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**1 Dependence of the Omori-Utsu law parameters on
2 mainshock magnitude: observations and modeling**

S. Hainzl

3 GFZ German Research Centre for Geosciences, Potsdam, Germany

D. Marsan

4 LGIT, Universite de Savoie, CNRS, Le Bourget du Lac, France

S. Hainzl, Helmholtz Zentrum Potsdam: Deutsches GeoForschungsZentrum, Section 2.1, Telegrafenberg, 14473 Potsdam, Germany. (hainzl@gfz-potsdam.de)

D. Marsan, Laboratoire de Geophysique Interne et Tectonophysique, Universite de Savoie, F-73376 Le Bourget du Lac, France. (david.marsan@univ-savoie.fr)

5 **Abstract.** We examine the dependence on mainshock magnitude m of
6 the p and χ parameters appearing in Omori-Utsu formula $\lambda(t, m) = \chi \times$
7 $(t+c)^{-p}$ relating the rate of aftershocks λ at time t after a mainshock. Ob-
8 servations point out to a significant increase of p with m , along with a scal-
9 ing relationship of the form $\chi \sim 10^{\alpha m}$. We here show that these observa-
10 tions can be explained within the framework of the rate-and-state friction
11 model, when accounting for realistic levels of coseismic stress heterogeneity
12 on the main fault. We constrain the model parameters in order to recover
13 the trends observed in previous and new analyses of aftershock sequences.
14 The expected ratio of the coseismic stress drop standard deviation to its mean
15 is found to be of the order of a few units for large ($m=7$) earthquakes, re-
16 sulting in a very rough stress field at the small scale, while it is much smoother
17 at small magnitudes (ratio $\simeq 0.1$ at $m=2$). Finally, the influence of afterslip
18 on parameters p and χ is studied, to highlight the fact that it can significantly
19 perturb the $p(m)$ and $\chi(m)$ relations obtained with the initial afterslip-free
20 model.

1. Introduction

Almost all larger earthquakes are found to trigger aftershocks with a temporal decaying probability. In particular, the occurrence rate of aftershocks λ can be well described by the modified Omori-Utsu law

$$\lambda(t, m) = \chi(t + c)^{-p} \quad (1)$$

where t indicates the elapsed time since the mainshock, see Utsu et al. (1995) for a review. The c -value is a constant typically much less than 1 day, and in most cases is related to changes in detection level of the operating seismic network. Recent attempts at finding a c -value of physical rather than instrumental origin have proposed that it could be of the order of one to several minutes (Kagan and Houston, 2005; Peng et al., 2006, 2007, Enescu et al. 2007), although there is no clear consensus on how the Omori-Utsu law actually breaks down below this cut-off. The p -value is in the range 0.8-1.2 in most cases (Utsu et al., 1995). While alternative models for describing the aftershock decaying rate have been proposed (Kisslinger, 1993; Gross and Kisslinger, 1994; Narteau et al., 2002), the Omori-Utsu law generally provides a very good fit to the data, and is an ubiquitous feature in seismicity dynamics.

We here analyze how parameters p and χ change with the magnitude m of the mainshock. A wealth of recent studies have addressed the dependence of χ (mostly) with m , generally showing that $\chi \sim 10^{\alpha m}$. The value of parameter α is however variable from one study to the other, mainly because of different assumptions regarding to the definition of what mainshocks and aftershocks are. Also, a significant increase of p with m , which was

37 not recognized before, has been recently observed by Ouillon and Sornette (2005), which
38 these authors explain by a multifractal model of stress interactions (Sornette & Ouillon,
39 2005).

40 The goal of this paper is to show that these observations are consistent with a model
41 based on rate-and-state friction (Dieterich, 1994), with a spatially heterogeneous coseismic
42 stress change at the length scale of earthquake nucleation. In Section 2, we recall results
43 of past analyses on the magnitude dependence of χ and p , and test these results by new
44 such analyses, probing different ways of selecting mainshocks and aftershocks. In Section
45 3, we detail our model, and explore its parameter space so to provide constraints on
46 what values of these parameters can reproduce the observations. Finally, in Section 4, we
47 study how the addition of afterslip can influence the p and χ values, still exploiting the
48 rate-and-state model with coseismic stress heterogeneity.

2. Observations

49 There is good evidence that the productivity χ grows exponentially with m , i.e., fol-
50 lowing a $\chi \sim 10^{\alpha m}$ relation. However, the exact value of α varies substantially between
51 studies: Helmstetter (2003) obtained that $0.7 < \alpha < 0.9$ for southern California, while
52 Felzer et al. (2004) and Helmstetter et al. (2005) found $\alpha = 1$ and $\alpha = 1.05 \pm 0.05$,
53 respectively, for the same region. Using space-time ETAS models and inverting for the
54 model parameters, Zhuang et al. (2004) found that $\alpha \simeq 0.6$ for Japan (1926-1999 $m \geq 4.2$
55 earthquakes), Zhuang et al. (2005) found $\alpha = 0.7 \pm 0.05$ for Taiwan (1987-2000 $m \geq 5.3$
56 earthquakes), while Console et al. (2003) obtained $\alpha = 0.42$ for Italy (1987-2000 $m \geq 2$
57 earthquakes).

Variations in the estimate of α can be due to differences in the seismogenic properties of the regions analyzed, but also to the different procedures used to select which earthquakes are mainshocks and which others are aftershocks. This selection is generally performed using space-time windows that define the 'aftershock domain' of a mainshock (cf. Molchan and Dmitrieva, 1992). Such methods rely on sets of parameters, that are largely arbitrary. The alternative approach of fitting ETAS model parameters to the data is computationally much more involved, and remains clearly model-dependent. Recently, a new probabilistic method (nicknamed MISD for Model-Independent Stochastic Declustering) for selecting mainshocks and aftershocks, that does not rely on any particular model nor specific parameterization, has been proposed (Marsan and Lengliné, 2008). This approach, based on the premises that seismicity dynamics result from a linear cascade of earthquake triggering, permits to distinguish between directly and indirectly triggered aftershocks. Applying this method to southern California data, Marsan and Lengliné (2008) found that the $\chi \sim 10^{\alpha m}$ is indeed a good representation of the data, with an α parameter equal to 0.6 for directly triggered aftershocks, while $\alpha = 0.66$ for all (i.e., direct and indirect) aftershocks, which is what the space-time window methods measure. In the context of the ETAS model, a single α parameter characterize both the direct and the overall aftershock populations. However, α -values inverted by cascading models with an isotropic spatial kernel are likely to underestimate the real value as recently demonstrated for the case of the space-time ETAS model (Hainzl et al., 2008). The reason is that real aftershock clusters are usually anisotropically distributed in space due to the spatial extension of mainshock ruptures. Indeed, relaxing the isotropy assumption, Marsan and Lengliné (2008) found $\alpha = 0.86$ ('bare' value for directly triggered events) and $\alpha = 0.73$ ('dressed' value for directly and

indirectly triggered events), instead of $\alpha = 0.60$ and 0.66 , respectively, when assuming isotropy.

To further study the p and χ dependence on m , we here analyze the global earthquake catalog provided by the ISC, focusing on the 1978-2005 period and $m \geq 5.5$ earthquakes. The starting date of 1/1/1978 is constrained by the fact surface wave magnitudes m_s only start to be reported at that date. We kept the maximum magnitude (whatever its type) to characterize the size of an earthquake. This choice is purely empirical, and was motivated by the requirement that the frequency-magnitude curve follows an exponential Gutenberg-Richter law. Indeed, no deviation to the Gutenberg-Richter law above magnitude 5.5 is found when examining the global seismicity, and when analyzing each year individually. Also, as shown in Fig.1, the b -value remains stable over the years, indicating that there is no statistically significant change in magnitude reporting in the 1978-2005 period.

We select mainshocks and aftershocks using several different published selection rules, for comparison purposes. An earthquake is characterized by its time of occurrence t , its location \underline{x} , and its magnitude m , which we use to define its rupture length as $L = 10^{0.45 \times (m-6)} \times 10$ km consistent with the analysis of Wells and Coppersmith (1994) (with a minimum $L = 10$ km, hence for all earthquakes with $5.5 \leq m \leq 6$, to account for location error). Namely, we use:

(1) a space-time window method, so that an earthquake $\{t, \underline{x}, m\}$ is not a mainshock if there exists too big and too close a previous earthquake $\{t', \underline{x}', m'\}$, with $m' \geq m - \Delta m$, at $t - \Delta t < t' < t$ and so that their rupture zones overlap, i.e., $|\underline{x} - \underline{x}'| < L(m) + L(m')$. The aftershocks of a mainshock are all the earthquakes that follows it in its rupture zone,

103 until a new mainshock occurs which rupture zone overlaps with the current one. To test
 104 the sensitivity of the method to Δm and Δt , we take either $\Delta t = 1$ year, $\Delta m = 1$, or
 105 $\Delta t = 3$ years, $\Delta m = 2$.

