (34). We choose this definition in order to have an expected value of $\xi = 1$ in the case of a well-estimated confidence limit. Fig. 5 shows the estimates of ξ for different periods. The good agreement of estimated and empirically determined errors proves the reliability of the estimation after eq. (34).

After applying the compensation method, all significant deviations from the mean disappeared in all components and for all frequency bands. From this we concluded that no significant resistivity change occurred within the observation time of 2 yr. Thus, the goal of establishing a method for a reliable monitoring of the electric resistivity was fulfilled (Müller 1997).

7 APPLICATION TO MAGNETOTELLURIC DATA FROM CENTRAL JAVA (INDONESIA)

Whilst it could be argued that the outcome of the Turkish measurements was due to our special configuration for the measurement of the magnetic field variations, the method has also been successfully applied to data obtained with different instrumentation in different environments. In June–July 1997, magnetotelluric measurements were carried out at 10 sites on a profile crossing Central Java in a N30°E direction (Ritter *et al.* 1998b). We used a sampling rate of 0.5 Hz for measurements with fluxgate magnetometers (MAGSON) and a RAP datalogger (Steveling 1996). Because of a high noise level due to the dense population and the corresponding dense electric power network in Central Java, impedance estimates in the period range 30–300 s were biased. Figs 6–8 show the results for the major components Z_{xy} and Z_{yx} . In each case the impedances have been estimated by four different methods:

- (1) minimization of noise in the electric components by eq. (4);
- (2) compensation according to eq. (32);
- (3) minimization of noise in the magnetic components by eq. (15);
- (4) calculation with a remote reference station after Gamble *et al.* (1979).

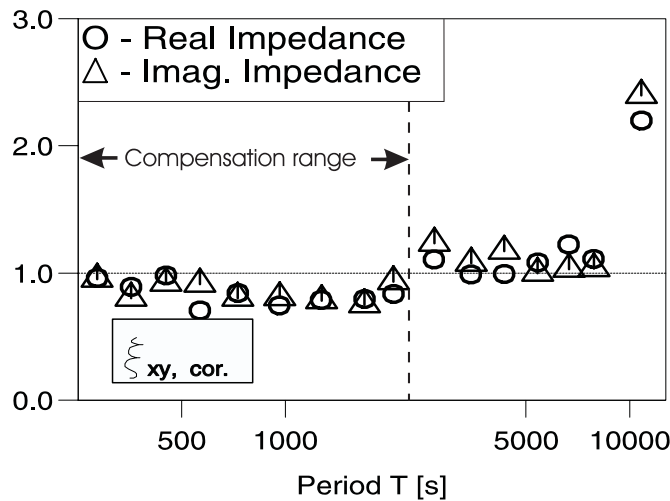


Figure 5. Relationship between the empirical variance and the estimated confidence limits. Values above 1 mean underestimated errors; values below 1 mean overestimated errors. After applying the compensation and the error estimation eq. (34), the estimated and empirically determined errors correspond well, except for one value for long periods, probably because of a low degree of freedom or departures from the assumption of a homogenous source field.

The results from the bias compensation (2) were plotted when a χ^2 -test proved the linear relationship according to eq. (31).

While Fig. 6 shows that the minimization methods (1) and (3) fail for impedances below 25 s (Z_{xy}) and 40 s (Z_{yx}), the estimates after the bias compensation scheme (2) remain stable towards shorter periods. The results are confirmed by the estimates from the remote reference calculation (4). However, the remote reference estimation supplies one additional reliable value for the period 15 s. As expected, the phases appear unbiased.

In contrast, the estimates from the bias compensation (2) at the station KAWR (Fig. 7) towards short periods (20 s) appear to be even better than the outcome from the remote reference (4).

