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Towards a One Percent Measurement of Frame Dragging by Spin with Satellite Laser Ranging to LAGEOS, LAGEOS 2 and LARES and GRACE Gravity Models

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Abstract During the past century Einstein's theory of General Relativity gave rise to an experimental triumph; however, there are still aspects of this theory to be measured or more accurately tested. Today one of the main challenges in experimental gravitation, together with the direct detection of gravitational waves, is the accurate measurement of the gravito-

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magnetic field generated by the angular momentum of a body. Here, after a brief introduction on frame-dragging, gravitomagnetism and Lunar Laser Ranging tests, we describe the past measurements of frame-dragging by the Earth spin using the satellites LAGEOS, LAGEOS 2 and the Earth's gravity models obtained by the GRACE project. We demonstrate that these measurements have an accuracy of approximately 10%.

We then describe the LARES experiment to be launched in 2010 by the Italian Space Agency for a measurement of frame-dragging with an accuracy of a few percent.

We finally demonstrate that a number of claims by a single individual, that the error budget of the frame-dragging measurements with LAGEOS-LAGEOS 2 and LARES has been underestimated, are indeed ill-founded.

Keywords General Relativity · Frame-dragging · Gravitomagnetism · Lense-Thirring effect

1 Introduction

A number of experiments have been performed and proposed to accurately measure the gravitomagnetic field (Ciufolini 2007a; Thorne et al. 1986; Ciufolini and Wheeler 1995) generated by the angular momentum of a body and frame-dragging, from the complex space experiment Gravity Probe B, launched by NASA in 2004 after more than 40 years of preparation (GRAVITY PROBE-B http://einstein.stanford.edu/), to the observations of the LAGEOS and LAGEOS 2 satellites (Ciufolini and Pavlis 2004; Ciufolini et al. 2006, 2009a) and from the LARES satellite, to be launched in 2010 by ASI (Italian Space Agency) (Ciufolini et al. 2009b) using the new launch vehicle VEGA of ESA (European Space Agency), to Lunar Laser Ranging (Williams et al. 2004a), binary pulsars (Stairs et al. 2004) and other astrophysical observations (Nordtvedt 1988; Cui et al. 1998). A number of other space experiments are also currently proposed to various international space agencies.

In Einstein's gravitational theory the local inertial frames have a key role (Misner et al. 1973; Weinberg 1972; Ciufolini and Wheeler 1995). The strong equivalence principle, at the foundations of General Relativity, states that the gravitational field is locally 'unobservable' in the freely falling frames and thus, in these local inertial frames, all the laws of physics are the laws of Special Relativity. The local inertial frames are determined, influenced and dragged by the distribution and flow of mass-energy in the Universe; the axes of these non-rotating, local, inertial frames are determined by torque-free test gyroscopes that are dragged by the motion and rotation of nearby matter, for this reason this phenomenon is called dragging of inertial frames or frame-dragging (Ciufolini and Wheeler 1995; Ciufolini 2007a).

In General Relativity, a torque-free test gyroscope defines an axis non-rotating relative to the local inertial frames; the orbital plane of a test particle is also a kind of gyroscope. The frame-dragging effect on the orbit of a satellite, due to the angular momentum vector **J** of a central body, is known as Lense-Thirring effect:

$$\boldsymbol{\Omega}^{\text{Lense-Thirring}} = \frac{2G\mathbf{J}}{c^2 a^3 (1 - e^2)^{3/2}} \tag{1}$$

where $\boldsymbol{\Omega}^{\text{Lense-Thirring}}$ is the rate of change of the longitude of the nodal line of the satellite, that is the intersection of its orbital plane with the equatorial plane of the central body, i.e., it represents the rate of change of the orbital angular momentum vector, a is the semimajor axis of the orbit of the test-particle, e its orbital eccentricity, G the gravitational constant and c the speed of light. Frame-dragging by the Earth spin has been measured using the LAGEOS satellites with about 10 percent accuracy (Ciufolini and Pavlis 2004; Ciufolini et al. 2006, 2009a; Ries et al. 2008) (Sects. 2, 3, 4, 6, 7 and 8, below), might be detected by further Gravity Probe B data analysis (GRAVITY PROBE-B http://einstein.stanford.edu/) and will be measured with an accuracy of a few percent by the LARES satellite (Ciufolini et al. 2009b) (Sects. 5, 6, 7 and 8 below).

In General Relativity there is another type of frame-dragging effect and precession of a gyroscope known as geodetic precession or de Sitter effect (Ciufolini and Wheeler 1995; Ciufolini 2007a). If a gyroscope is at rest with respect to a non-rotating mass, it does not experience any drag. However, if the gyroscope starts to move with respect to the non-rotating mass it acquires a precession that will again disappear when the gyroscope will stop relative to the non-rotating mass. The geodetic precession, due to the velocity $\mathbf{v}$ of a test gyroscope, is $\boldsymbol{\Omega}_{\text{geodetic}} = \frac{3}{2}\frac{GM}{c^2 r^3}\mathbf{x} \times \mathbf{v}$, where M is the mass of the central body and $\mathbf{x}$ and r are position vector and radial distance of the gyroscope from the central mass.

A basic difference between frame-dragging by spin and geodetic precession is that in the case of the former (the Lense-Thirring effect) the frame-dragging effect is due to the additional spacetime curvature produced by the rotation of a mass, whereas in the case of the latter (the de Sitter effect) the frame-dragging effect is due to the motion of a test gyroscope on a static background and its motion produces no spacetime curvature, (see below and Sect. 6.11 of Ciufolini and Wheeler 1995; for a discussion on frame-dragging and geodetic precession see Ashby and Shahid-Saless 1990; O'Connell 2005; Ciufolini 2007a).

The geodetic precession has been measured on the Moon's orbit by LLR with an accuracy of the order of 0.6 percent (Williams et al. 2004a), see also Bertotti et al. (1987), Williams et al. (1996), by Gravity Probe B with approximately 1 percent accuracy (GRAVITY PROBE-B http://einstein.stanford.edu/) and has been detected on binary pulsars (Weisberg and Taylor 2002; Stairs et al. 2004). Recently, a number of authors have debated whether the gravitomagnetic interaction and frame-dragging by spin have also been accurately measured on the Lunar orbit by Lunar Laser Ranging (Murphy et al. 2007a, 2007b; Kopeikin 2007; Ciufolini 2007a). This is a recent chapter of a long debate on the meaning of frame-dragging and gravitomagnetism (Ashby and Shahid-Saless 1990; O'Connell 2005; Barker and O'Connel 1979; Khan and O'Connell 1976; Murphy et al. 2007a, 2007b; Kopeikin 2007; Ciufolini 1994, 2007a; Ciufolini and Wheeler 1995); a basic issue treated in Murphy et al. (2007a, 2007b), Kopeikin (2007) is whether the effect detected by LLR is a frame-dependent effect or not.