106 (2) the method by Helmstetter (2003): an earthquake $\{t, \underline{x}, m\}$ is not a mainshock
 107 if there exists a previous, larger earthquake $\{t', \underline{x}', m' > m\}$ within 1 year and 50 km
 108 independent of the magnitude. Then, all the earthquakes within a rupture length $L(m)$
 109 and 1 year after the mainshock are its aftershocks.

110 (3) the method by Helmstetter et al. (2005), which mimics the declustering algorithm
 111 of Reasenberg (1985). Here, an earthquake $\{t, \underline{x}, m\}$ is not a mainshock if there exists
 112 a previous earthquake $\{t', \underline{x}', m'\}$ with $m' \geq m - 1$ that occurred within 1 year and a
 113 distance $L(m')$. Then, an earthquake is an aftershock of a given mainshock $\{t, \underline{x}, m\}$ if
 114 it occurs within $L(m)$ and 1 year of it, or within $L(m')$ and 1 year of any of its previous
 115 aftershocks $\{t', \underline{x}', m'\}$.

116 (4) the algorithm by Gardner and Knopoff (1974). An earthquake is an aftershock of
 117 a given mainshock if it occurs within a time $T(m)$ and distance $R(m)$ of it, with both T
 118 and R increasing with m . We extend the magnitude range of Gardner and Knopoff (1974)
 119 up to magnitude 9, by setting $T = 1000$ days for $m \geq 8.5$ and keeping the $R \sim 10^{0.12m}$
 120 scaling. All the earthquakes that are not aftershocks are considered as mainshocks.

121 (5) the model-independent stochastic declustering (MISD) method by Marsan and
 122 Lengliné (2008). This method assumes that all the previous earthquakes have an in-
 123 fluence on a subsequent earthquake, and that those influences sum up. The method then
 124 amounts to running an iterative algorithm converging towards the mean-field influences
 125 (i.e., mean-field in the sense that two earthquakes of equal magnitudes will be considered

126 as having equal influences at the same inter-event distances and times). This gives the
 127 'bare' (i.e., direct) influences. The 'dressed' (i.e., both direct and indirect) influences are
 128 obtained by considering the full cascade of aftershocks triggering other aftershocks and
 129 so on, and summing over the various bare influences. There is no parameterization in
 130 this method. Notice that all the other methods exploited here only probe the dressed
 131 aftershock sequences.

132 Figure 2 displays the aftershock rates for all these methods, along with the best power-
 133 law fits $\lambda(t, m) = \chi \times t^{-p}$ which amounts to the Omori-Utsu law after neglecting the
 134 cut-off time c . These fits are computed for $0.1 \leq t \leq 100$ days (i.e., over 3 decades).
 135 No correction for the loss of aftershocks due to detection issues at short time scales is
 136 introduced. Given the quality of all the fits, we believe the scaling interval is appropriate
 137 for this 'no-correction' choice, given these fitting time intervals. Table 1 summarizes the
 138 various estimates related to Fig. 2.

139 All the space-time window methods (1), (2) and (3) yield very similar rates. The method
 140 using Gardner and Knopoff (1974) is also quite similar to the dressed rates of the MISD
 141 method. Although the general aspect is well preserved from one method to the other,
 142 with the notable exception of the bare rates using the MISD method (i.e., because all the
 143 other rates are 'dressed'), subtle differences can however be seen. The parameters p and
 144 χ obtained with the best fits are reported on Figure 3. As can be observed, there exists a
 145 significant dispersion of the parameters at all magnitudes, especially for the p -value, and
 146 even for one method by just changing its parameters (i.e., method (1), triangles). The
 147 p -values proposed by Ouillon and Sornette (2005) are occasionally significantly different
 148 from the ones obtained here. This could be due to the fact that they analyzed a very

149 different dataset than ours (southern California earthquakes), and also to the way they
150 selected their time intervals for fitting the decays.

151 The productivity is effectively found to follow a $\chi \sim 10^{\alpha m}$ scaling, although parameter
152 α ranges between 0.66 (bare and dressed rates using MISD) and 1.15 (for most space-time
153 window methods). This confirms that the productivity scaling is unfortunately strongly
154 dependent on the selection method, as already discussed above. Low α values are typically
155 obtained with ETAS inversions and the MISD method, which both perform space-time
156 analyses and estimate the bare influences by assuming that the observed seismicity re-
157 sults from cascading. The other methods do not account for this cascading, and could
158 therefore be biased towards large α values as a result. On the other hand, the inversion of
159 cascading models with isotropic spatial kernel can lead to significant underestimation of
160 the α parameter as recently shown for the space-time ETAS model (Hainzl et al., 2008).

161 In the following, we will use the results shown in Figure 3 as a constraint for our model
162 parameters. As there is yet no clear consensus on the 'correct' values of p and α , we will
163 ask our model to output values that are within the ranges shown in Fig.3 and proposed
164 in past analyses, rather than attempting to reproduce one particular set of values.

3. Model of earthquake occurrence

165 Many aftershocks occur on-fault where quasi-static stress is expected to decrease after
166 the mainshock, resulting in an apparent paradox. However, earthquake slip is known to
167 be heterogeneous, leading locally to an increased shear stress after mainshock slip (i.e.,
168 loading rather than unloading). This has been observed by Mikumo and Miyatake (1995),
169 Bouchon (1997), Bouchon et al. (1998), Day et al. (1998), Dalguer et al. (2002), Zhang

170 et al. (2003), and Ripperger and Mai (2004) for a number of earthquakes. Examples
171 of stress drop heterogeneity images at large scales can be found in Day et al. (1998).
172 Heterogeneous fault stress has also been found in simulations by Parsons (2008) to be
173 a long-lasting feature, with a spatial distribution of stress gaps showing persistence over
174 tens of years.

175 Recently, it has been shown that coseismic stress heterogeneities are able to explain
176 aftershock activity, especially those observed in stress shadows such as within the main-
177 shock rupture (Helmstetter and Shaw, 2006; Marsan, 2006). At the scale of the nucleation
178 of seismic instability (typically meters to tens of meters as predicted by rate and state
179 friction; cf. Fig.11, Dieterich, 1992), the stress drop is dominated by this spatial variabil-
180 ity: numerous nucleation patches are then loaded rather than unloaded by the mainshock,
181 resulting in the occurrence of aftershocks.

182 As it is shown later, the situation is significantly different for smaller mainshocks, i.e.,
183 characterized by rupture lengths not too large compared to the nucleation length. Then,
184 scale invariance of the coseismic slip implies that the stress drop is much smoother (still at
185 the scale of nucleation) than that of large mainshocks. The ruptured fault is then mostly
186 unloaded, and no aftershocks occur. The direct observation of this shadowing effect for
187 small mainshocks has been made by Rubin (2002), for relocated earthquakes on the San
188 Andreas fault, and by Fischer and Horalek (2005) for relocated swarm earthquakes in the
189 Vogtland area. In both cases, the stacked seismicity showed a significant gap within the
190 rupture area.

191 We here study how a model based on rate-and-state friction with a magnitude-dependent
 192 distribution of coseismic stress change can recover the aftershock decay characteristics
 193 described in the preceding section.

3.1. Description of the model

194 In the following treatment, we postulate that:

- 195 • Seismicity can be well described by the rate-and-state model of Dieterich (1994) with
 196 the ageing evolution law, in the localized nucleation regime for which healing is negligible
 197 (Rubin and Ampuero, 2005).
- 198 • Static stress triggering dominates the production of aftershocks.
- 199 • The coseismic slip is fractal, causing the stress drop to be fractal as well.
- 200 • Spatial fluctuations in stress drop can be modeled with Gaussian statistics.
- 201 • There exists a finite, time-independent nucleation length ℓ that characterize the size
 202 of fault patches self-accelerating to failure (Dieterich, 1992).
- 203 • All earthquakes initially nucleate at scale ℓ , their final size being controlled by the dy-
 204 namic propagation of the instability outside the nucleation patch rather than by processes
 205 occurring within this nucleation zone (Lapusta and Rice, 2003).

206 **Rate-and-state model:** According to Dieterich (1994), in the no-healing approxima-
 207 tion, the seismicity rate λ is inversely proportional to the state variable γ describing the
 208 creep velocities on the faults, namely $\lambda(t) = \frac{r}{\dot{\tau}\gamma(t)}$, where r is the stationary background
 209 rate of earthquakes and $\dot{\tau}$ the tectonic loading rate. The evolution of the state variable γ
 210 is given by

$$d\gamma = \frac{dt - \gamma d\tau}{A\sigma} \quad (2)$$

211 with A being a dimensionless fault constitutive parameter usually ~ 0.01 and σ the effective
 212 normal stress. A sudden stress jump of τ for a background stationary rate r leads to a
 213 time-dependence of the activity according to

$$\lambda(t, \tau) = \frac{r}{1 + (e^{-\frac{\tau}{A\sigma}} - 1)e^{-\frac{t}{t_a}}} \quad (3)$$

214 with $t_a = A\sigma/\dot{\tau}$. For simplicity, we will give hereinafter all stress jumps in units
 215 of $A\sigma$ and the time in units of t_a , unless stated otherwise, leading to the expression
 216 $\lambda(t, \tau) = r/[1 + (e^{-\tau} - 1)e^{-t}]$.

