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On the double-peak spectrum of the Chandler wobble

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Abstract. It is well known that two close spectral peaks with comparable amplitudes exist in the Chandler wobble (CW) band in the observed polar motion spectrum during the first half of the past century. However, it is not generally accepted that the CW is an oscillation with two closely lying frequencies, and some authors attribute the appearance of these two peaks to some unusual phase variation (180° shift) of the CW during 1920s-1930s. In this note, the phase variation of the CW is explored using the longest Earth orientation parameter (EOP) time series currently available—the International Earth Rotation and Reference System Service (IERS) EOP-C01 from 1846 to 2003. The observation time series mainly containing the CW obtained by removing the annual wobble (AW) from the original EOP time series is compared with a synthetic double frequency CW times series constructed using the frequencies, amplitudes and initial phases of the two peaks precisely determined using spectral analysis. The result shows reasonable agreement between their peak times, a feature not shown before. This feature is in agreement with the hypothesis of a double frequency CW. While the result shown here is inconclusive that the CW really has two frequencies, this note is expected to serve as a reminder for its renewed interest.

Key words. Chandler wobble – Double frequency – Phase shift

1 Introduction

The Chandler wobble (CW) is well known to be one of the most important and enigmatic components of polar motion. Spectral analysis of the polar motion time series during the first half of the past century revealed two closely lying spectral peaks with comparable amplitudes in the CW frequency band, and some authors attempted to interpret this in terms of a double frequency CW (e.g., Labrousse and Labrousse 1946; Yashkov 1965; Gaposchkin 1972). Colombo and Shapiro (1968) tried to provide a theoretical model for a double frequency CW. Chao (1983) argued that, from the point of view of numerical analysis rather than geophysical interpretation, the then International Latitude Service (ILS) data support the multiple component hypothesis of the CW, and used four components to represent the polar motion in the CW band.

However, some authors showed that the appearance of the two peaks does not necessarily imply a double frequency CW from a statistical point of view (e.g., Pedersen and Rochester 1972; Lenhardt and Groten 1985; Okubo 1982), and that the double peak feature could generally be explained based on a single frequency oscillation. One such explanation is that the phase of the CW has been subjected to some unusual variation (a phase shift of 180°) during 1920s-1930s (Fedorov and Yatskiv 1965; Guinot 1972; McCarthy 1974; Dickman 1981; Vondrák 1985). Another explanation is to accept a variable CW frequency. In fact, the variability of the CW frequency has been a subject of debate. Some authors assumed or proved the frequency to be invariant (e.g., Ooe 1978; Kuehne and Wilson 1996; Vicente and Wilson 1997; Gross et al. 2003; Liao et al. 2003; Seitz et al. 2004). Some authors suggested a variable frequency (e.g., Melchior 1954, 1957; Sekiguchi 1972, 1976; Guinot 1972; Carter 1981; Nastula et al. 1993; Liu et al. 2000; Schuh et al. 2001; Höpfner 2003, 2004). Jochmann (2003) argued that, as the CW is a proper mode of the Earth, the variation of its frequency should depend on the variation of the dynamical structure of the Earth, and showed that the variation due to changes in the structure of the Earth is insignificant. Here we accept this opinion and ignore the possible explanation in terms of frequency variation.

In this note, we examine how the phase of the CW has shifted during the 1920s-1930s using the longest Earth orientation parameter (EOP) time series currently available: the International Earth Rotation and Reference System Service (IERS) EOP-C01 from 1846 to 2003. Special attention is focused on a comparison between the observed CW obtained by removing the annual wobble (AW) from the EOP time series and a double frequency synthetic CW time series built with the frequencies,

amplitudes and initial phases of the two peaks precisely determined by spectral analysis. The result shows reasonable agreement between their peak times, a feature not shown before. We do not try to conclude here that the CW is a double frequency oscillation based on this agreement alone. However, we suggest nevertheless that this double peak feature is not necessarily due to numerical artifacts, and merits further investigation for understanding its geophysical cause.