In order to answer to this question, we have proposed a distinction between gravitomagnetic effects generated by the translational motion of the frame of reference where they are observed, e.g., by the motion of a test gyroscope with respect to a central mass (not necessarily rotating), and those generated by the rotation of a mass or by the motion of two masses (not test-particles) with respect to each other, without any necessary motion of the frame of reference where these effects are observed. The geodetic precession is a translational effect due to the motion of the 'Earth–Moon gyroscope' in the static field of the Sun. The Lense-Thirring effect measured by the LAGEOS satellites, that might also be detected by further Gravity Probe B data analysis and by LARES, is due to the rotation of a mass, i.e., by the rotation of the Earth's mass. In order to distinguish between the Lense-Thirring effect, that we call an 'intrinsic' gravitomagnetic effect, and 'translational' ones, such as the geodetic precession, we have proposed to use spacetime curvature invariants (Ciufolini and Wheeler 1995; Ciufolini 1994, 2009). For example, in Ciufolini (2009) we have shown that the phenomena measured by Lunar Laser Ranging are translational gravitomagnetic effects. In general, one cannot derive intrinsic gravitomagnetic effects from translational ones unless making additional theoretical hypotheses, such as the linear superposition of the translational gravitomagnetic effects, i.e., the linear superposition of the terms contained in the non-diagonal part of the metric tensor (the so-called gravitomagnetic potential) in the standard PPN (Post-Newtonian-Parametrized) coordinates; for example, the magnetic field generated by the intrinsic magnetic moment (Bohr magneton) is an intrinsic phenomenon due to the intrinsic spin of a particle that cannot be explained and derived as a translational effect by any Lorentz and frame transformation.

Recently, rotational frame dragging has been derived for all linear cosmological perturbations of Friedmann–Robertson–Walker cosmologies with $k = 0$ (Schmid 2006). Finally, we mention that the uncertainty in proposed determination of the gravitomagnetic field of Mars (Iorio 2006) was shown, by several authors (Krogh 2007; Sindoni et al. 2007; Felici 2007) to be underestimated by a factor of at least ten thousand.

2 The Measurement of Gravitomagnetism with LAGEOS, LAGEOS 2 and the GRACE Earth Gravity Models

The orbital plane of a planet, moon or satellite is like a huge gyroscope that feels general relativistic effects, e.g., the Lense-Thirring effect (1), that is the dragging of the whole orbital plane of a test-particle due to the angular momentum J of the central body. The Lense-Thirring effect is extremely small for Solar System objects, so in order to measure its effect on the orbit of a satellite we need to measure the position of the satellite to extremely high accuracy.

Laser-ranging is the most accurate technique to measure distances to the Moon (Bender et al. 1973; Williams et al. 2004b) and to artificial satellites such as LAGEOS (Cohen and Dunn 1985). Short-duration laser pulses are emitted from lasers on Earth and then reflected back to the emitting laser-ranging stations by retro-reflectors on the Moon or on artificial satellites. By measuring the total round-trip travel time we are today able to determine the instantaneous distance of a retro-reflector on the LAGEOS satellites with a few millimeters accuracy (Noomen et al. 2003).

LAGEOS, LAser GEOdynamics Satellite (Cohen and Dunn 1985), was launched by NASA in 1976 and LAGEOS 2 by the Italian Space Agency (ASI) and NASA in 1992. The LAGEOS satellites are heavy brass and aluminum satellites, of about 406 kg weight, completely passive and covered with retro-reflectors, orbiting at an altitude of about 6000 km above the surface of Earth. LAGEOS and LAGEOS 2 have an essentially identical structure but they have different orbits. The semimajor axis of the orbit of LAGEOS is $a \cong 12270$ km, the period $P \cong 3.758$ hr, the orbital eccentricity $e \cong 0.004$ and the orbital inclination $I \cong 109.9°$. The orbital inclination is the angle between the satellite orbital plane and the Earth equatorial plane. The semimajor axis of LAGEOS 2 is $a_{II} \cong 12163$ km, the orbital eccentricity $e_{II} \cong 0.014$ and the orbital inclination $I_{II} \cong 52.65°$.The LAGEOS satellites position can be predicted, over a 15-day period, with an uncertainty of just a few centimetres (Ciufolini et al. 2006; Noomen et al. 2003).

The Lense-Thirring drag (1) of the orbital plane of LAGEOS and LAGEOS 2 is (Ciufolini 1986, 1996) respectively 31 and 31.5 milliarcsec/yr (a milliarcsec is a thousandth of a second of arc), corresponding at the LAGEOS altitude to approximately 1.9 meters/yr. Since by laser-ranging we can determine their orbits with a few centimeters accuracy, the Lense-Thirring effect can be very accurately measured on the LAGEOS satellites orbit if all other orbital perturbations can be modelled well enough (Ciufolini 1989, 1996, 1998c; Tapley et al. 1989). Indeed, the precession of the node of LAGEOS and LAGEOS 2 can be measured with an accuracy of a fraction of milliarcsec per year (one milliarcsec corresponds to about 6 centimeters at the LAGEOS altitude).

On the other hand, the LAGEOS satellites are very heavy, small cross-sectional area, spherical satellites, therefore atmospheric particles and photons can only slightly perturb their orbit (Rubincam 1990) and especially they can hardly change the orientation of their orbital plane (Ciufolini 1989; Tapley et al. 1989; Rubincam 1990; Lucchesi 2002). Indeed, by far the main perturbation of their orbital plane is due to the Earth's deviations from spherical symmetry. The deviations of the Earth's gravitational potential from spherical symmetry are described by a spherical harmonics expansion of the potential (Kaula 1966). However, the only secular perturbations of the node of a satellite are due to the so-called even zonal harmonics, J_{2n}, i.e., the spherical harmonics terms of even degree and zero order (axially symmetric deviations from spherical symmetry of the Earth's gravitational potential that are also symmetric with respect to the Earth's equatorial plane). In particular, the flattening of the Earth's gravitational potential, described by the quadrupole moment, produces a large perturbation of the LAGEOS node (Ciufolini 1986, 1989; Tapley et al. 1989). The rate of change of the nodal longitude of a satellite $\dot{\Omega}$, due to the quadrupole moment, J_2, and to the second largest even zonal harmonic of degree four, i.e., J_4 (see Fig. 1) are described by