217 **Fractal coseismic slip and stress-drop heterogeneity:**

218 The stress variations induced by an earthquake are expected to be spatially hetero-
 219 geneous due to coseismic slip as well as material heterogeneities. Thus, for any given
 220 crustal volume, the actual stress experienced by nucleation patches must be described by
 221 a probability density function $f(\tau)$, and the earthquake activity of the volume must be
 222 calculated by

$$\lambda(t) = \int \lambda(t, \tau) f(\tau) d\tau \quad (4)$$

223 On or close to the main fault, stress heterogeneity is dominated by slip variability. Scale-
 224 invariant slip models have been proposed by several authors (Andrews, 1980; Frankel,
 225 1991; Herrero and Bernard, 1994; Mai and Beroza, 2002). For a two-dimensional fractal

226 model, the slip $u(k)$ is proportional to $k^{-1-H}g(k)$ with H the Hurst exponent related to
 227 the fractal dimension $D = 3 - H$, where g is a realization of a Gaussian white noise, and
 228 k the wave number. In their extended analysis of the slip distributions of 44 earthquakes,
 229 Mai and Beroza (2002) found that $H = 0.71 \pm 0.23$. Since the stress drop scales as
 230 $\tau(k) \sim k u(k) \sim k^{-H}g(k)$, e.g. see Schmittbuhl et al. (2006), the scaling of the standard
 231 deviation σ_τ of the stress change at the length scale of the nucleation sites, ℓ , is given by
 232 Marsan (2006)

$$\sigma_\tau = C \sqrt{\left(\frac{L}{\ell}\right)^{2-2H} - 1} \quad (5)$$

233 for $H < 1$, where L is the rupture length of the earthquake. The standard deviation,
 234 hence the variability of the stress drop, thus diverges for $\ell \rightarrow 0$ when $H \leq 1$ (Helmstetter
 235 and Shaw, 2006). Figure 4 shows an example of how the stress drop roughness depends on
 236 the scale ratio between the fault size L and the nucleation scale ℓ . A 10×10 km² fault is
 237 simulated, which roughly corresponds to a magnitude 6 earthquake: we generate a fractal
 238 (scalar) slip $u(x, y)$ with Hurst exponent $H = 0.7$, such that the stress drop, defined as
 239 $(\partial_x + \partial_y)u$, has a mean value of 3 MPa. We vary the scale of observation, thus changing
 240 the scale ratio between the rupture size L and the cut-off scale ℓ . As this scale ratio is
 241 increased, the roughness of the stress drop is enhanced, with the emergence of patches
 242 undergoing stress loading (i.e., negative stress drops).

243 We calibrate the intensity of the stress fluctuation by considering that the induced stress
 244 variability of large earthquakes is typically of the order of the average stress drop $\bar{\tau}$, when
 245 observed at the $\simeq 5$ km scale. Using $H = 0.7$, this gives that the stress variability at the

246 nucleation length scale of approximately 10 m would be of the order of $6\bar{\tau}$. In the following,
 247 we define the stress variability σ_7 induced by a $m = 7$ event as an input parameter,
 248 typically ranging between 0.1 and 10 times the mean stress drop: $0.1 < CV = \frac{\sigma_7}{\bar{\tau}} < 10$,
 249 where CV is the coefficient of variation of the stress distribution.

250 The dependence of the stress drop heterogeneity on the magnitude is given by Eq.(5),
 251 together with $L = \ell 10^{\beta(m-m_0)}$, where the empirical value of β is close to 0.45 (Wells and
 252 Coppersmith, 1994, assuming the rupture length as the square root of the rupture area).
 253 We denote by m_0 the magnitude corresponding to a rupture size of ℓ , i.e., the minimum
 254 magnitude for friction-controlled earthquakes.

255 **Stress drop modeled with Gaussian statistics:**

256 A Gaussian model for τ is only a first-order approximation. There is evidence for an
 257 asymmetric stress drop in some instances (Day et al., 1998), with pronounced peaks of high
 258 stress drop embedded in large zones of low, negative stress drop. Elaborating even further
 259 away from a Gaussian model, Lavallée and Archuleta (2003, 2005) have proposed that the
 260 slip distribution of both the 1979 Imperial Valley and the 1999 Chi-Chi earthquakes are
 261 better modeled by Lévy-stable statistics. In this model, $\tau(k) \sim k^{-H} g_\alpha(k)$, where g_α is a
 262 Lévy noise with stability index α , typically with α close to 1 (hence g_α close to a Cauchy
 263 noise). The difficulty in handling this type of model is that the stress drop distribution
 264 can no longer be characterized by its standard deviation, as it is not defined anymore.
 265 Clearly, Lévy-distributed stress drops will generate even rougher fields, and the results
 266 presented in this manuscript, that are based on normal (Gaussian) laws, can therefore be
 267 seen as a 'most-conservative', i.e., least-heterogeneous, limit case.

Nucleation size:

So far, nucleation zones lack direct observation. Therefore, we assume the simplest case that the nucleation size is independent of the aftershock magnitude. This is in agreement with the well-known cascade model for earthquake ruptures (e.g. Kilb and Gomberg, 1999) and numerical simulations (Lapusta and Rice, 2003). In particular, we assume that the magnitude m_0 , which corresponds to the nucleation size ℓ , is constant. In the case of $m_0(m)$, our results would be directly applicable only for each magnitude band of the aftershocks separately. However, because of the weak dependence of our results on m_0 (see Fig.5), our general results are expected to remain valid even in this case.

3.2. Model predictions vs. observations

We calculate the seismicity rate within the rupture zone of the mainshock by solving Eq.(4) numerically with a magnitude-dependent Gaussian probability distribution, i.e., $f(\tau)$ is Gaussian with mean $-\bar{\tau}$ and standard deviation σ_τ . $\bar{\tau}$ is the (Coulomb) stress drop on the main fault which can be seen further away from the fault (King and Cocco, 2001; Freed, 2005; JGR special issue on stress triggering, 2005): adding stress heterogeneities allows to go beyond usual Coulomb stress modeling by introducing a stochastic term to the deterministic stress field. This stochastic term is here viewed as accounting for the small scale variability that is not accessible to direct measurement nor computation. It can alternatively be seen as modeling the error on the large-scale stress field: as well as a mean stress drop $\bar{\tau}$, we also need its uncertainty σ_τ . Accounting for such an uncertainty is not a 2nd-order refinement: as already shown by Helmstetter and Shaw (2006) and Marsan (2006), it can significantly alter the seismicity.

289 The model has a number of parameters, which have a direct influence on the Omori-Utsu
 290 parameters p and χ . We summarize these parameters in Table 2, along with their values.
 291 For an earthquake of magnitude m , the distribution of stress drops on the main fault is
 292 thus a Gaussian distribution with mean $\bar{\tau}$ independent of m , and standard deviation as
 293 given by Eq.(5). The crucial point here is that this standard deviation increases with
 294 the magnitude m , this increase being constrained by parameters C (or equivalently σ_7 or
 295 CV), ℓ (or m_0), and H . Changing these three key parameters amounts to changing the
 296 dependence of p and χ on m .

297 The standard value of the Hurst exponent H is set to 0.7 because it was the mean
 298 value obtained by Mai and Beroza (2002). Letting H vary within the acceptable range
 299 $0.5 \leq H \leq 0.9$ strongly affects the results, as decreasing H causes the stress field to
 300 become more heterogeneous. This will be further discussed in subsection 3.4.

301 A first point is to note that the aftershock decay depends very little on ℓ and m_0 , as
 302 long as they remain very small compared to the sampled rupture lengths and magnitudes.
 303 Figure 5 illustrates the increasing stress drop heterogeneity for increasing earthquake
 304 magnitudes for the range of Hurst-exponents inverted from slip data and for three different
 305 m_0 -value. The tested values of $m_0 = -4, -2, \text{ and } 0$ correspond to nucleation length of
 306 approximately 0.3 m, 2.2 m, and 18 m (Wells and Coppersmith, 1994). Given that the
 307 dependence on the assumed m_0 -value is very weak, we (arbitrarily) set m_0 to -2 for the
 308 remainder of this study.

309 The only parameters left to vary are therefore the mean stress drop and the calibration
 310 constant C (or σ_7, CV). We calculate the aftershock rate as a function of the mainshock

311 magnitude for different values of these parameters. Figure 6 shows the aftershock decay for
 312 different mainshock magnitudes in the case of a coefficient of variation $CV \equiv \sigma_{\bar{\tau}}/\bar{\tau} = 2.3$
 313 and a stress drop of $\bar{\tau} = 1$ MPa.