2 The double-peak spectral property

We use essentially the same spectral analysis method as Yashkov (1965). This method was re-studied by Guo et al. (2004) for precisely determining the frequency of periodical oscillation using a time series of observations, and was referred to as the folding-averaging algorithm (FAA). The basic principle of the FAA is to rebuild for every test period a new short time series by cutting the original time series to shorter ones of equal length identical to the test period, and then averaging them. In the averaged short time series, the amplitude of the possible signal with a period equal to the test period remains the same, but signals of different periods are averaged out and the random error is reduced. The amplitude and phase of the possible signal with a period equal to the test period can then be estimated using the averaged short time series. By searching for the extremes of the amplitude by varying the test period, the periods (and the amplitudes and phases) of the signals which may be present in the time series can be precisely determined. See the Electronic Supplementary Material (ESM) of this paper for detail.

In the IERS EOP-C01 time series, the data are in 0.1 year intervals during 1846.0 to 1890.0, and in 0.05 year intervals during 1890.0 to 2004.0. We have adopted the interval of 0.1 year by dropping one data over two during 1890.0 to 2004.0. We have also removed the long term variation that is estimated by spline interpolation of the non-overlapping means of 13 years (about twice the length of the beating period between the CW and AW) (Jeffreys 1968; Vicente and Wilson 1997).

First we compute the amplitude spectra for the components X_p and Y_p by using the FAA to an array of selected frequency/period (Guo et al. 2004; See also the ESM), that is similar to the fast Fourier transform (FFT) with zero padding. These spectra are then used to construct the spectrum for the complex-valued EOP time series $\tilde{P} = X_p - iY_p$. We express the harmonic signals with an angular frequency ω in the components X_p and Y_p in the form of

$$X_p = A_x \sin(\omega t + \phi_x), \quad Y_p = A_y \sin(\omega t + \phi_y). \quad (1)$$

They correspond to a prograde-retrograde pair of harmonic signals in the complex-valued EOP time series $\tilde{P}$:

$$\tilde{P} = A_+ e^{i(\omega t + \phi_+)} + A_- e^{-i(\omega t + \phi_-)}. \quad (2)$$

It can be shown that

$$A_+ \cos \phi_+ = (A_x \sin \phi_x - A_y \cos \phi_y)/2, \quad (3)$$

$$A_+ \sin \phi_+ = -(A_x \cos \phi_x + A_y \sin \phi_y)/2, \quad (4)$$

$$A_- \cos \phi_- = (A_x \sin \phi_x + A_y \cos \phi_y)/2, \quad (5)$$

$$A_- \sin \phi_- = -(A_x \cos \phi_x - A_y \sin \phi_y)/2. \quad (6)$$

These are the basic formulas for computing the spectrum of $\tilde{P}$ using the spectra of X_p and Y_p . We show in Fig. 1 the amplitude spectrum in the AW and CW band. A similar diagram may be found in Pedersen and Rochester (1972) (Fig. 1) who used the FFT, and more recently Jochmann and Felsmann (2001) (Fig. 1) who used a modified version of the Fourier transform to obtain a very clean and sharp peak spectrum.

Figure 1 near here

Table 1 near here

The precise frequencies of the wobbles are first determined by extensively searching for the maximum amplitudes using a very dense test frequency/period near the peaks found in the spectrum (Fig. 1). The amplitudes and phases are then estimated by applying the least squares method to the average short time series built according to the precise values of frequencies of the wobbles (Guo et al. 2004; See also the ESM). We adopt here an iterative process to achieve additional accuracy. At each iteration, before building the averaged short time series for estimating the amplitude and phase for one wobble, the other wobbles are first removed from the original time series using their amplitude and phase estimated in the previous iteration. In this step, the amplitudes and phases of the spectrum of the complex-valued EOP time series $\tilde{P}$ are also determined using the amplitudes and phases of the spectra of the components X_p and Y_p according to Eq. (3)-(6). The frequencies, amplitudes and initial phases of the AW and the two peaks in the CW band (denoted with CW_1 and CW_2) precisely determined are listed in Table 1. As the annual wobble is only slightly elliptical (Vondrák 1985), we approximately treat it as a circular mode. Our estimates of the two frequencies in the CW band are 0.8495 and 0.8400 cycle per year (cpy) (the periods are, respectively, 1.1771 and 1.1905 years or 429.9 and 434.8 days). The difference between the two frequencies in the CW band is about 0.01 cpy, giving a beating period of about 100 years.