$$\begin{aligned}\dot{\Omega}^{\text{Class}} &= -\frac{3}{2} n \left(\frac{R_\oplus}{a}\right)^2 \frac{\cos I}{(1-e^2)^2} \\ &\quad \times \left\{ J_2 + J_4 \left[\frac{5}{8} \left(\frac{R_\oplus}{a}\right)^2 \times (7\sin^2 I - 4) \frac{(1+\frac{3}{2}e^2)}{(1-e^2)^2} \right] + \cdots \right\} \end{aligned} \tag{2}$$

where $n = 2\pi/P$ is the orbital mean motion, P is the orbital period, $R_\oplus$ is the Earth equatorial radius and J_{2n} are the even zonal harmonic coefficients. The orbital parameters n, a, e and I in (2), i.e., mean motion, semimajor axis, orbital eccentricity and orbital inclination are determined with sufficient accuracy via LAGEOS laser ranging (Cohen and Dunn 1985; Noomen et al. 2003; Ciufolini 1989; Tapley et al. 1989; Ciufolini et al. 1989, 2004), see Sect. 7. Any other quantity in (2) can be determined or is known with sufficient accuracy, apart from the J_{2n}.

We stress that what is *critical for the measurement of the Lense-Thirring effect* is that the *modeling* of this classical *node precession* (i.e., the prediction of its behavior on the basis of the available physical models) must be accurate enough (i.e., at the level of a few milliarcsec) compared to the Lense-Thirring effect (of size of about 31 milliarcsec Ciufolini 1986). What is critical is *not* that all the quantities entering this equation, i.e., the Earth parameters and the orbital parameters, and in particular the Earth spherical harmonic coefficients and the semimajor axis and the inclination, must be *predicted* in their variations, but instead what is critical is that they must be *determined* with sufficient accuracy via satellite laser ranging and other techniques (such as GRACE). For example if the variations of the *inclination* and of the *semimajor axis* of LAGEOS are not well modeled because the effect of particle drag (i.e., atmospheric drag) is not known with sufficient accuracy, this is *not* critical for our measurement of the Lense-Thirring effect because we are able to *measure* the variations of inclination and semimajor axis accurately enough with satellite laser ranging (see Sect. 8 on the uncertainty in the determination of the orbital inclination due to the atmospheric refraction modeling errors) and we are thus able to precisely quantify the effect of these variations on the nodal rate, (2) (with the orbital estimators GEODYN, EPOS-OC and UTOPIA). Indeed, in Sect. 8 we show that the average measurement error in the inclinations of LAGEOS

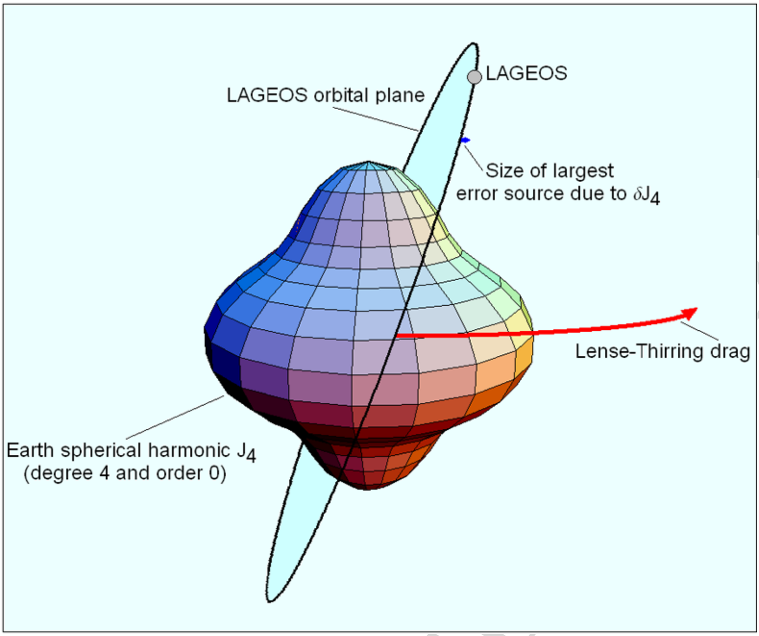


Fig. 1 The Lense-Thirring effect on the orbital plane of a test-particle. The Lense-Thirring precession of the orbital plane of a test-particle by the spin of a central body is represented by the *big red arrow*. Also shown is the Earth deviation from spherical symmetry (enhanced, and so not to scale) described by the so-called even zonal harmonic of degree four, J_4. The uncertainty in its static part is the largest source of error in the measurement of Earth's frame-dragging using the LAGEOS satellites. The maximum precession of the LAGEOS orbital plane due to the uncertainty in J_4 of the EIGEN-GRACE02S model, that is, the nodal precession error due to δJ_4, is represented by the *blue arrow*; this error and the Lense-Thirring effect are drawn to scale

and LAGEOS 2 (that are roughly constant) can be estimated to be at the level of a few tens of μarcsec for LAGEOS and LAGEOS 2, and this error, when propagated in the nodal rate, (2), corresponds to less than one percent of the Lense-Thirring effect only.

In fact, the only quantities in (2) for $\dot{\Omega}$ that are not measured with sufficient accuracy, in order to accurately measure the Lense-Thirring effect, are the even zonal harmonic coefficients J_{2n} and therefore the main uncertainty in the measurement the Lense-Thirring effect is due to the uncertainty in these J_{2n} coefficients. For example, a relative uncertainty $\frac{\delta J_{2n}}{J_2}$ of the quadrupole coefficient, J_2, of the order of Reigber et al. (2005): $\frac{\delta J_{2n}}{J_2} \sim 10^{-7}$, corresponds, from (2) to an uncertainty in the nodal precession of about 45 milliarcsec/year, i.e., to a systematic error in the measurement of the Lense-Thirring effect (that has a predicted size of 31 milliarcsec) of about 150%. In addition, we need to include the uncertainty due to the higher J_{2n} coefficients.

In order to solve the problem of the systematic error due to the uncertainty in the Earth's even zonal harmonics coefficients, such as Earth's flattening, the following main techniques were proposed.