314 The Omori-Utsu law is fitted to each of these curves in the time interval $[10^{-4} - 10^{-1}]$
 315 yielding an estimate of the p -value as a function of the mainshock magnitude. For a stress
 316 field variation of $CV = 2.3$, the magnitude-dependence is found to be in good agreement
 317 with the observed p -value dependence in California (Ouillon & Sornette, 2005), and to our
 318 global analysis of section 2. This is shown in Fig.7. Note that for significantly stronger
 319 heterogeneities, the magnitude dependence becomes quite weak and would be difficult to
 320 detect in real data (see the curve for $CV = 8.0$ in the same figure): in this case, the stress
 321 heterogeneity is large enough even at magnitude 2 to push the p -value close to 1.

3.3. Aftershock productivity as a function of mainshock magnitude

322 In the case that ruptures produce a stress drop variability which is independent of the
 323 earthquake magnitude, our model would predict an aftershock productivity which would
 324 simply scale with the mainshock rupture area, i.e. $\sim 10^{0.9m}$. However, as another con-
 325 sequence of the scaling of stress heterogeneity with mainshock magnitude, the aftershock
 326 productivity is not simply scaling with mainshock area anymore. For the previous exam-
 327 ples, the productivity values χ are shown in Fig.8. For moderate stress heterogeneities,
 328 the increase of the aftershock productivity is close to $\sim 10^{1.05m}$ which is the empirical
 329 scaling exponent found by Helmstetter et al. (2005) for California and in agreement with
 330 our own investigations of the global earthquake catalog with methods (1)-(3) (Fig. 3).

331 For significantly larger heterogeneities ($CV = 8.0$), the scaling exponent is smaller and
 332 becomes almost $\sim 10^{0.9m}$. Thus for large heterogeneities, the model predicts an almost
 333 constant $p \approx 1$ -value and an aftershock productivity which simply scales with the rupture
 334 area.

335 We have implicitly assumed that aftershocks can occur everywhere on and close to the
 336 mainshock fracture. However, some studies indicate that aftershocks occur on spatial
 337 fractals with dimension $D < 2$ (Turcotte 1997, Helmstetter et al., 2005). Assuming that
 338 aftershocks are restricted to such fractal subsets of the fault plane, we would get a smaller
 339 theoretical cutoff-value $\alpha_{min} = \beta \cdot D = 0.45 D$ instead of 0.9.

3.4. Dependence on the Hurst-exponent and the stress drop

340 Our general findings are independent of the assumed value of the mean stress drop
 341 $\bar{\tau}$. The increase of the p -value and the aftershock productivity is found to be preserved
 342 for other values of $\bar{\tau}$. However, changing $\bar{\tau}$ impacts on the degree of the stress field
 343 heterogeneity which is needed to produce the same magnitude dependence. For example,
 344 practically the same curve as shown in Fig.7 for $\bar{\tau}=1$ MPa and $CV = 2.3$ is found for
 345 $\bar{\tau}=0.5$ MPa with $CV = 4.0$ and $\bar{\tau}=2$ MPa with $CV = 1.6$. These results depend also on
 346 the Hurst-exponent. Figure 9 shows for the case of $\bar{\tau}=1$ MPa the same characteristics for
 347 the lower and upper limits of the observed Hurst-exponents, $H=0.5$ and $H=0.9$. In each
 348 case, the standard deviation σ_7 is chosen such that the p -value dependence on magnitude
 349 fits the observation best. It is found that higher Hurst-exponents underestimate the
 350 observed magnitude-dependence whereas lower Hurst-exponents seem to overestimate the
 351 trend. Thus the value $H=0.7$ which is independently found to best describe observed

352 slip-distributions is also found to give consistently the best description of the aftershock
 353 decay. This is another indication of the applicability of the rate-and-state friction model
 354 for aftershocks.

355 To examine the whole parameter space more systematically, we calculate, for stress
 356 drops varying between 0.1 MPa and 10 MPa, the stress field variability CV which leads
 357 to a p -value increase of 0.1 from $m=3.0$ to 7.0 mainshocks. The resulting curves are shown
 358 for the different Hurst exponents in Fig. 10. These curves can be seen as the boundary
 359 delineating the parameter region where significant p -changes should be detectable from
 360 the analysis of empirical data sets: For lower CV -values, the p -value change is larger
 361 than 0.1 while, for higher CV -values, p -value changes (smaller than 0.1) could be hardly
 362 detected in empirical data sets. It is found that for the same CV -value, the p -value change
 363 becomes more significant for smaller stress drop values.

4. Influence of afterslip on the p and χ dependence on m

364 There is growing evidence that large mainshocks are followed by significant amounts
 365 of afterslip (e.g., Miyazaki et al., 2004; Chlieh et al., 2007). It has been proposed that
 366 this afterslip, which typically decays as $1/t$ (see Montesi, 2004, for analysis and modeling
 367 of afterslip decay), could be the driving force in producing aftershocks (Perfettini and
 368 Avouac, 2004). Dieterich (1994) derived, in the context of rate-and-state friction, the
 369 earthquake rates that would be triggered by a $1/t$ -decaying afterslip following a coseismic
 370 stress change. Addition of afterslip is indeed seen to substantially modify the aftershock
 371 decay, both in terms of decay exponent (p -value) and of aftershock productivity. We
 372 therefore consider in this section how afterslip could further change the conclusions reached
 373 in the previous section.

For a coseismic stress change τ followed by a afterslip-induced stress of the form

$$\tau_1 \times \ln(1 + t/t^*), \quad (6)$$

solving equation (2) leads to the seismicity rate

$$\lambda(t, \tau) = \frac{r}{e^{-\frac{\tau}{A\sigma}} \left(1 + \frac{t}{t^*}\right)^{-a} + \frac{t^*}{(1+a)t_a} \left[1 + \frac{t}{t^*} - \left(1 + \frac{t}{t^*}\right)^{-a}\right]} \quad (7)$$

in place of Eq.(3), see Dieterich (1994). Parameter a equals $\frac{\tau_1}{A\sigma}$. This solution ignores
the constant, tectonic stressing rate $\dot{\tau}$ contribution to the post-seismic stress. Account-
ing for it affects the aftershock decay $\lambda(t, \tau)$ (as given by Eq. 7) only when t becomes
comparable to t_a , and amounts to a convergence of the rate to the background rate r . As
an illustration, Fig. 11 compares the solution of Eq.(7) that ignores the tectonic loading,
with the numerical solution of Eq.(2) that includes this loading.

We analyze the effect of afterslip on the p -value variations and the aftershock pro-
ductivity for the previous example of $\bar{\tau}= 1$ MPa and $CV=2.3$. Parameter t^* is set to
 $10^{-7} t_a$. The strength of the stress changes induced by afterslip is characterized by the
ratio between the cumulative stress change by afterslip within time t_a and the mean of the
coseismic stress drop $\bar{\tau}$. The results are shown in Fig.12. For additional loading (positive
values of τ_1), the p -values slightly decrease and the productivity increases. Vice versa for
an unloading (negative values of τ_1): p -values increase and the productivity decreases.
 p -values larger than 1 are found in the case of very strong unloading when the afterslip
induced stress is of the order of the coseismic mean stress drop. However, in all cases, the
consideration of afterslip does not change the general shape of both the p -value change
and the scaling of the productivity with mainshock magnitude.

Summary and Conclusions

391 In this paper, we investigate the mainshock magnitude dependence of the aftershock
 392 activity which results from fractal slip and frictional nucleation of earthquakes. Fractal
 393 earthquake slip directly leads to a rupture size-dependent heterogeneity of the induced
 394 stress field. The larger the earthquake, the stronger is the expected variance of the stress
 395 changes. Thus larger earthquakes will typically produce strongly loaded patches within
 396 the rupture zone, even though the average stress level dropped significantly. In such loaded
 397 patches, which are for smaller events less frequent, aftershocks will nucleate rapidly. We
 398 systematically studied the predicted aftershock characteristics and compared them with
 399 observations. Firstly, the model predicts that small earthquakes should be followed by
 400 an immediate on-fault seismicity shadow, while larger earthquakes should not because
 401 of the induced stress drop heterogeneity. Direct observation of this shadowing effect for
 402 small mainshocks has been made by Rubin (2002), for relocated earthquakes on the San
 403 Andreas fault, and by Fischer and Horalek (2005) for relocated swarm earthquakes in the
 404 Vogtland area. Secondly, the Omori-Utsu's p -value increases with mainshock magnitude
 405 as a consequence of enlarged stress field heterogeneity. The aftershock productivity χ is
 406 also affected, although less significantly, by the stress heterogeneity: its scaling $\chi \sim 10^{\alpha m}$
 407 with mainshock magnitude m is made steeper by a rough stress field ($\alpha \simeq 1.05$ compared
 408 to $\alpha = 0.9$ when there is no heterogeneity). Both predictions are in good agreement with
 409 recent observations by Ouillon and Sornette (2005), Helmstetter et al. (2005), and our
 410 own observations of section 2. In particular, we find that the Hurst exponent deduced
 411 from slip inversions, $H = 0.7$, gives the best fit to the data which supports the model.