It is generally accepted that the AW is relatively stable over time (e.g., Pedersen and Rochester 1972; Vondrák 1985, 1999; Höpfner 2004). Considering the fact that the AW and the CW are the most important constituents of polar motion, for both the

components X_p and Y_p , by removing the AW as a simple harmonic signal according to the parameters listed in Table 1, we obtain a time series that principally contains the CW, which will be loosely called the observed CW. As the annual wobble is only slightly elliptical (Vondrák 1985), removing it as a circular mode is not expected to distort the result. We also construct a synthetic time series composed of two harmonic waves for both the components X_p and Y_p using the parameters of the two peaks in the CW band as listed in Table 1, which will be referred to as the double frequency synthetic CW. As an example, we show a comparison between the observed CW and the double frequency synthetic CW for the component X_p in Fig. 2 (The figure for the component Y_p is similar. It is provided in the ESM).

Figure 2 near here

We begin to discuss Fig. 2 by considering the properties of the sum of two harmonic signals of very close frequencies and amplitudes, i.e. the two peaks in the CW band shown in Table 1. We take again the component X_p as example, and write

$$X_p = A_1 \sin(\omega_1 t + \phi_1) + A_2 \sin(\omega_2 t + \phi_2) \quad (7)$$

where we assume, according to the data listed in Table 1,

$$\omega_1 > \omega_2, \quad \omega_1 - \omega_2 \ll \omega_1 \text{ and } \omega_2, \quad (8)$$

$$A_1 > A_2, \quad A_1 - A_2 \ll A_1 \text{ and } A_2. \quad (9)$$

For a simple interpretation, we rewrite Eq. (7) in the form

$$X = 2A_2 \cos(\omega' t + \phi') \sin(\omega + \phi) + \Delta A \sin(\omega_1 t + \phi_1), \quad (10)$$

where

$$\Delta A = A_1 - A_2, \quad (11)$$

$$\omega = (\omega_1 + \omega_2)/2, \quad \omega' = (\omega_1 - \omega_2)/2, \quad (12)$$

$$\phi = (\phi_1 + \phi_2)/2, \quad \phi' = (\phi_1 - \phi_2)/2. \quad (13)$$

In Eq. (10), the first term at the right hand side is equal to $A_2 \sin(\omega_1 t + \phi_1) + A_2 \sin(\omega_2 t + \phi_2)$ that is the sum of two harmonic signals. So this term can be understood either as a double frequency signal or a single frequency signal with modulated amplitude and a phase shift of 180° at around $\omega' t + \phi' = \pm\pi/2$ where it vanishes. From this point of view, it is not possible to distinguish between a single frequency signal and a double frequency signal with only one phase shift of 180° .

At a time span not covering the vicinity of $\omega' t + \phi' = \pm\pi/2$, Eq. (10) is dominated by the first term, thus behaving like a single frequency signal with modulated amplitude; its frequency is ω defined in Eq. (12) that has a numerical value of 0.8448 cpy, corresponding to a period of 1.1838 years (432.4 days). In Fig. 2, the distance between the vertical grid lines is just this period. We see that before 1920, the grid lines coincide with the higher peaks of both the observed CW and the double frequency synthetic CW, and after 1940, at the lower peaks. If we interpret both the observed CW and the double frequency synthetic CW as a single frequency signal with phase shift, this property indicates that there is a phase shift of 180° during 1920-1940. Furthermore, it also indicates that the peak times of the observed CW and the double frequency synthetic CW are in good agreement before 1920 and after 1940; the peak-to-peak distance being around the period of the CW if it is considered as a single frequency wobble, i.e. 1.1838 years.