A technique would be to use polar satellites; in fact, from formula (2), for a polar satellite, since $I = 90°$, $\dot{\Omega}^{\text{Class}}$ is equal to zero. Yilmaz proposed the use of polar satellites in 1959 (Yilmaz 1959) and in 1976, Van Patten and Everitt (1976) proposed an experiment with two drag-free, guided, counter–rotating, polar satellites. The reason to propose *two* counter-rotating polar satellites was to avoid the inclination measurement errors.

Another solution (Ciufolini 1984, 1986, 1989; Tapley et al. 1989; Ciufolini et al. 1989, 2004) would be to orbit a new satellite of LAGEOS type (called LAGEOS III), with the same semimajor axis, the same eccentricity, but the inclination supplementary to that of LAGEOS. With this choice, the classical precession $\dot{\Omega}^{\text{Class}}$, (2), would be equal and opposite for the two satellites. By contrast, since the Lense-Thirring precession, (1), is independent of the inclination, $\dot{\Omega}^{\text{Lense-Thirring}}$ would be the same in magnitude and sign for both satellites. Therefore, by combining the measured nodal precessions of LAGEOS and LAGEOS III we could eliminate the uncertainties due to all the even zonal harmonics in $\dot{\Omega}^{\text{Class}}$ and very accurately measure $\dot{\Omega}^{\text{Lense-Thirring}}$.

Another technique, that was proposed in Ciufolini (1989), Ciufolini and Wheeler (1995), is to orbit several high-altitude, laser-ranged satellites, similar to LAGEOS; this is the method that we used in Ciufolini and Pavlis (2004), Ciufolini (1996) and in this paper, and that is described below in Sect. 3. A similar technique, proposed in Ciufolini (1996), is the use of three observables: both nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II, in order to remove the error due to the uncertainties of the Earth's quadrupole moment, J_2, and of the next largest even zonal harmonic, J_4 (see Fig. 1), and to measure the Lense-Thirring effect, i.e., in order to use three observables for the three unknowns: Lense-Thirring effect, δJ_2 and δJ_4 (Ciufolini 1996). This technique led to the observation of the Lense-Thirring effect in 1996–1998 (Ciufolini et al. 1997a, 1997b, 1998b). However, the accuracy of these earlier observations of the Lense-Thirring effect could not be easily assessed, the two limiting factors were (a) the knowledge of the Earth's gravity field in 1996, indeed the Earth gravity models JGM-3 and EGM96, even though representing the state of the art in 1995–1996, were not accurate enough for a precise measurement of the Lense-Thirring effect, thus forcing to use a third observable, i.e., the perigee; (b) the use of the perigee of LAGEOS 2, in fact the perigee of a satellite is less stable than the node under non-gravitational perturbations; in classical mechanics the node (orbital angular momentum) is conserved under any central force, however, the perigee (Runge-Lenz vector) is conserved only under a central force of the type $\sim 1/r^2$. Thus, the perigee of an Earth satellite such as LAGEOS 2 is affected by a number of non-gravitational perturbations whose impact in the final error budget is not easily assessed.

Therefore, since 1996, in order to use the nodes only, our effort was to find a new observable to replace the perigee of LAGEOS 2 (see the LARES Sect. 5) and to eliminate the perigee from our analysis using a more accurate Earth gravity field model.

Overcoming the problem (Ries et al. 2003a, 2003b) of the Earth's gravity field uncertainties came in March 2002 when NASA's two identical GRACE (Reigber et al. 2002; Tapley 2002) spacecraft (Gravity Recovery and Climate Experiment) were launched in a polar orbit at an altitude of approximately 450 km and at a mutual distance of about 200–250 km. The spacecraft range to each other via a radar and they are tracked by the GPS satellites. The GRACE satellites have provided dramatic improvements in the knowledge of the Earth's gravitational field. Indeed, by using the two LAGEOS satellites and GRACE Earth gravity models (Reigber et al. 2005), the orbital uncertainties due to the modelling errors in the non-spherical Earth gravitational field are only a few percent of the Lense-Thirring effect (Ciufolini et al. 2006), see Sects. 3, 4 and 6. In 2004, nearly eleven years of laser-ranging data were analyzed. This analysis resulted in a measurement of the Lense-Thirring effect, described in the Sects. 3 and 4, with an accuracy (Ciufolini and Pavlis 2004; Ciufolini et al. 2006, 2009a) of approximately 10%; the main error source was the uncertainty in some axially symmetric Earth's departures from sphericity (see Figs. 1, 2).

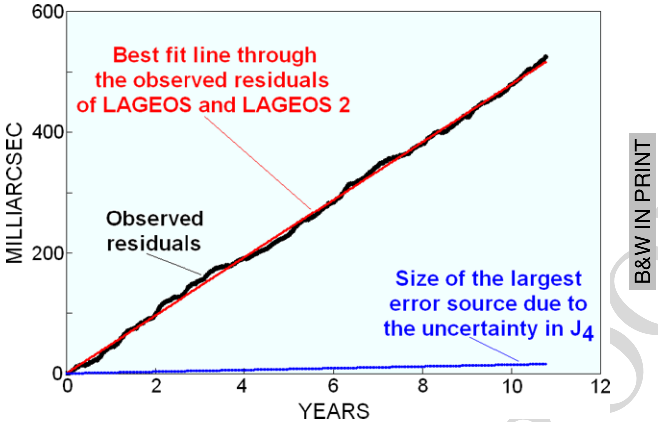


Fig. 2 The Lense-Thirring effect measured via the LAGEOS satellites in 2004. The *red solid line* is the best-fit line through the observed residuals (in *black*) and the *blue solid line* represents the uncertainty in the combined nodal longitudes of the LAGEOS satellites from the largest error source due to the uncertainty in the Earth's even zonal harmonic of degree four, J_4, of the EIGEN-GRACE02S model; see Fig. 1. The observed slope of the *red line* is 0.99 ± 0.1, where 1 is the prediction of General Relativity and the ± 0.1 uncertainty is the estimated total systematic error (see Sects. 2, 3, 4, 7 and 8)

After 2004, other accurate Earth gravity models have been published using longer periods of GRACE observations. The LAGEOS analyses have been recently repeated with these models, over a longer period and by using different orbital programs independently developed by NASA Goddard and GFZ of Potsdam/Munich. These recent frame-dragging measurements (Ciufolini et al. 2009a), by a team of the universities of Salento (Lecce), Sapienza (Rome), Maryland BC, NASA-Goddard and GFZ of Potsdam/Munich, have improved the precision of the 2004 LAGEOS determination of the Lense-Thirring effect. No deviations from the predictions of General Relativity have been observed.