412 To prove the robustness of the recently observed mainshock magnitude dependence
413 of the p -value and the scaling of the aftershock productivity χ , we have performed an
414 independent analysis of the global earthquake catalog for mainshock magnitudes $M \geq 5.5$.
415 For a number of different declustering algorithms, we could confirm a systematic increase
416 of the p -value with mainshock magnitude. On the other hand, we found that the apparent
417 productivity value is strongly dependent on the selection algorithm, resulting in a broad
418 interval of possible values between 0.6 and 1.15. By means of a systematic parameter
419 analysis, we have used these empirical observations to constrain the expected degree of
420 the stress drop variability.

421 Within the same model framework, p -values larger than 1 cannot be explained if only
422 coseismic mainshock-induced stress changes and tectonic loading are considered. This
423 is in contradiction with empirical observations of $p > 1$ aftershock decays. However,
424 Dieterich (1994) already showed that $\log(t)$ -unloading in agreement with frequently ob-
425 served afterslip can explain $p > 1$. We have checked numerically that this result remains
426 true if tectonic forcing is additionally taken into account. Our analysis shows, however,
427 that afterslip does not change the general characteristics of the mainshock-magnitude
428 dependence of the p -value and the aftershock productivity.

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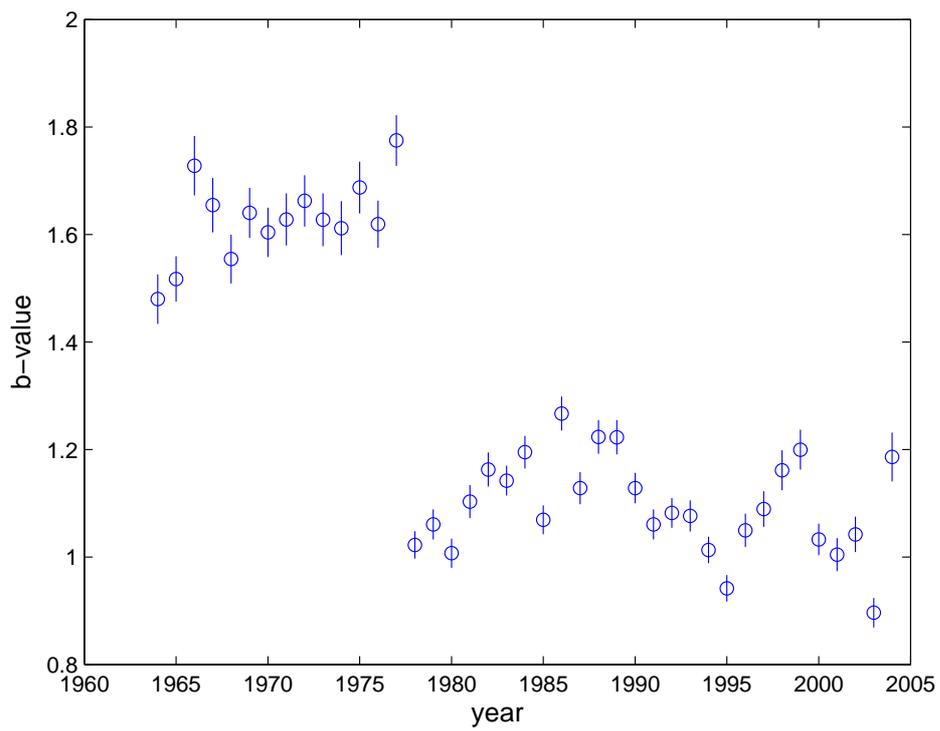


Figure 1. b-value estimated for each year individually in the ISC catalog. A clear change in b-value is observed in 1978, when M_s magnitudes started to be reported.

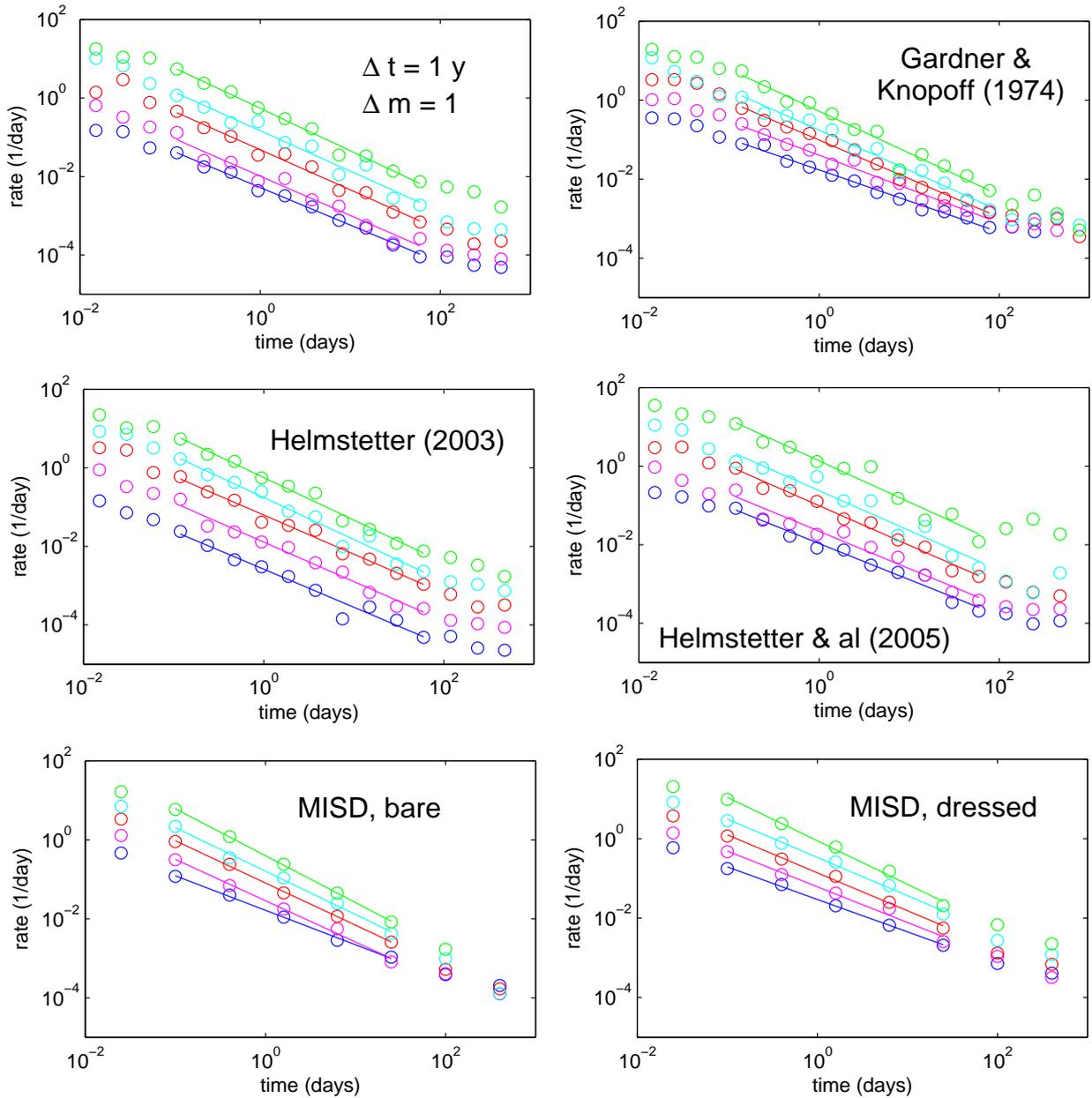


Figure 2. Aftershock rates following mainshocks of magnitude $5.5 \leq m < 6$, $6 \leq m < 6.5$, $6.5 \leq m < 7$, $7 \leq m < 7.5$, and $m \geq 7.5$ (from bottom to top), using the various methods described in the text for selecting mainshocks and aftershocks. The top, left graph corresponds to method (1) with $\Delta t = 1$ year and $\Delta m = 1$. The best power-law fits performed in the interval between 0.1 day and 100 days are shown in continuous lines.