The phase shift, which may be understood as the peak-to-peak distance variation, can be seen in more detail in the vicinity of $\omega' t + \phi' = \pm\pi/2$, i.e. during 1920-1940, where the influence of the second term is also important. In general, we see from Fig. 2 that, during the period 1920-1940, the phase has gradually shifted 180° in advance for both the observed CW and the double frequency synthetic CW if we interpret both of them as single frequency signals with phase shift. We also notice that the peak times of the observed CW and the double frequency synthetic CW are in reasonably good agreement, considering the small amplitude of the CW and the presence of error of observation and variations with shorter periods not removed from the observed CW that we obtained by removing only the annual wobble from the EOP time series. This agreement may in fact be understood as a fact supporting the hypothesis of a double frequency CW, though not sufficient to draw a conclusion for its definite existence.

Particularly, during the short time nearest to $\omega' t + \phi' = \pm\pi/2$, i.e. around 1930, Eq. (10) is dominated by the second term that has a frequency ω_1 , slightly higher than ω discussed in the last paragraph. The peak-to-peak distance at this time is just the period corresponding to the higher frequency ω_1 , the numerical value being 1.1771 years (429.9 days). This is, in fact, the smallest peak to peak distance (at least for the double frequency synthetic CW).

To see the phase shift in more detail, we give an alternative expression of (7) (i.e., again only considering the component X_p):

$$X_p = A(t) \sin[\omega t + \phi + \beta(t)] \quad (14)$$

where the amplitude $A(t)$ and the phase shift terms $\beta(t)$ are

$$A(t) = [A_1^2 + A_2^2 + 2 A_1 A_2 \cos(2\omega' t + 2\phi')]^{1/2}, \quad (15)$$

$$\beta(t) = \arctan \left[\frac{A_1 - A_2}{A_1 + A_2} \tan(\omega't + \phi') \right]. \quad (16)$$

Eqs. (14)-(16) express the sum of two sinusoidal signals in terms of only one with amplitude and phase modulated. The phase shift is given by (16). From these expressions, we again see that the amplitude is smallest near $\omega't + \phi' = \pm\pi/2$. It can also be seen that the phase-shift term $\beta(t)$ varies fastest at these locations, since $(A_1 - A_2)/(A_1 + A_2)$ is of very small order of magnitude, only 0.11 according to the data given in Table 1. For example, $\beta(t)$ varies from 0 to $1/5 \times \pi/2$ and then to $\pi/2$ when $\omega't + \phi'$ varies from 0 to $4/5 \times \pi/2$ and then to $\pi/2$. Although Eqs. (14)-(16) give accurate phase shift, they are not as simple as Eq. (10) when examining the peak-to-peak distance during the short time nearest to $\omega't + \phi' = \pm\pi/2$, as discussed in the last paragraph.

3 Discussion

We have compared the observed CW and the double frequency synthetic CW built using the parameters of the two peaks in the CW band precisely determined using spectral analysis (Table 1). We have focused on the period around 1930 when a phase shift of 180° responsible for the appearance of the double peak in the CW band is believed to have happened. The peak times of the observed CW and the double frequency synthetic CW agree reasonably well considering the small amplitude of the CW during that period and the presence of error of observation and variations with shorter periods that we have not removed from the observed CW we have defined. This agreement supports the hypothesis of a double frequency CW, though not enough to draw a definitive conclusion.

The CW is a free mode of the Earth, which has been decaying and re-excited. Such a free mode cannot be expected to be purely harmonic; amplitude variation and phase shift are natural. The large phase shift during 1920s-1930s may well be attributed to the excitation process while adopting a single frequency CW (e.g. Jeffreys 1940; Vondrák 1999). However, a phase shift of 180° during a very short time is something peculiar, though the small amplitude of the CW during that time may render a large phase shift to be caused more easily by the excitation process. Guinot (1982) pointed out that the cause of the event is probably quite different from the excitation which modifies the amplitude. Examination of the process of this large phase shift may potentially be helpful for understanding the cause of the event.