In 2008, these measurements of the Lense-Thirring effect were repeated and extended (Ries et al. 2008) by an independent group of CSR-University of Texas at Austin, with an independent orbital estimator called UTOPIA and using more recent and more accurate GRACE Earth gravity field models. Their results, reported in the next section, confirmed our measurements of the Lense-Thirring effect with an accuracy of about 12%. The laser-ranged satellite LARES (ASI) will provide a future improved test of Earth's gravitomagnetism with accuracy of a few percent, see Sects. 5 and 6.

3 Method of the 2004 Analysis of LAGEOS and LAGEOS 2 Data Using the GRACE Models

The accurate measurement of the Lense-Thirring effect, obtained in 2004 and described in this paper, has been obtained using the laser–ranging data of the satellites LAGEOS and LAGEOS 2 and the Earth gravity field models EIGEN-GRACE02S (Reigber et al. 2005), EIGEN-GRACE03S and JEM03G. An independent GRACE group of the Center for Space Research (CSR) of the University of Texas at Austin has extended these measurements using GGM02S, EIGEN-CG03C, GIF22a, JEM04G, EIGEN-GL04C, JEM01-RL03B, GGM03S, ITG-GRACE03S, EIGEN-GL05C. The analysis covered an observational period between about 11 years and 14 years, i.e. more than 2.5 times longer than in any previous analysis.

Table 1 Models used in the orbital analysis with EIGEN-GRACE02S, EIGEN-GRACE03S, JEM03G

geopotential (static part)	EIGEN-GRACE02S, EIGEN-GRACE03S, JEM03G
geopotential (tides)	Ray GOT99.2 and FES2002, FES2004
lunisolar and planetary perturbations	JPL ephemerides DE-403
general relativistic corrections	PPN except L–T
Lense-Thirring effect	set to zero
direct solar radiation pressure	cannonball model
albedo radiation pressure	Knocke–Rubincam model
Yarkovsky–Rubincam effect	GEODYN model
spin axis evolution of LAGEOS satellites	LOSSAM 2004 (Andrès et al. 2004)
station positions (ITRF)	ITRF2000
ocean loading	Scherneck model with GOT99.2 and FES2002 tides
polar motion	estimated
Earth rotation	VLBI + GPS

We have analyzed the laser–ranging data using the principles described in International Earth Rotation Service (1996) and adopted the underlying IERS conventions in our modeling, except that, in the 2004 analysis and following ones, we used the GRACE Earth's static gravity models listed above. Our analysis was performed using 15-day arcs. For each 15-day arc, initial state vector (position and velocity), coefficient of reflectivity (C_R) and polar motion were adjusted. Solar radiation pressure, Earth albedo, and anisotropic thermal effects were modeled according to Rubincam (1988, 1990), Rubincam and Mallama (1995), Martin and Rubincam (1996). In modeling the thermal effects, the orientation of the satellite spin axis was obtained from Andrès et al. (2004). We have applied a $\dot{J}_4 = -1.41 \cdot 10^{-11}$ correction (Reigber et al. 2005; Ciufolini and Pavlis 2005). Lunar, solar, and planetary perturbations were also included in the equations of motion, formulated according to Einstein's general theory of relativity with the exception of the Lense-Thirring effect, which was purposely set to zero. Polar motion was adjusted and Earth's rotation was modeled from the very long baseline interferometry-based series SPACE (Gross 1996) which are extended annually. We analyzed the laser-ranging data and the orbits of the LAGEOS satellites using the orbital analysis and data reduction software GEODYN II (NASA Goddard) (Pavlis et al. 1998) and EPOS-OC (GFZ) (the CSR-UT team used UTOPIA). The models used in the GEODYN II and EPOS-OC analysis are listed in Table 1.

As we have pointed out in the previous section, the perigee of an Earth satellite such as LAGEOS 2 is affected by a number of perturbations whose impact in the final error budget is not easily assessed and this was one of the two main points of concern of Ries et al. (2003a). The other point of concern was some favorable correlation of the errors of the Earth's spherical harmonics for the EGM96 model that might lead to some underestimated error budget. However, these points of concern do not exist in our 2004–2009 analyses with the GRACE models; using the previous models JGM-3 and EGM96, we were forced to use three observables to eliminate the J_4 uncertainty (Figs. 1, 2) and thus we needed to use the perigee of LAGEOS 2. However, for a measurement of the Lense-Thirring effect with accuracy of the order of 10%, using a number of 2004–2008 GRACE models, thanks to the dramatic improvement in the determination of the Earth's gravity field due to GRACE, it is just enough to eliminate the uncertainty in the Earth's quadrupole moment and thus to use two observables only, i.e. the two nodes of the LAGEOS satellites. Nevertheless, for a measurement of the Lense-Thirring effect with accuracy of a few percent, it is necessary to use one additional observable that will be provided by the node of the LARES satellite (see the LARES Sects. 5 and 6).

To precisely quantify and measure the gravitomagnetic effects we have introduced the parameter μ that is by definition 1 in general relativity (Ciufolini and Wheeler 1995) and zero in Newtonian theory (thus, our approach is not based on the metric gravitational theories described by the PPN, Post-Newtonian-Parametrized, approximation).

The main error in this measurement is due to the uncertainties in the Earth's even zonal harmonics and their time variations. The unmodeled orbital effects due to some harmonics of lower degree are of order of magnitude comparable to the Lense-Thirring effect (Ciufolini 1996). However, analyzing the GRACE models and their uncertainties in the even zonal harmonics, and propagating these errors on the nodes of LAGEOS and LAGEOS 2, we find that by far the main source of error in the determination of the Lense-Thirring effect is only due to the first even zonal harmonic, J_2 (Ciufolini and Pavlis 2004; Ciufolini et al. 2006).

We can therefore use the two observable quantities $\dot{\Omega}_I$ and $\dot{\Omega}_{II}$ to determine μ (Ciufolini 1989, 1996, 2002), thereby avoiding the largest source of error arising from the uncertainty in J_2. We can do this by solving the system of the two equations for $\delta\dot{\Omega}_I$ and $\delta\dot{\Omega}_{II}$ in the two unknowns μ and J_2, obtaining for μ:

$$\begin{aligned}
&\delta\dot{\Omega}^{\mathrm{Exp}}_{\mathrm{LAGEOS\ I}} + c\,\delta\dot{\Omega}^{\mathrm{Exp}}_{\mathrm{LAGEOS\ II}} \\
&\quad = \mu(31 + c31.5)\ \text{milliarcsec/yr} + \text{other errors} \\
&\quad \cong \mu(48.2\ \text{milliarcsec/yr}),
\end{aligned} \tag{3}$$

where $c = 0.545$.

