Quick-look gravity field analysis of formation scenarios selection

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1. Introduction
The development of an advanced future gravity field mission for time variable gravity field recovery is a difficult task. A lot of options exist to improve the performance, accuracy and sensitivity compared to GRACE. Besides technological progress in satellite system and metrology as for instance the application of laser and advanced drag-free and orbit control systems also a variety of geodetic parameters exist which can be tuned in order to improve the mission. These geodetic parameters include for instance the selection of the orbit height, the satellite distance, the inclination, the repeat mode, the sensor type, the formation and a possible multi-satellite/formation mission. Especially the last two options – advanced formations and multi-formation mission – are regarded as the key instruments to improve the main weaknesses of GRACE, which are i) North-South striations caused by anisotropy of the measurements and ii) aliasing due to temporal undersampling of time-variable signals. By means of advanced formations, which are able to detect signals different from the along-track-direction a higher isotropy is reached and striations might be reduced or avoided. Aliasing might be reduced by multi-formation missions, which enlarge the temporal/spatial sampling.

In order to identify suitable missions from the huge variety of options a huge amount gravity simulations have to be performed. However, a time-consuming full-scale gravity retrieval ist the only possibility to take all effects, especially the severe aliasing into account. In order to reduce the search space for the full-scale simulations dramatically, quick-look tools have been developed. They enable a sensitivity analysis by means of error propagation. The influence of the following parameters can be investigated: orbit height, satellite distance, inclination, duration, sensor type and combinations, measurement noise (as PSD (power spectral density)). By means of a new quick-look-tool additionally the sensitivity of advanced formations as pendulum, cartwheel, LISA-type, trailing cartwheel and helix (trailing LISA) can be estimated. However, quick-look tools are not capable of investigating aliasing effects, since they are based purely on error propagation. Despite of this, quick-look tools are regarded as a useful tool for the sensitivity analysis of future mission options.

In this paper the sensitivity of the six basic formations, including the standard inline formation and the five advanced formations, is investigated. However, from technological side of view, constraints on some formation parameters as maximum range-rate or maximum variation of the yaw/pitch angle exist, which seems to be problematic for all five advanced missions. Therefore a set of advanced formations is additionally investigated, which fulfils the mission constraints.

2. Quick-look-tools
Under the assumption of a circular nominal orbit with constant inclination ($r = r_0$, $I = I_0$) a fast and efficient block-diagonal error propagation (order wise with even/odd degree separation) from the observational and stochastic model to gravity field errors can be performed

with the original semi-analytic quick look tool (QLT). It is based on *Sneeuw (2001)* and described here briefly. The lumped coefficient representation of a gravitational signal $f(t)$ along the orbit reads

$$f(r,u,I,\Lambda) = \sum_m \sum_k A^f_{mk}(r,I) e^{i\dot{\psi}_{mk}(t-t_0)}$$

$$A^f_{mk}(r,I) = \sum_l \underbrace{\frac{GM}{R}\left(\frac{R}{r}\right)^{l+1} \overline{F}_{lmk}(I)}_{H^f_{lmk}(r,I)} K_{lm}$$