One may argue that the double peak feature of the CW is not real because only one peak appears in the spectra of the EOP time series before 1920, after 1940, or between 1920 and 1940 (e.g., Pedersen and Rochester 1972; Lenhardt and Groten 1985). We have computed the spectra for the time series of the double frequency synthetic CW during 1846-1920, 1940-2003 and 1920-1940 respectively. The double peak feature did not appear in the resulting spectra, although the time series really contain two harmonic signals. This implies that, even if the CW really has two frequencies, one can only expect to obtain them by spectral analysis of an EOP time series long enough and including the period around 1930 when the variation of the CW is most dramatic. Thus, we believe that the results of spectral analysis of a short EOP time series, or an EOP time series not covering the period around 1930 cannot be used to deny the possibility of a double frequency CW.

The comparison made in this paper is qualitative. No statistical test has been made. In fact, it is not evident that certain firm criteria may be found to really judge if the CW is single or double frequency. As already mentioned, the CW is not purely harmonic. So it is basically inappropriate to describe a single or double frequency CW with 3 or 6 parameters (frequency, amplitude and initial phase). Although we believe that the correlation between the phases of the observed CW and the synthetic double frequency CW is in favor of a double frequency CW, we do not think that a definite conclusion may be drawn based on this correlation alone. In summary, we suggest that, although it is generally believed that the EOP data available up to now may be explained using a single frequency CW, it seems that the possibility of a double frequency CW could not definitely be ruled out. We also suggest that the definitive answer may only be drawn in about 30 years to see whether another variation similar to that around 1930 will happen again, since the beat time between the two frequencies in the CW band is about 100 years.

The main reason for which the idea of a double frequency CW is not accepted is that there exists no reliable theoretical model of the free wobble of the Earth based on the Earth's shape and internal constitution which predict this phenomena. Recently, Rogister and Vallete (2004) proposed a wobble model of the Earth with non-neutrally stratified outer core capable of producing two wobbles at the CW band with periods 2-5 days apart assuming equal amplitudes. As compared to this model, the minor difference in the amplitudes of the two peaks in the CW band obtained from the EOP time series in this paper is insignificant, while the difference between the periods of these two peaks is 5 days, that is within the limit of the model. However, this coincidence is still insufficient to conclude that the CW really has two frequencies. Excitation based on the model of Rogister and Vallete (2004) need to be further studied. This is evidently another work.

Furthermore, besides a phase shift that is generally accepted, a double frequency CW may not be the only explanation of the double peak. Perhaps some nonlinearity may make the CW behaves in a more complicated way that appears like a double frequency harmonic signal.

In addition to the comparison of phases, we expect this note to serve as a reminder for the double frequency feature of the EOP time series in the CW band for potential renewed interest in explaining this peculiar phenomena.

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Fig. 1. The amplitude spectrum in the AW and CW band computed using the IERS EOP-C01 time series during 1846-2003, which shows the double peak of the CW.

Fig. 2. Comparison between the observed CW (gray thick line) and the double frequency synthetic CW (black thin line) for the component X_p . The lower figure is a portion of the upper one amplified when the phase shift happened. We see that the peak-to-peak correspondences between the observed and the synthetic CWs are in general good, keeping in mind that variation of the amplitude and phase of the observed CW is a natural behavior due to decay and excitation of the CW. Long-term variation in the EOP time series has been fitted out with a quadric line.

Table 1. Periods, amplitudes and initial phases of the AW and the two peaks in the CW band. Initial time: 1846.0.

Wobble	Frequency cycle/year	Amplitude mas	Initial phase ($^{\circ}$)
AW	0.99866	88.540	166.73
CW ₁ (First peak)	0.84953	104.759	45.89
CW ₂ (Second peak)	0.84001	83.333	153.91