The use of the nodes of two laser-ranged satellites of LAGEOS type to measure the Lense-Thirring effect by eliminating in this way the Earth spherical harmonics uncertainties was first proposed and published in Ciufolini (1984, 1989) and further studied in Tapley et al. (1989), Ciufolini et al. (1989, 2004), Ries (1989). The calculation of the standard relativistic perigee precession of LAGEOS was carried out in Rubincam (1977) and the proposal to use laser ranging to artificial satellites to detect relativistic effects, among which the Lense-Thirring effect, was published in Cugusi and Proverbio (1978), in this paper the LAGEOS Lense-Thirring precession was calculated to be 4 arcsec/century, i.e. 40 milliarcsec/yr, instead of the correct 31 milliarcsec/yr figure calculated in Ciufolini (1986) and the problem of the Earth's even zonal harmonics errors was not treated in Cugusi and Proverbio (1978). A solution to the problem of the Earth's spherical harmonics using polar satellites was proposed in Yilmaz (1959) and then in Van Patten and Everitt (1976), see Sect. 2. The use of the nodes of N high-altitude, laser-ranged satellites, similar to LAGEOS, to determine the first $N-1$ even zonal harmonics J_2, J_4, $J_6, \ldots,$ and to measure the Lense-Thirring effect was first published in Ciufolini (1989), p. 3102 (see also Ciufolini and Wheeler 1995, p. 336). T he use of the nodes of LAGEOS and LAGEOS 2, together with the explicit expression of the LAGEOS satellites nodal equations, was proposed in Ciufolini (1996). A detailed study of the various possibilities to measure the Lense-Thirring effect using LAGEOS and other laser-ranged satellites was presented in Peterson (1997). The use of GRACE-derived gravitational models, when available, to measure the Lense-Thirring effect with accuracy of a few percent was published by Ries et al. (2003a, 2003b) and Pavlis (2000). In the proceedings of the 1998 William Fairbank conference and of the 13th Int. Laser Ranging Workshop, Ries et al. (2003a) concluded that, in the measurement of the Lense-Thirring effect using the GRACE gravity models and the LAGEOS and LAGEOS 2 satellites, a more current error assessment is probably at the few percent level; see also the paper by Pavlis in the proceedings of the SIGRAV 2000 conference (Pavlis 2000).

Equation (3) for μ does not depend on J_2 nor on its uncertainty, thus, this value of μ is unaffected by the largest error, due to δJ_2, and it is sensitive only to the smaller uncertainties due to δJ_{2n}, with $2n \geq 4$.

The next largest error source is due to uncertainty in J_4 that may be as large as 10% of the Lense-Thirring effect. To eliminate this error source we need a new observable that will indeed be provided by the node of LARES, allowing a measurement of the Lense-Thirring effect with accuracy of a few percent.

The various error sources that can affect the measurement of the Lense-Thirring effect using the nodes of the LAGEOS satellites have been extensively treated in a large number of papers by several authors, see, e.g., Ciufolini (1986, 1989, 1996), Ciufolini et al. (1997a, 1997b, 1998b, 2006, 2009a), Rubincam (1988, 1990), Rubincam and Mallama (1995), Martin and Rubincam (1996), Lucchesi (2001, 2002); the main error sources are treated in details in Ciufolini et al. (2006, 2009a). We refer to these papers for a detailed error analysis and error budget. Here, we only point out that a single author has claimed in a number of papers (Iorio 2005a; Iorio 2008a, 2008b, 2008c) that the error analysis and error budget of the frame-dragging measurements with LAGEOS-LAGEOS 2 and LARES have been quite underestimated. Therefore, in Sect. 7 we describe the main misunderstandings and miscalculations that led to these ill-founded claims; any interested reader should study Sects. 2, 7 and 8, and Ciufolini and Pavlis (2005), Ciufolini et al. (2006), Lucchesi (2005), Ries et al. (2008).

4 Results of the Measurement of the Lense-Thirring Effect

In this section we present the results in the measurement of the Lense-Thirring effect on the basis of a number of different GRACE Earth gravitational models, through the analysis of the nodal rates of the LAGEOS and LAGEOS 2 satellites. We first report the results we obtained using EIGEN-GRACE02S, EIGEN-GRACE03S and JEM03G (Figs. 2, 3, 4, 5) over a period of about 11 years. We also separately plot the integrated residuals of the node of LAGEOS, of the node of LAGEOS 2 and of their J_2-free combination Fig. 3). In Fig. 4d, we report the result of the measurement of the Lense-Thirring effect with EIGEN-GRACE02S using the orbital estimator EPOS-OC by the German GRACE group of GFZ, this program is independent of GEODYN; thus this independent result confirms our previous analyses obtained with GEODYN. Then, in Fig. 5, we present the results with these three models, including an error bar representing the total estimated error in our measurement including systematic errors. Finally, in Fig. 6 we report the result of Ries et al. (2008) for the measurement of the Lense-Thirring effect using a number of GRACE models, including EIGEN-GRACE02S, GGM02S, EIGEN-CG03C, GIF22a, JEM04G, EIGEN-GL04C, JEM01-RL03B, GGM03S, ITG-GRACE03S and EIGEN-GL05C.

Figure 3 shows (in blue) the integrated residuals of the node of LAGEOS from January 1993 to October 2003 using the model EIGEN-GRACE02S, the integrated node residuals of LAGEOS 2 (in green), using EIGEN-GRACE02S, and the integrated combination of the nodes residuals of LAGEOS and LAGEOS 2 (in red), according to formula (5), using EIGEN-GRACE02S; this figure displays that the large residuals of the node of each satellite (in blue and green) due to the error in J_2 are eliminated in the combined residuals (in red), see also Ries et al. (2008).