with the inclination function $\overline{F}_{lmk}(I)$, the frequency $\dot{\psi}_{mk} = k\dot{u} + m\dot{\Lambda}$ and the complex SH coefficients K_{lm}. For a nominal orbit the transfer coefficient $H^f_{lmk}(r,l)$ and the lumped coefficient $A^f_{lmk}(r,I)$ become constant and the normal equation gets a orderwise blockdiagonal structure. The transfer coefficient $H^\rho_{lmk}(r,I)$ of the low-low-SST for an inline-formation (leader-follower) is derived as $H^\rho_{lmk} \approx 2i\sin(\eta\beta_{mk})H^{\Delta x}_{lmk}$ with $\sin\eta = 0.5\rho_0/r$. In our case, we are not interested in the solution for the SH coefficients but in their accuracy (variance-covariance matrix $\mathbf{Q}_{\hat{x}}$), which can be estimated by means of blockwise variance-covariance propagation $\mathbf{Q}_{\hat{x}}=(\sum_i A_i^T Q^{-1}_{yi} A_i)^{-1}$ from the variance-covariance matrix Q_{yi} of the observations. The design matrix $\mathbf{A}$ is composed by the transfer coefficients H_{lmk} and the variance-covariance matrix Q_{yi} of the corresponding block can easily be derived as a diagonal matrix from the PSD of the functional f. Here the psd-value belonging to the frequency $\dot{\psi}_{mk} = k\dot{u} + m\dot{\Lambda}$ of the lumped coefficient A^f_{mk} has to be inserted. From the estimated variance-covariance matrix the error measures used for visualisation are derived. With the semi-analytic QLT the influence of the parameters (i) measurement type, (ii) measurement noise (as PSD), (iii) orbit height, (iv) inclination, (v) mission period, (vi) intersatellite distance can be studied. The estimated formal errors are represented as (i) degree-RMS, (ii) triangle plots, (iii) geoid errors per latitude and (iv) covariance functions (at the equator).

The influence of two basic mission parameters, the intersatellite distance ρ and the orbit height h is studied in Figure 1 for a future standard mission (polar and circular orbit, time span of $T = 15$ days, PSD of future laser and accelerometer). As visible, the best geodetic sensitivity is reached for a large intersatellite distance and a low orbit height. However, a low orbit height is problematic due to a higher air drag (high energy and propulsion consumption) and a large satellite distance faces problems with the laser technology (pointing, signal strength, noise). Thus, an orbit height of $h = 350$ km and a satellite distance of $\rho = 100$ km seem to be a good compromise between geodetic sensitivity and technological feasibility.

However, a constant transfer-coefficient for other formations than the standard inline formation could not be derived so far, thus another strategy was used for the formal error simulation of the advanced formations. This formation-QLT can be regarded as some kind

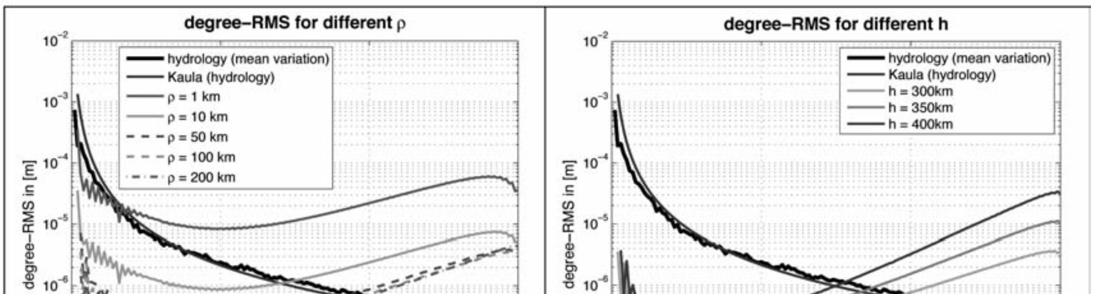

Figure 1: Impact of different intersatellite distance and orbit height on the accuracy of the gravity field recovery.

of pseudo-QLT, i.e. in between QLT and full scale simulation. It is based on the formulation of the equation for range-accelerations:

$$\ddot{\rho}(t)-\frac{1}{\rho(t)}\left(\left(\Delta\dot{\mathbf{X}}_1^2(t)\right)^2-\dot{\rho}^2(t)\right)=\mathbf{e}_1^2\left(\operatorname{grad}V\left(\mathbf{X}_2(t)\right)-\operatorname{grad}V\left(\mathbf{X}_1(t)\right)\right)$$