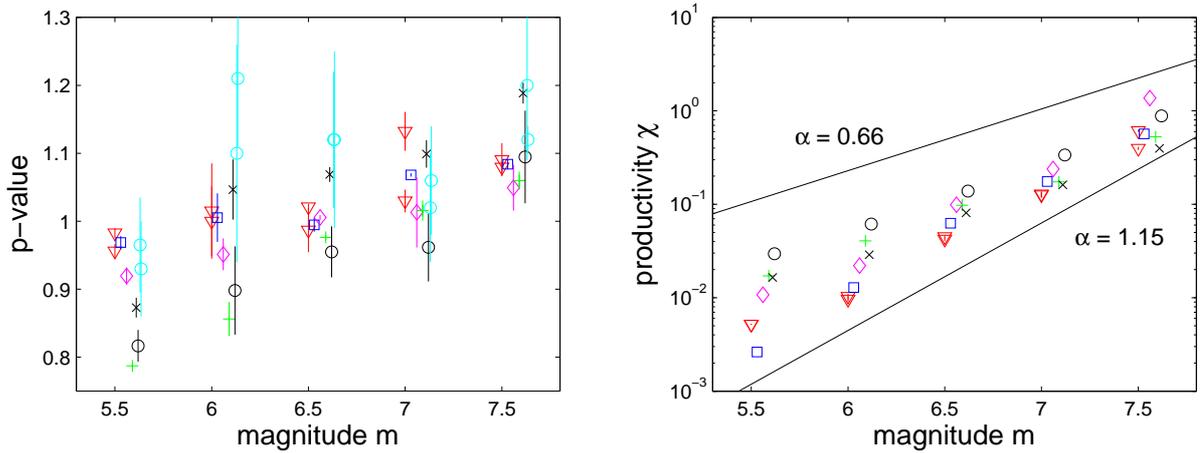


Figure 3. Parameters p and χ of Omori-Utsu law obtained from the best fits shown in Figure 2. The symbols distinguish the various methods described in the text: (1) triangles, (2) squares, (3) diamonds, (4) pluses, (5, dressed) circles, (5, bare) crosses. For method (1), the 2 combinations using $\Delta t = 1$ year, $\Delta m = 1$ and $\Delta t = 3$ years, $\Delta m = 2$ are shown. The p -values obtained by Ouillon and Sornette (2005) for southern California earthquakes, using their two methods for selecting mainshocks and aftershocks, are displayed for comparison (light gray). We added a small shift for presentation purposes. The two lines on the right hand graph show a $10^{0.66m}$ and a $10^{1.15m}$ scaling. Table 1 summarizes all parameter estimates.

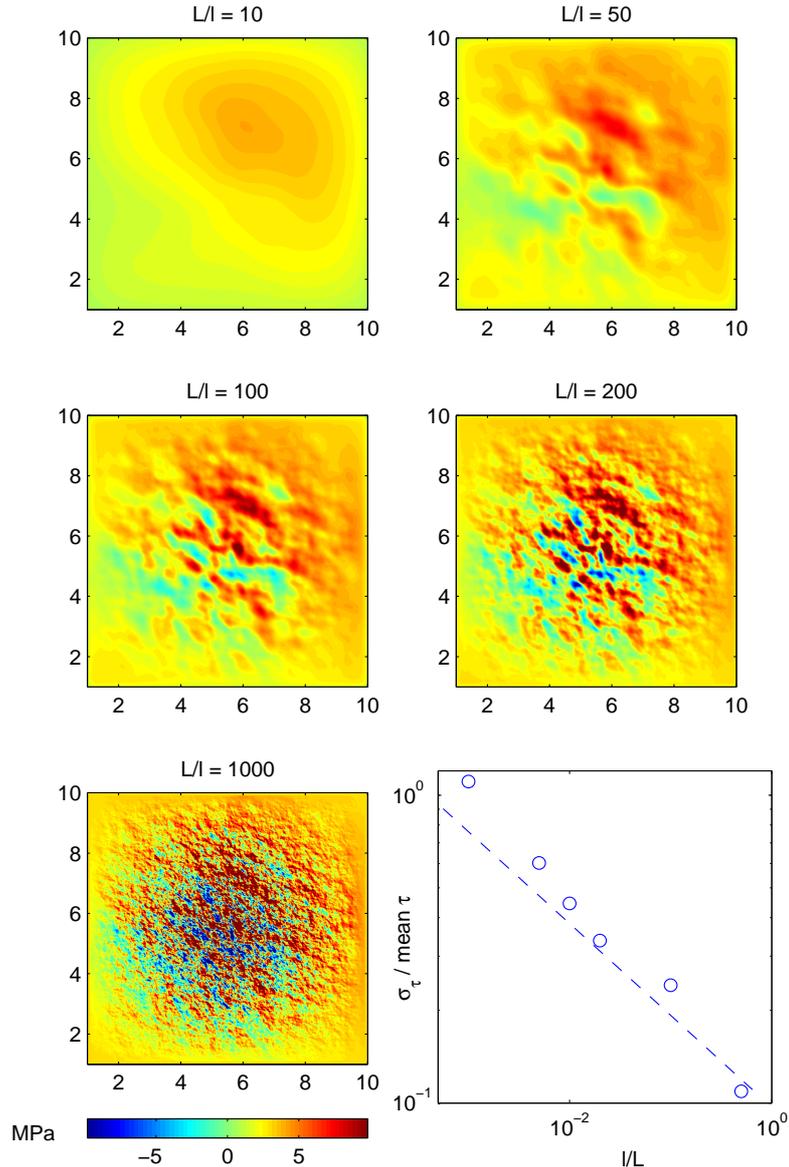


Figure 4. Stress drop distribution, in MPa, on a 10×10 km² simulated fault, seen at a varying nucleation length ℓ . The fractal stress field becomes rougher as the scale ratio L/ℓ grows. The mean stress drop is 3 MPa whatever ℓ . Stress loading corresponds to negative stress drops, and is observed at places starting at $L/\ell = 100$. Bottom right: standard deviation σ_τ normalized by the mean stress drop 3 MPa, function of the inverse ratio ℓ/L . The power-law trend (dashed line) follows the expected exponent $-1 + H = -0.3$ as predicted by Eq.(5). This can be compared to Figure 5 of Marsan (2006).

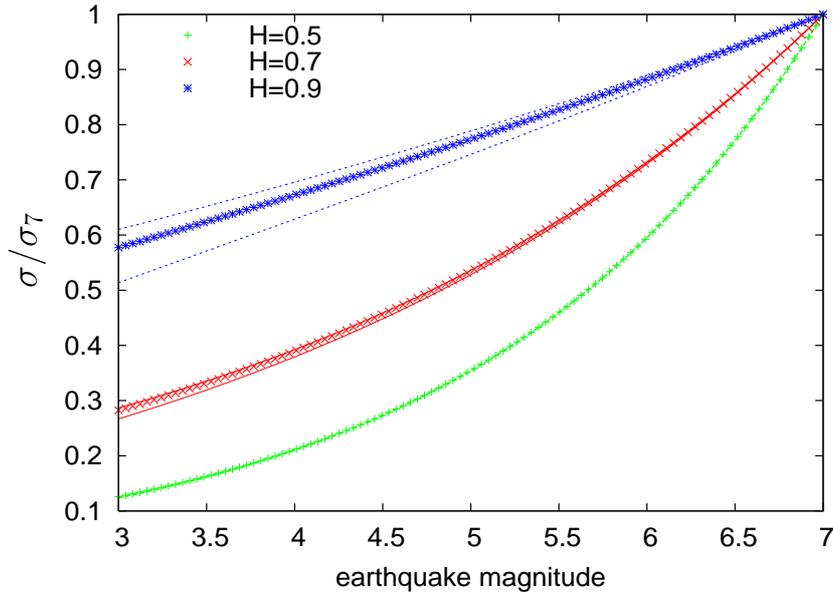


Figure 5. The dependence of the expected stress drop variability σ/σ_7 on the mainshock magnitude m for Hurst exponents inverted from observations. For each Hurst-exponent, the symbols refer to $m_0=-2$ while the lines refer to $m_0=-4$ and $m_0=0$, respectively. We use $\beta = 0.45$ in the $L \sim 10^{\beta \times m}$ relation, according to Wells and Coppersmith (1994).

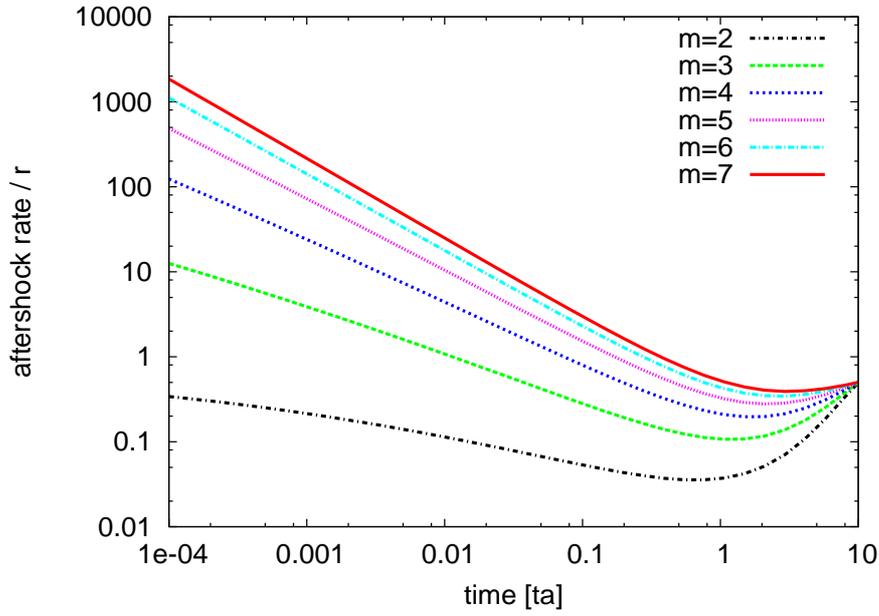


Figure 6. The aftershock decay as a function of the mainshock magnitude in the case of $H=0.7$, $CV = \sigma_7/\bar{\tau}=2.3$, and $\bar{\tau}=1$ MPa. The aftershock rate is normalized by the background rate. At long time scales (i.e., t/t_a typically greater than 1), the aftershock rate becomes less than the background rate, indicating the onset of a seismic quiescence. This is caused by the overall stress drop, see Marsan (2006).