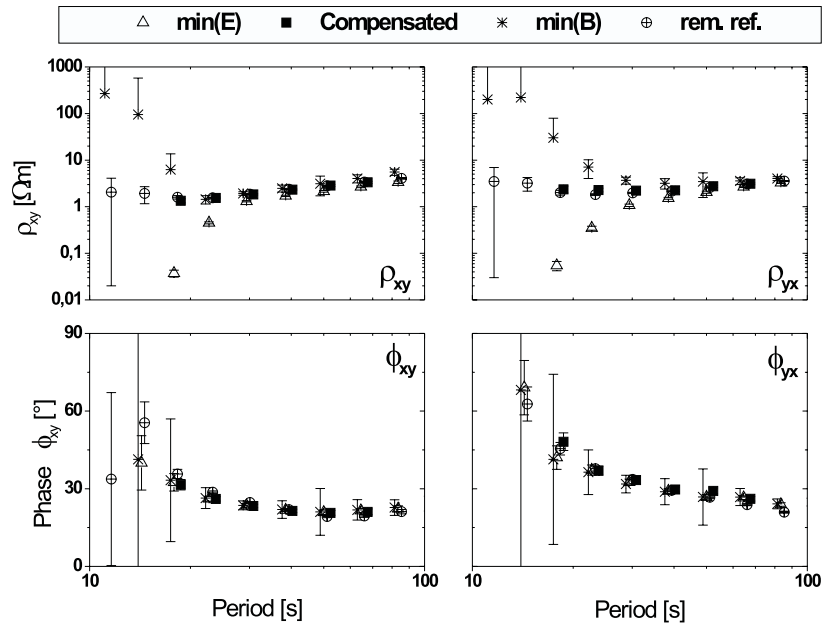


Figure 6. Comparison of impedances estimated by four different methods from the site TELO (period positions have been slightly shifted in order to make the different error bars visible): (1) minimization of noise in the electric components by eq. (4) [min(E), triangles]; (2) compensation according to eq. (32) (compensated, squares); (3) minimization of noise in the magnetic components by eq. (15) [min(B), asterisks]; (4) calculated with a remote reference station after Gamble *et al.* (1979) (rem. ref., circles with crosses). The results from the bias compensation (2) have been plotted when a χ^2 -test proves the linear relationship according to eq. (31).

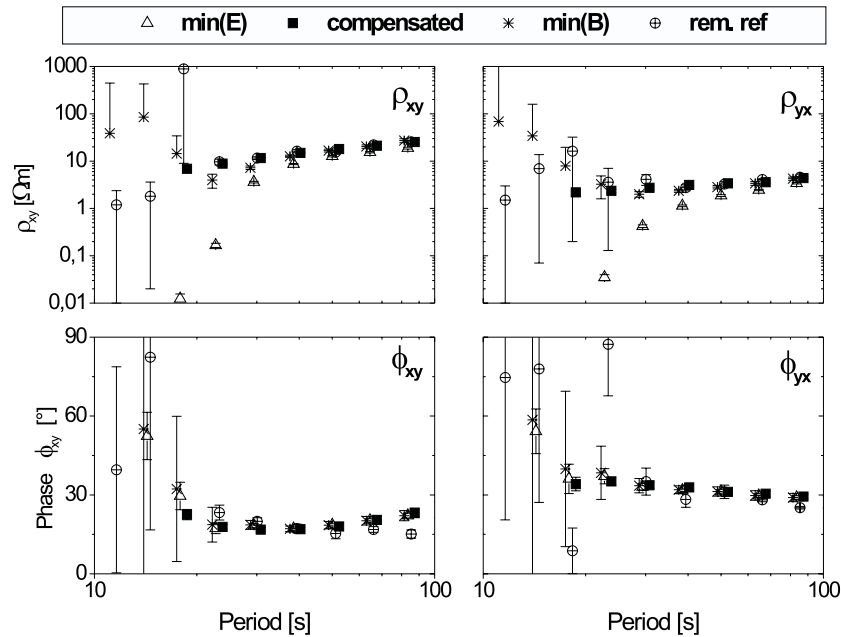


Figure 7. Comparison of impedances estimated by four different methods from the site KAWR. In this case the estimates from the bias compensation scheme (2) are apparently even better than the results calculated with a remote reference (4).

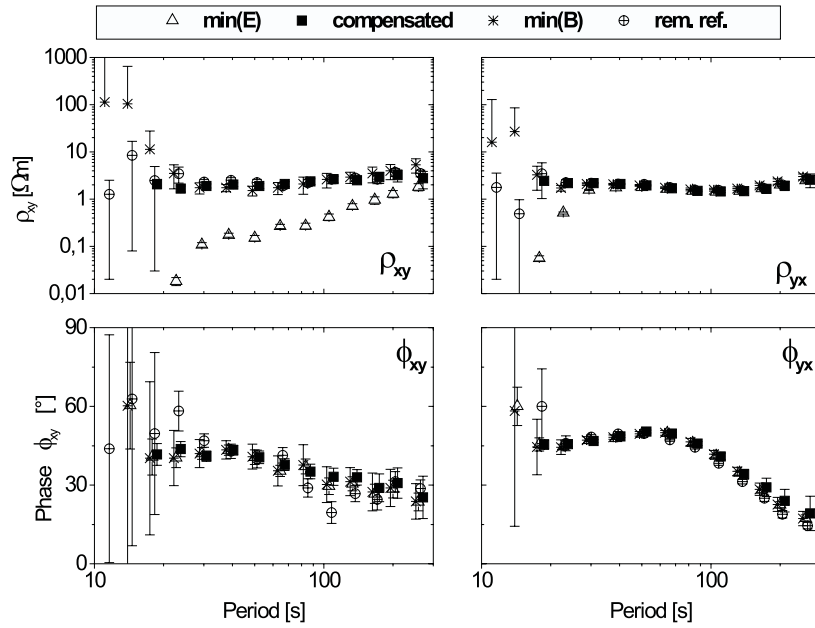


Figure 8. Comparison of impedances estimated by four different methods from the site SCHI. Again, the estimates from (2) appear to be the most reliable results.