The designmatrix is composed from the right hand side of this equation. The needed positions of the two satellites $\mathbf{X}_2(t), \mathbf{X}_1(t)$ are computed by (i) computation of circular (β/α)-repeat orbits ($I = I_0$, $r = r_0$) for the center $(\mathbf{X}_2+\mathbf{X}_1)/2$ of both satellites and (ii) computation of the relative movement of the two satellites by means of the homogeneous solution of the Hill-equations (e.g. *Sharifi et al., 2007*):

$$x(t)=-2A\sin(nt+\alpha)-\frac{3}{2}ntz_{\text{off}}+x_{\text{off}}$$
$$y(t)=B\cos(nt+\beta)$$
$$z(t)=A\cos(nt+\alpha)+z_{\text{off}}$$

with the initial conditions

$$A=\frac{1}{n}\sqrt{\dot{z}_0^2+(2\dot{x}_0+3nz_0)^2}\quad \tan\alpha=\frac{\dot{z}_0}{2\dot{x}_0+3nz_0}\quad x_{\text{off}}=x_0-\frac{2}{n}\dot{z}_0$$
$$B=\frac{1}{n}\sqrt{\dot{y}_0^2+(ny_0)^2}\quad \tan\beta=\frac{\dot{y}_0}{ny_0}\quad z_{\text{off}}=\frac{2}{n}(\dot{x}_0+2nz_0)$$

The following initial elements have to be chosen for the formations (start point at t_0 is over the equator):

- inline (leader-follower, GRACE-like): $x_0 = \rho$
- Pendulum: $x_0 = \rho_x$, $y_0 = \rho_y$ (along-track distance ρ_x, maximum cross-track-distance over equator ρ_y)
- Cartwheel: $z_0 = \rho_r$, $\dot{x}_0 = -2n\rho_r$ (maximum radial distance: ρ_r (maximum along track-distance $\rho_x = 2\rho_r$))
- LISA: $y_0=-\sqrt{3}\rho/2$, $z_0=\rho/2$, $\dot{x}_0=-n\rho$, (constant satellite distance ρ)
- trailing Cartwheel: $x_0 = \rho_{\text{x-offset}}$, $z_0 = \rho_r$, $\dot{x}_0 = -2\rho_r$ (Cartwheel with shift $\rho_{\text{x-offset}}$ in along-track direction)
- Helix: $x_0 = \rho_{\text{x-offset}}$, $y_0 = -\sqrt{3}\rho/2$, $z_0 = \rho/2$, $\dot{x}_0 = -n\rho$ (trailing LISA with shift $\rho_{\text{x-offset}}$ in along-track direction)

The angular velocity of the reference orbit is n, here also secular effects caused by J_2 on the angular velocity are considered (). In contrast to the semi-analytic QLT the formation QLT is slower since analysis for a complete repeat orbit is performed. Furthermore the formation QLT so far works only for white noise of range accelerations. In the following sections the sensitivity of the advanced formations is investigated.

3. Formation analysis

It is expected that the advanced formations will lead to a higher sensitivity and isotropy compared to the standard inline-formation since signal components apart from the along-track direction are measured. The six formations are investigated using the following parameters: orbit height $h = 334$ km (corresponding to $\beta/\alpha = 503/32$ repeat mode), inclination $I = 90°$, average satellite distance $\rho_{\text{avg}} = 100$ km, time interval $T = 30$ d, range acceleration white noise of 10^{-10} [m/s²/sqrt(Hz)] corresponding to an average combined laser/accelerometer noise level. Figure 2 shows the resulting formations in the Hill-system as well as the range, the range-rates and the yaw-/pitch-angles. As visible, the pendulum adds cross-track information over equatorial regions while the cartwheel is sensitive for radial information. The LISA-formation is a combination of both and thus gathers additionally both, cross-track and radial information. In general, the feasibility of cartwheel- and LISA-formations is regarded as problematic due to the rotating yaw-/pitch-angle (360° per revolution). Thus LISA- and cartwheel-formations with an along-track shift are also regarded (named as helix, trailing cartwheel), where the maximum yaw-/pitch-angle can be controlled to stay between certain limits. Of course the main measurement-direction now is again along-track, which will probably lead to reduced sensitivity and isotropy compared to the non-trailing formations.