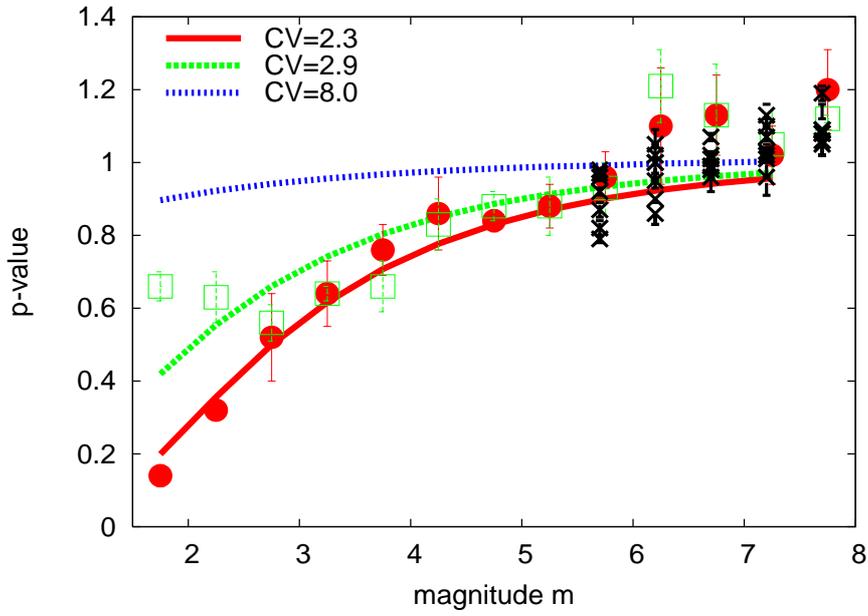


Figure 7. The p -value as a function of the mainshock magnitude in the case of $H=0.7$ and $CV = \sigma_7/\bar{\tau}=2.3, 2.9,$ and 8.0 . The curves are compared with the observed p -value dependence in California (data from Ouillon and Sornette 2005: results based on their declustering method 1 (dots) and declustering method 2 (squares)), and with the range of p -values reported in section 2 (crosses).

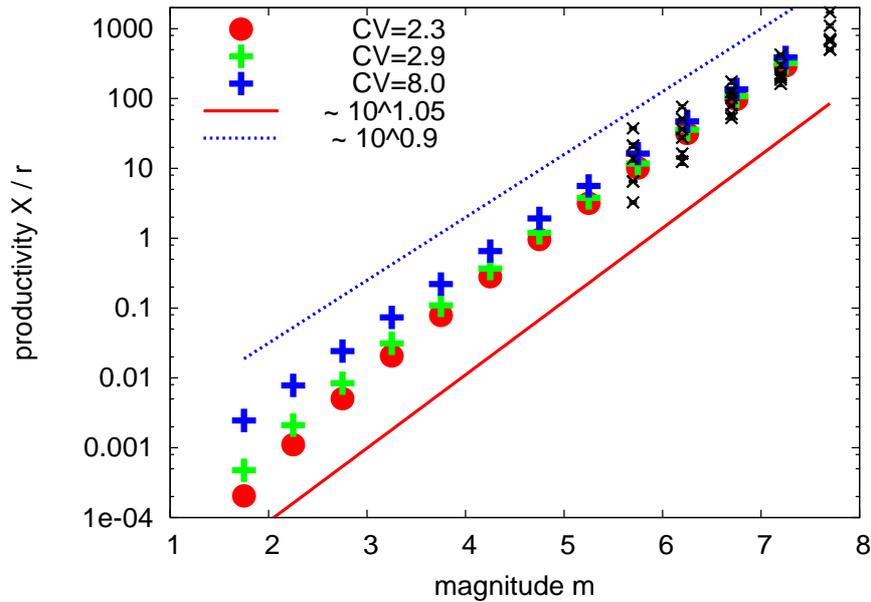


Figure 8. The aftershock productivity as a function of mainshock magnitude. Note that the productivity is normalized by the background rate r . The results are compared with the two scaling laws $\sim 10^{1.05m}$ and $\sim 10^{0.9m}$. For a comparison, the observational χ -values reported in section 2 have to be rescaled by the unknown factor t_a/r (t_a measured in units of days). For a factor of 1250, the observations are represented by small crosses.

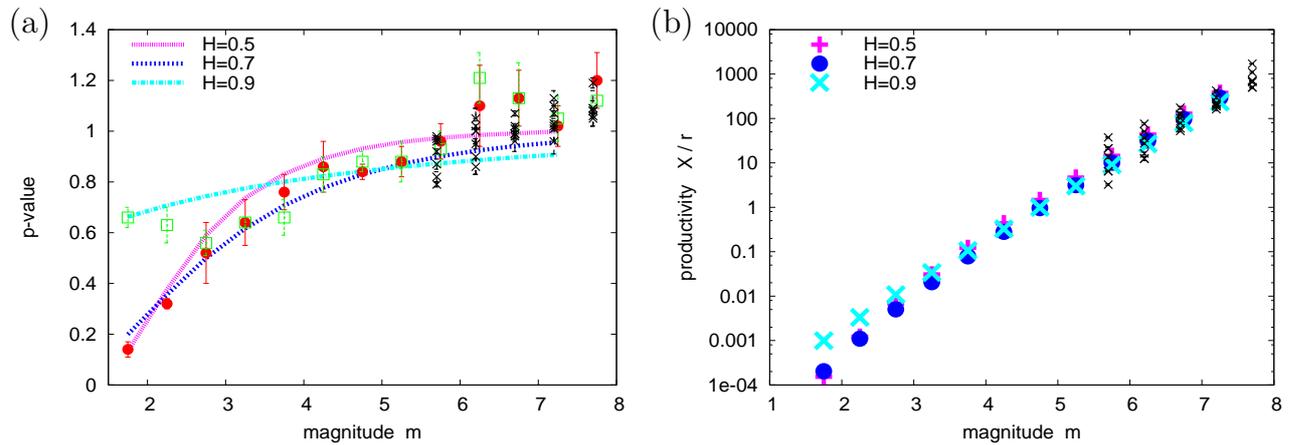


Figure 9. (a) The p -value as a function of the mainshock magnitude in comparison for the three different values $H=0.5$, 0.7 , and 0.9 . In each case the stress field heterogeneity is chosen in a way that the empirically data points (for description see Fig.7) are best fitted: $CV=6.5$ for $H=0.5$; $CV=2.3$ for $H=0.7$, and $CV=1.5$ for $H=0.9$. (b) The aftershock number as a function of mainshock magnitude for the same cases. In both cases, the data reported in section 2 are added (small crosses).

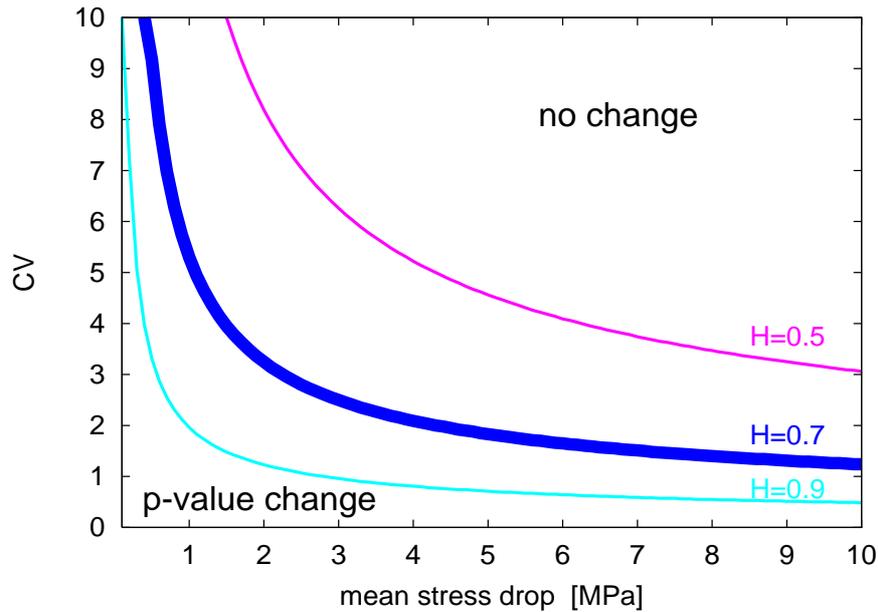


Figure 10. Phase diagram for significant p -value changes. The plot shows the stress field variability CV which leads to a p -value increase of 0.1 from $m=3.0$ to 7.0 mainshocks, considered as an observable change: For lower CV -values, the p -value change with mainshock magnitude is significant, while p -value changes could be hardly detected in empirical data sets for higher CV -values.