In Fig. 8 the apparent resistivities after (1) for the XY -component at site SCHI are strongly biased for periods below 200 s. Because the decrease is moderate towards 30 s, it would probably not be recognized from the commonly used method of minimization of noise in the electric field alone, and instead the resistivity structure would be interpreted as 2-D. However, the ρ_{xy} component is close to the better determined ρ_{yx} component, which can be seen from the outcome of the three other methods.

The impedances from the minimization of the electric noise (1) are underestimated, and those from minimization of the magnetic noise (3) are overestimated. Results using (3) are better than those of (1), indicating a larger amount of noise in the magnetic components. The large error bars for impedances calculated with a remote reference (4) indicate that the reference stations are far from perfect. In all cases the phases are not affected by bias.

Out of nine long-term sites, the compensation method worked well for five stations. In three cases it could not be applied for one component because the electric component was dominated by noise. For one site, all methods failed because of the high noise level. The assumption for the tensor's structure was valid in all cases. We found that the method works when the a smaller tensor element is about 30 per cent of the corresponding larger element. We have found no cases where the assumption of a constant relationship between the noise in electric and magnetic components failed.

The results from Java prove that the applicability of the method is more general than expected. The bias effect appeared at shorter periods (30–300 s) than for the Turkish measurements (2500 s). This was expected because of the less efficient use of the magnetic signal with respect to the noise for the measurements in Turkey. The behaviour of the impedances, their noise dependence and therefore the noise model seem to be similar in both cases, for different instrumentation as well as for a different site environment.

8 CONCLUSIONS

The ANDI α expressing the relationship between noise in one magnetic and the electric channels seems to be constant for measurements with one instrument at one site. How can we understand this result?

(1) If we have instrumental noise from the sensors, preamplifiers or the digitization one can expect this noise to be constant in each channel. Therefore, the relation is also constant amongst channels.

(2) If we have an external noise source with a constant transfer function different from the transfer function of the magnetotelluric signal, the relationship of noise amongst different channels still remains constant. This is the noise model for the two-source method of Larsen *et al.* (1996). They used data from a second station not contaminated by the correlated noise in order to compensate for errors in the impedance estimates. However, in contrast to Larsen's two-source method we assume uncorrelated noise only. In the case of correlated noise the cross-powers in eq. (4) are also overestimated, and the derivation for eq. (31) fails.

(3) If we have many external noise sources with different transfer functions due to different directions and distances of the sources, the stacked power estimates might be unbiased although the single events are correlated. This kind of background noise

might be the dominant noise source in either the electric or the magnetic channels depending on the observed period band and therefore may explain the outcome of our observations. However, this model does not explain the systematic increase of noise in the magnetic field towards short periods.

We assume the fluxgate magnetometers as our major noise source for the measurements in Turkey as well as in Java. Their resolution is limited for small magnetic variations at short periods. This hypothesis is supported by the finding that the relative amount of noise in the magnetic channels increases towards shorter periods. Thus we can explain our finding with argument (1).

One important advantage of the method is stability. Only a few values of the ANDI α differ significantly from 0 and 1. Therefore, the assumptions of noise-free electric or magnetic signals work over a broad range of periods. However, usually we do not know which one of these assumptions we should use. The compensation method gives more accurate results than the estimates with the assumption of magnetic noise, even for the ANDI close to unity. An attempt to compensate the impedances with the assumption of a constant magnetic noise level appeared to be less stable, and therefore failed.

The effect of bias may be as large as the misfit of the CMP, e.g. a CMP limit of 80 per cent allows for 20 per cent biased impedance estimates. The amount of bias may change only continuously with period. Therefore, the biased curve is smooth. Thus, the criterion of a smooth curve is insufficient to detect bias. In order to detect bias one should at least calculate the impedances by both minimization methods, with the minimization of noise in either the electric or the magnetic component. If the resulting impedances differ much the bias must be accounted for. Whereas there can be no general solution for this problem, the bias compensation method complements methods such as the stacking algorithm or the remote reference method. It might be preferably applied where neither the CMPs are close to unity nor (good) data from a reference station are available.

ACKNOWLEDGMENTS

I would like to thank J. Zschau to enabling me to carry out the work on the data from Turkey in his working group. E. Erkul performed most of the practical work in carrying out these measurements. The measurements in Central Java were carried out in the working group of V. Haak. I would like to thank him for his patience as well as for his efforts to motivate this publication. O. Ritter, F. Echernacht, A. Hoffmann-Rothe and S. Byrdina participated in the measurements in Central Java. I like to thank A. Junge and an anonymous reviewer for their valuable comments. This work has been supported by the GFZ Potsdam and the German Science Foundation DFG (HA-1210/18–1).

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