The results for the six formations are displayed in Figures 3-5 in terms of triangle plots, covariance functions, degree-RMS and geoid-errors per latitude. As the figures show, the advanced formations of pendulum, cartwheel and LISA lead to a significant improvement in sensitivity and isotropy compared to the inline-formation with a similar performance for all three cases. As shown by the degree-RMS, the improvement is almost one order of magnitude. The geoid error is reduced mainly over regions of lower latitude since the cross-track and radial components are largest over the equator, while over the poles the measurements of the advanced formations still contain mainly along-track information. The advanced formations show a high isotropy, as the covariance functions show and thus might be able to reduce the striping effects well known from GOCE (LISA here has even a stronger pronunciation of East-West structures). The higher isotropy is depicted also in the triangle plots of Figure 3 where the accuracy of higher orders is improved compared to the inline case. The adapted trailing formations of helix and trailing cartwheel show a significant improvement compared to the inline case, although it is not as pronounced as for the other three non-trailing formations. Especially the helix shows a good performance which is only slightly worse than for the cartwheel if degree-RMS and geoid-errors are concerned. The trailing cartwheel still leads to an improvement of a factor of three compared to the inline-case. However the stronger impact of the along-track component in the trailing formations is clearly visible in the covariance functions and triangle plots with a lower isotropy indicated North-South structures and a lower accuracy of higher orders compared to the non-trailing formations. Taking into account that cartwheels and LISA formations (and thus also their trailing variants) are unstable due to the perigee drift caused by the Earth flattening a pendulum seems to be a very promising option for a future formation.

3.1. Mission constraints

Although the advanced missions seem to be of great benefit for geodesy, their technical feasibility is regarded as critical. From the technological side, two crucial constraints for the formation design exist up to now:

1) the maximum range-rate has to be kept within ±10 m/s. This is a constraint from the laser link in order to keep the Doppler effect sufficiently small
2) the line-of-sight angle between the two satellites is allowed to change only within ±30° in yaw-/pitch-direction around the mean axis in order to guarantee spacecraft- or beamsteering-pointing.

As it can be seen in Figure 2, neither the advanced non-trailing formations nor the trailing formations fulfil these mission constraints (critical parameters are shaded dark grey, less critical bright grey). Within a pendulum, the maximum range-rate and yaw angle can be controlled by reducing the cross-track component and or the average satellite distance in general. For the cartwheel, the pitch-angle can only be kept within the limits if it is regarded within the space-fixed system. Then, it stays within ±20°, but this option might afford cylindrical satellites with a similar drag coefficient in every possible air-drag direction. But then still the cartwheel shows a strong dynamical range such that the intersatellite-distance has to be reduced significantly (this is also important for the reduction of the differential air-drag of both satellites). The LISA-formations seems to be perfect regarding the range-rate since it has almost no dynamical range. However, the yaw- or pitch-angles can not be kept both within the limits, neither in the Hill-system nor in the space-fixed system. Thus this formation doesn't seem feasible at the moment. The two trailing formations can be trimmed within the limits by downscaling the shown formations to a shorter intersatellite-distance.

3.2. Realistic formations

In this chapter the formations adapted to the mission constraints are investigated. In case of the cartwheel, the trailing cartwheel and the helix the intersatellite is scaled down significantly to average values of approximately 15 km such that the constraints are met. In case of the pendulum different options exist to keep the formation within the mission constraints, e.g. (i) by applying the maximum yaw angle of 30° and downscaling the average range to a value < 100 km such that the range-rate limits are met, or (ii) selecting a maximum yaw angle <30° such that an average range of 100 km can be hold and the range-rate can be kept within the limits. Here the more promising case (ii) with the larger satellite-distance is