Figure 4 shows the result of the measurement of the Lense-Thirring effect using the three GRACE Earth models EIGENGRACE02S (Fig. 4a), EIGENGRACE03S (Fig. 4b) and

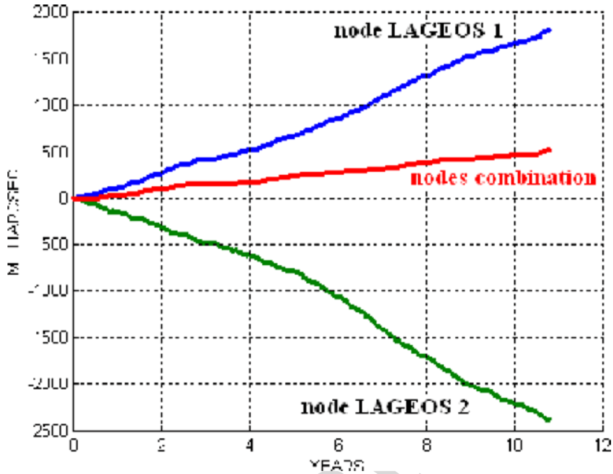


Fig. 3 Residuals of the node of LAGEOS (in *blue*), node of LAGEOS 2 (in *green*) and combination of the nodes of LAGEOS and LAGEOS 2 using formula (3) (in *red*), JEM03G is the GRACE Earth gravity model used here

JEM03G (Fig. 4c) by fitting the orbital residuals with a secular trend together with *six* periodic effects. Figure 4d represents the result of the measurement of the Lense-Thirring effect using *the orbital estimator EPOS-OC* of the German GRACE team of GFZ, with EIGEN-GRACE02S and by fitting the orbital residuals with a secular trend together *six* periodic effects.

In conclusion, by fitting our combined residuals with a secular trend plus 6 periodic signals, using EIGENGRACE02S, we found over an observational period of 11 years:

$$\mu_{\text{EIGENGRACE02S}} = 0.994 \pm 0.10$$

with RMS of the post–fit residuals of 5.98 milliarcsec using GEODYN II however, using EPOS-OC and the corresponding GFZ set-up describing the satellites orbital perturbations, we found: $\mu_{\text{EIGENGRACE02S}} = 1.0 \pm 0.10$ (with RMS of the post–fit residuals of 6.92 milliarcsec); this small 0.6% difference for μ using the two orbital estimators is due to a different modeling of the orbital perturbations in the two cases.

By fitting our combined residuals with a secular trend plus six periodic signals, using EIGENGRACE03S and JEM03G, we found respectively:

$$\mu_{\text{EIGENGRACE03S}} \simeq 0.93 \pm 0.13$$

and

$$\mu_{\text{JEM03G}} \simeq 0.99 \pm 0.18,$$

where these uncertainties include all systematic errors. The static gravitational errors for the models EIGEN-GRACE02S, EIGEN-GRACE03S and JEM03G have been calculated by simply adding the absolute values of the errors in our combined residuals due to each even zonal harmonics uncertainty and by then multiplying this total error for a factor 2 to take into account possible underestimations of the published uncertainties of each model (see Sect. 7). However, in the case of JEM03G, only the formal uncertainties of the J_{2n} were available to us but *not* their calibrated uncertainties, i.e., the uncertainties including the systematic errors, and we have then tentatively used the uncertainties of the GRACE model GGM02S; then, using the published, calibrated, uncertainties of GGM02S we obtain

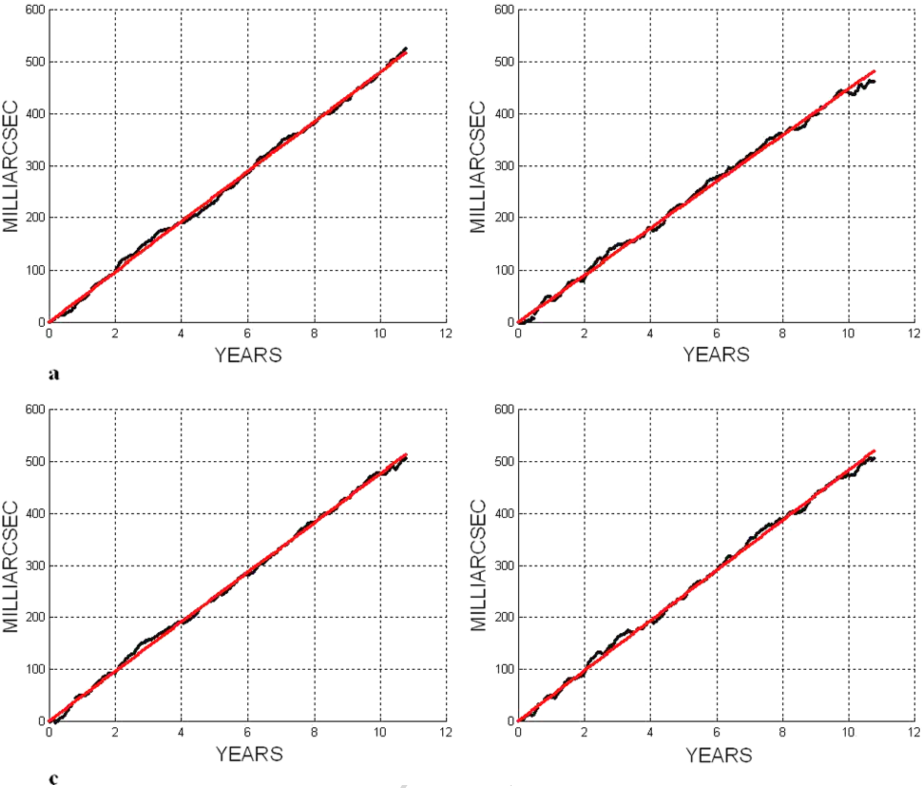


Fig. 4 Linear fit of the residuals of the nodes of LAGEOS and LAGEOS 2, using the combination (3) with: **a** the model EIGEN-GRACE02S, **b** EIGEN-GRACE03S, **c** JEM03G. In **d** is shown the fit of the nodes of LAGEOS and LAGEOS 2, using *the orbital estimator EPOS-OC* of the German GRACE team of GFZ and the model EIGEN-GRACE02S. The fits in **a** to **d** are with a secular trend plus *six* periodic terms. The slope in **a** is $\mu \simeq 0.99$, in **b** $\mu \simeq 0.93$, in **c** $\mu \simeq 0.99$ and in **d** $\mu \simeq 1.0$. The scale of the axes is different with respect to Fig. 3

a total error budget of the order of 10%, or by doubling the static even zonal harmonics uncertainties, we get a total error of the order of 18%

In the case of EIGENGRACE02S, by fitting our combined residuals with 2, 6, or 10 periodic terms we practically get the same value for the Lense-Thirring effect and by analyzing the data with the NASA orbital estimator GEODYN II and with the GFZ orbital estimator EPOS-OC with their corresponding different set-up for the orbital perturbations, we practically obtain the same result. Furthermore, these different measurements of the Earth frame-dragging effect obtained with EIGEN-GRACE02S, EIGEN-GRACE03S and JEM03G are in agreement with each other within their uncertainties. Therefore, our measured value of the Lense-Thirring effect with the Earth gravity models EIGEN-GRACE02S, EIGEN-GRACE03S and JEM03G agree with the general relativistic prediction and the corresponding uncertainty of our measurement using the more recent GRACE models is of the order of 10% (see Appendix of Ciufolini et al. 2009a).