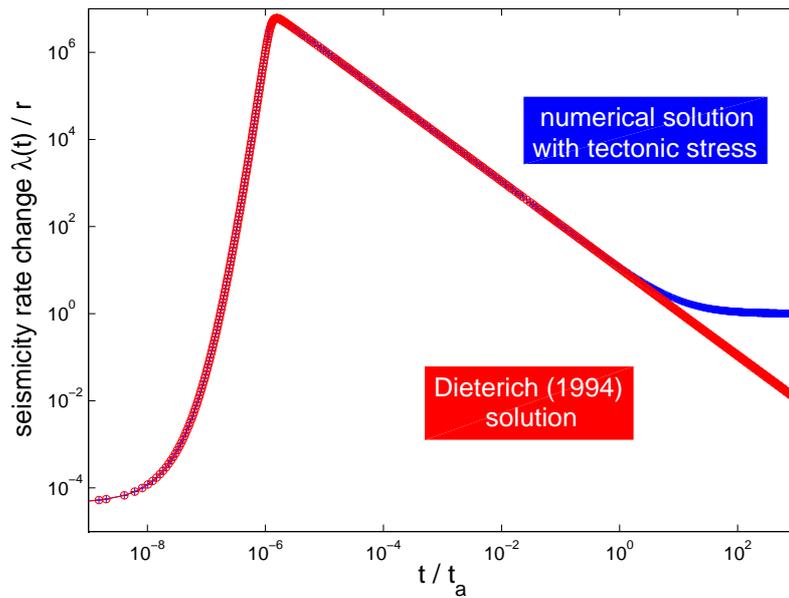


Figure 11. Seismicity rate normalized by background rate, for a 1 MPa coseismic stress drop followed by a stress increase due to afterslip $\tau_1 \times \ln(1 + t/t^*)$. Here $\tau_1 = 1$ MPa, $t^*/t_a = 10^{-7}$, $A\sigma = 0.1$ MPa, and the tectonic stress rate is $\dot{\tau} = 0.1$ MPa per unit t_a . Both solutions are identical as long as $t < t_a$.

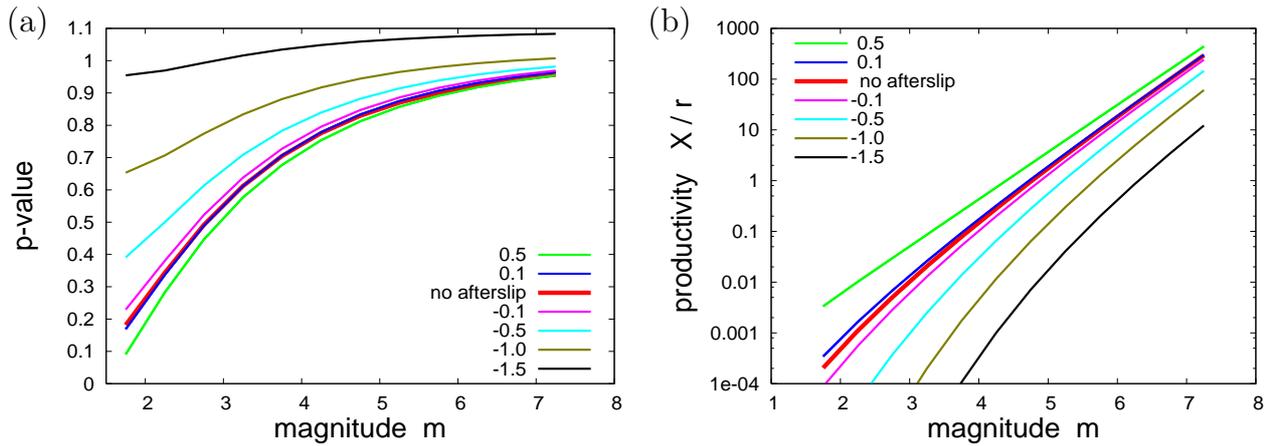


Figure 12. Consideration of postseismic stress changes due to afterslip: (a) the p -value and (b) the productivity as a function of the mainshock magnitude. The number for each curve gives the fraction of the coseismic mean stress drop which is added (positive sign for loading and negative sign for unloading) within time t_a to the coseismic stress value according to Eq.(6). All curves are based on $\bar{\tau}=1$ MPa and $CV=2.3$.

Method	Magnitude	p -value	χ -value	r^2
Method 1 ($\Delta t=1$ year, $\Delta m=1$)	$5.5 \leq m < 6$	0.96 ± 0.01	0.0052 ± 0.0001	0.99
	$6 \leq m < 6.5$	1.00 ± 0.05	0.010 ± 0.001	0.96
	$6.5 \leq m < 7$	1.02 ± 0.01	0.048 ± 0.001	0.98
	$7 \leq m < 7.5$	1.03 ± 0.02	0.15 ± 0.01	0.96
	$m \geq 7.5$	1.08 ± 0.01	0.55 ± 0.01	0.99
Method 1 ($\Delta t=3$ years, $\Delta m=2$)	$5.5 \leq m < 6$	0.98 ± 0.001	0.0052 ± 0.0001	0.99
	$6 \leq m < 6.5$	1.02 ± 0.07	0.010 ± 0.001	0.96
	$6.5 \leq m < 7$	0.99 ± 0.03	0.042 ± 0.002	0.96
	$7 \leq m < 7.5$	1.13 ± 0.03	0.13 ± 0.01	0.94
	$m \geq 7.5$	1.09 ± 0.03	0.40 ± 0.02	0.95
Method 2 Helmstetter (2003)	$5.5 \leq m < 6$	0.97 ± 0.01	0.0026 ± 0.0001	0.97
	$6 \leq m < 6.5$	1.00 ± 0.04	0.013 ± 0.001	0.98
	$6.5 \leq m < 7$	0.99 ± 0.01	0.063 ± 0.002	0.99
	$7 \leq m < 7.5$	1.07 ± 0.01	0.18 ± 0.01	0.97
	$m \geq 7.5$	1.08 ± 0.01	0.57 ± 0.01	0.99
Method 3 Helmstetter et al. (2005)	$5.5 \leq m < 6$	0.92 ± 0.01	0.011 ± 0.001	0.98
	$6 \leq m < 6.5$	0.95 ± 0.02	0.022 ± 0.001	0.96
	$6.5 \leq m < 7$	1.01 ± 0.01	0.10 ± 0.01	0.98
	$7 \leq m < 7.5$	1.01 ± 0.05	0.24 ± 0.02	0.94
	$m \geq 7.5$	1.05 ± 0.03	1.37 ± 0.01	0.95
Method 4 Gardner & Knopoff (1974)	$5.5 \leq m < 6$	0.79 ± 0.01	0.017 ± 0.001	0.99
	$6 \leq m < 6.5$	0.86 ± 0.03	0.041 ± 0.001	0.98
	$6.5 \leq m < 7$	0.98 ± 0.01	0.098 ± 0.001	0.99
	$7 \leq m < 7.5$	1.02 ± 0.02	0.17 ± 0.01	0.98
	$m \geq 7.5$	1.06 ± 0.01	0.53 ± 0.02	0.96
Method 5 Marsan & Lengliné (2008), bare	$5.5 \leq m < 6$	0.87 ± 0.02	0.017 ± 0.001	0.99
	$6 \leq m < 6.5$	1.05 ± 0.04	0.029 ± 0.001	0.99
	$6.5 \leq m < 7$	1.07 ± 0.01	0.081 ± 0.002	0.99
	$7 \leq m < 7.5$	1.10 ± 0.02	0.16 ± 0.01	0.99
	$m \geq 7.5$	1.19 ± 0.02	0.40 ± 0.01	0.99
Method 5 Marsan & Lengliné (2008), dressed	$5.5 \leq m < 6$	0.82 ± 0.02	0.030 ± 0.001	0.99
	$6 \leq m < 6.5$	0.90 ± 0.07	0.061 ± 0.003	0.98
	$6.5 \leq m < 7$	0.96 ± 0.04	0.14 ± 0.01	0.99
	$7 \leq m < 7.5$	0.96 ± 0.05	0.34 ± 0.01	0.99
	$m \geq 7.5$	1.09 ± 0.07	0.88 ± 0.04	0.99

Table 1. Estimates for the p and χ values of the best fits as shown in Fig. 2, along with their errors, and the r^2 value giving the goodness of fit.

parameter	description	value
$A\sigma$	constitutive parameter times effective normal stress	0.1 MPa
$\bar{\tau}$	mean stress drop on the main fault (the negative of the mean stress change)	variable
σ_{τ}	stress drop standard deviation at the nucleation length scale	variable
ℓ, m_0	nucleation length (ℓ) and equivalent nucleation magnitude (m_0)	$\ell = 2.2\text{m}, m_0 = -2$
C, σ_7, CV	calibration constants for σ_{τ} , see Eq.(5); $\sigma_7 = \sigma_{\tau}$ for $m = 7$ earthquakes, and $CV = \frac{\sigma_7}{\bar{\tau}}$	variable
H	Hurst exponent of fractal slip distribution	0.7 ± 0.2

Table 2. Summary of the model parameters that affect the aftershock decay characteristics.