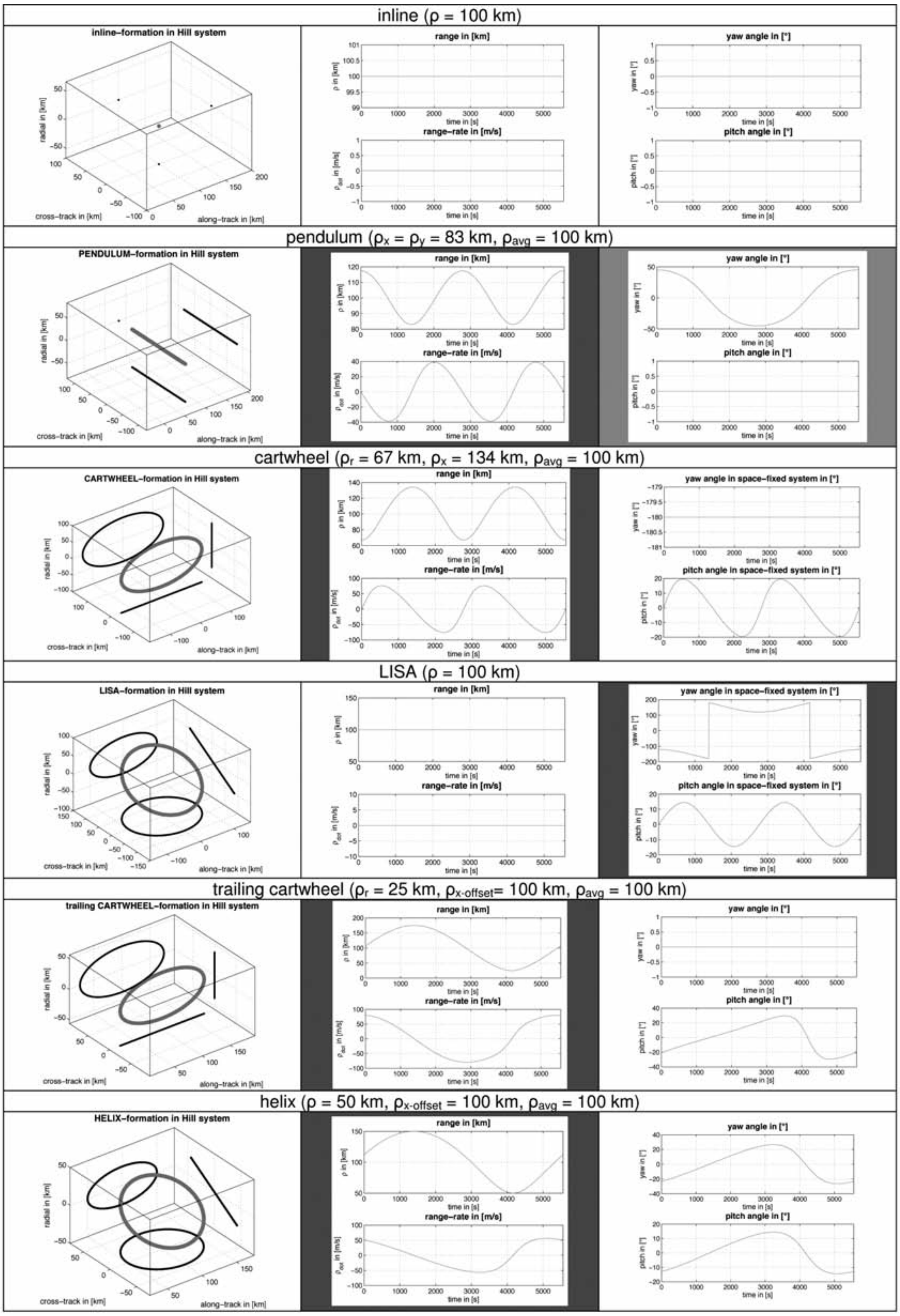

Figure 2: representation of the 6 basic formations with an average intersatellite distance of 100 km; left: relative movement of satellite 2 in the Hill-system; middle: range and range-rate; right: yaw and pitch angle. Critical values concerning the mission constraints are coloured dark grey, less critical are light grey.

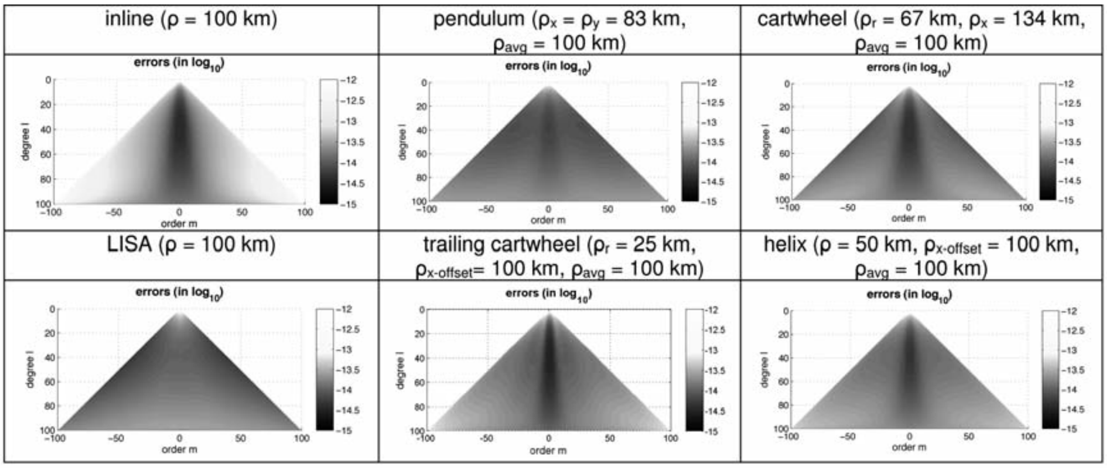

Figure 3: formal errors of the six basic formations.

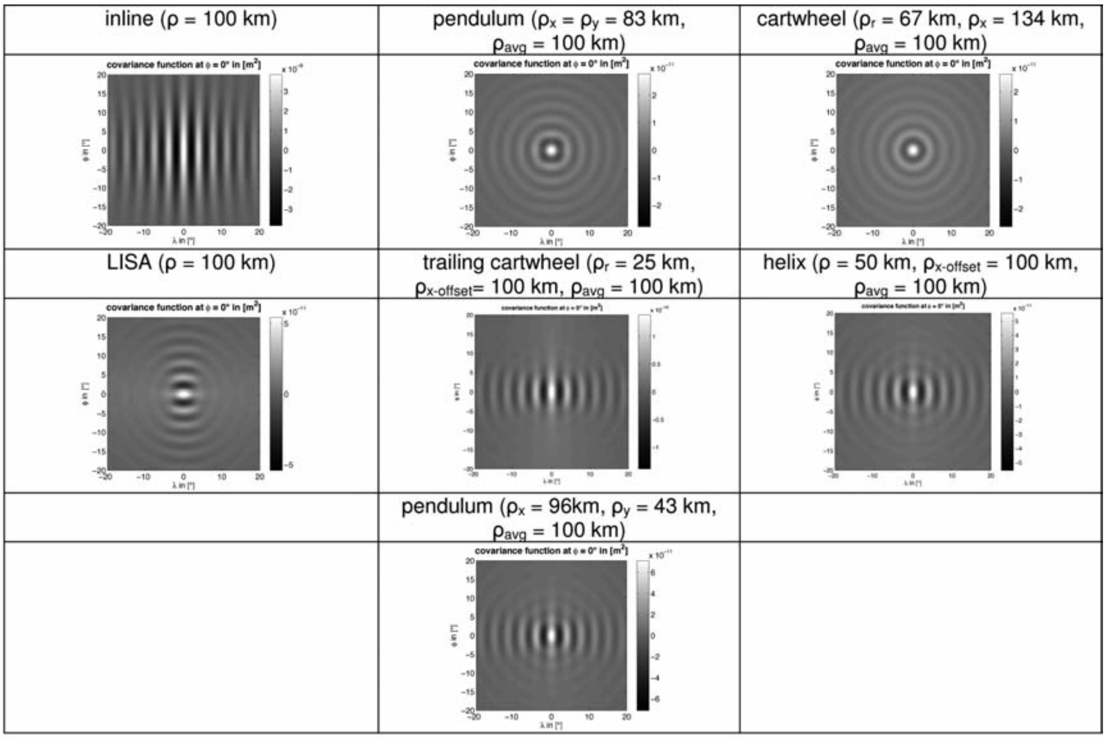

Figure 4: covariance functions of the six basic formations as well as of a pendulum adapted to the mission constraints.

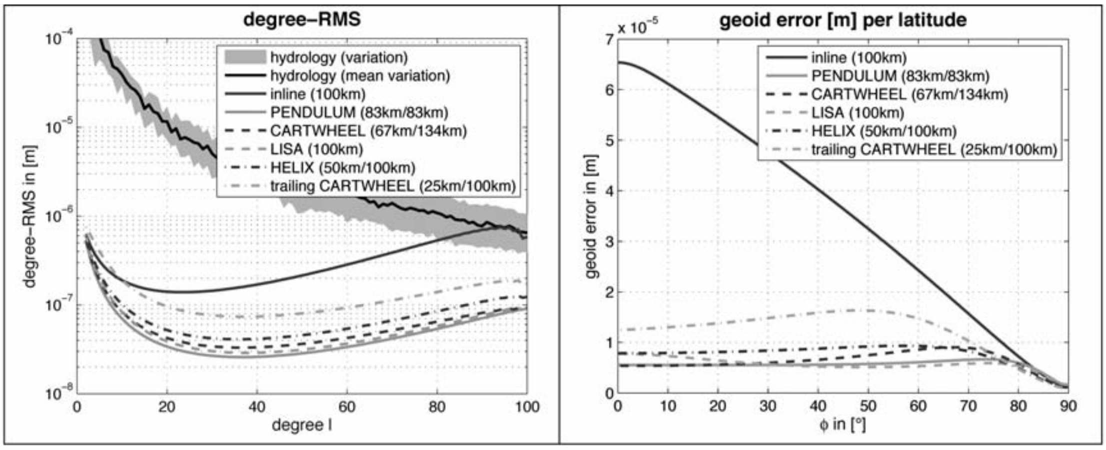

Figure 5: degree-RMS and geoid errors per latitude for the six basic formations.

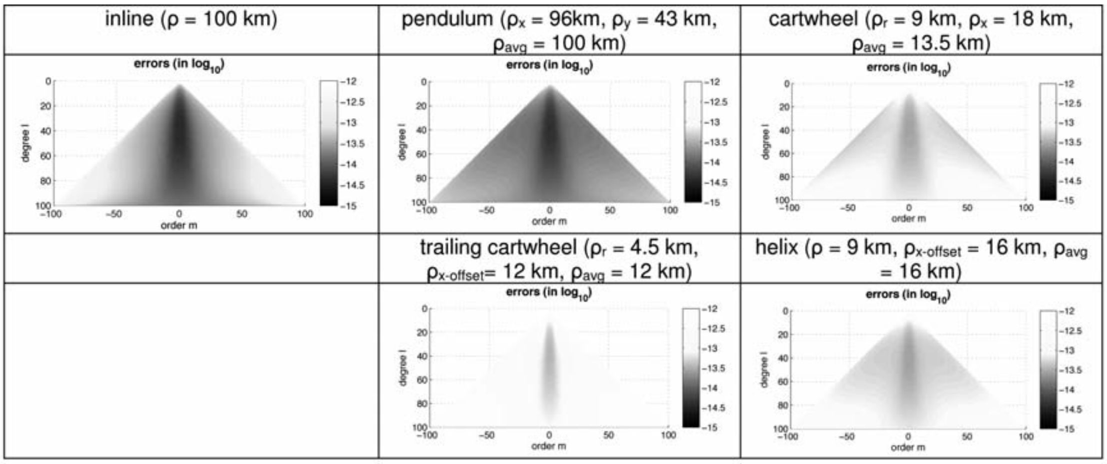

Figure 6: formal errors of the basic formations adapted to the mission constraints.

chosen due to the strong dependence of the sensitivity on the satellite distance shown in Figure 1. Of course such a pendulum leads to less isotropy than by application of a larger yaw-angle, as shown in the covariance functions of Figure 4, where now North-South structures are more pronounced.

The results for the more realistic formations concerning the mission constraints are shown in Figures 6 and 7 in terms of triangle plots, degree-RMS and geoid errors per latitude (the covariance functions are not shown, their shape corresponds to those of the »wanted« formations in Figure 4). As it can be seen the best results are obtained with the »realistic« pendulum. The improvement of this pendulum still is approximately a factor of 4 in terms of degree-RMS compared to the inline case. Due to the higher isotropy compared to the inline-case an improvement of the higher order coefficients and the geoid errors in regions of lower latitude is reached. The other „realistic" formations of the cartwheel, the helix and especially the trailing cartwheel perform even worse as the inline formation, which is caused by the loss of sensitivity due to a significant shorter intersatellite distance.

4. Results and outlook

The quick-look tools has proven to be a fast and efficient tool for the investigation of the influence of basic mission parameters and of the formation type on the sensitivity of a future

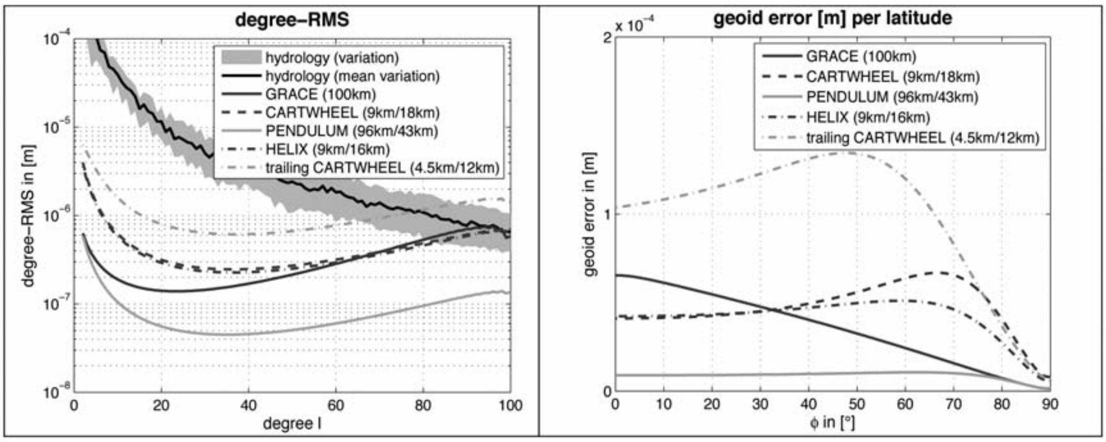

Figure 7: degree-RMS and geoid-errors per latitude of the basic formations adapted to the mission constraints.

mission. However, further improvements of the formation quick-look tool are desirable, e.g. the application of coloured noise (as PSD) instead of white noise of the derivation of transfer-coefficients for the advanced formations.

Advanced formations as pendulum, cartwheel or helix are able to improve the sensitivity and especially the isotropy significantly compared to the inline-case so that striping patterns known from GRACE can be avoided. However, the advanced formations make great demands on satellite technology/control systems and metrology which will make their realisation difficult. Even worse, due to restrictions on range rate and yaw/pitch angles coming from the laser and beamsteering-/satellite-pointing the advanced formations in their wanted design are not feasible at the present state of technology. By means of tuning parameters most of the suggested formations can be tailored to the given mission constraints. However, under these constraints, the »realistic« pendulum proposed seems to be the only formation which is able to improve sensitivity compared to the standard inline-case.

Under the current mission constraints and ll-SST sensor noise assumptions pendulum formations seem to be the most promising option. However, with advancements in technology also the other formations might become interesting options.

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References

Sharifi MA, Sneeuw N, Keller W (2007) Gravity recovery capability of four generic satellite formations. In: Kiliçoglu A and R Forsberg (Eds.) Gravity field of the Earth. General Command of Mapping, ISSN 1300-5790, Special Issue 18, pp 211-216. 1st Int. Symp. of the International Gravity Field Service, 28.8.-1.9.2006, Istanbul, Turkey

Sneeuw N (2001) A Semi-Analytical Approach to Gravity Field Analysis from Satellite Observations, DGK, Reihe C, Dissertationen, Heft Nr. 527, Verlag der Bayerischen Akademie der Wissenschaften, München.