Ries et al. (2008) have extended the measurement of the Lense-Thirring effect to a number of more recent models, including EIGEN-GRACE02S, GGM02S, EIGEN-CG03C, GIF22a, JEM04G, EIGEN-GL04C, JEM01-RL03B, GGM03S, ITG-GRACE03S

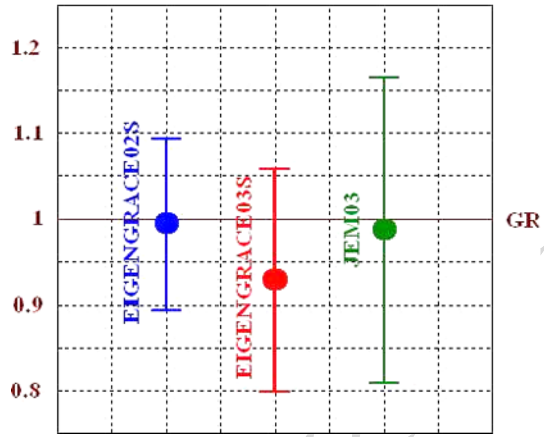


Fig. 5 Measurement of the Lense-Thirring effect with the GRACE models EIGEN-GRACE02S (Ciufolini and Pavlis 2004), EIGEN-GRACE03S, and JEM03G, obtained using GEODYN and EPOS-OC. The error bar includes systematic errors calculated using the published uncertainties of each model (see also Sect. 6)

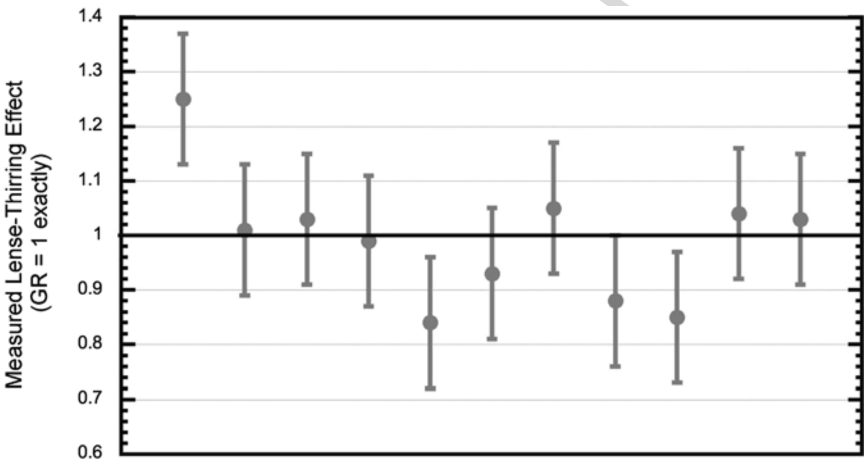
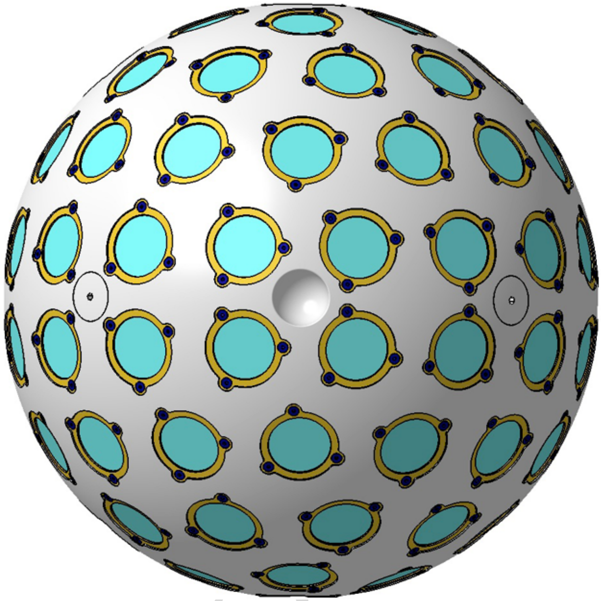


Fig. 6 Measurement of the Lense-Thirring effect with the GRACE models: EIGEN-GRACE02S, GGM02S, EIGEN-CG03C, GIF22a, JEM04G, EIGEN-GL04C, JEM01-RL03B, GGM03S, ITG-GRACE03S and EIGEN-GL05C, obtained using UTOPIA (Ries et al. 2008)

and EIGEN-GL05C, using the orbital estimator UTOPIA and the corresponding set-up for the orbital perturbations, the results are presented in Fig. 6. Ries et al. have concluded that the mean value of the Lense-Thirring effect using these models is 0.99μ with a total error budget in the measurement of the Lense-Thirring effect of about 12%.

In conclusion, the analysis of the University of Salento, Sapienza University of Rome, University of Maryland Baltimore County and GFZ Potsdam/Munich (using the orbital estimators GEODYN and EPOC-OC), and of the Center for Space Research of the University of Texas at Austin (using the orbital estimator UTOPIA) have confirmed the general relativistic prediction for Lense-Thirring effect using the LAGEOS and LAGEOS 2 orbital data with an accuracy of about 10% (*see* Figs. 5, 6).

Fig. 7 Drawing of one version of the LARES satellite



5 LARES

The LARES space experiment, by the Italian Space Agency (ASI), is based on the launch of a new laser-ranged satellite, called LARES (LAser RElativity Satellite), using the new launch vehicle VEGA (ESA-ELV-ASI-AVIO), Fig. 7. LARES will have an altitude of about 1450 km, an orbital inclination of about 71.5 degrees and a nearly zero eccentricity. The LARES satellite together with the LAGEOS (NASA) and LAGEOS 2 (NASA and ASI) satellites and with the GRACE (NASA-CSR and DLR-GFZ) Earth gravity field models will allow a measurement of the Earth gravitomagnetic field and of Lense-Thirring effect with an uncertainty of a few percent.

In this section, after a description of the LARES experiment and of its orbit, we discuss the main error sources affecting the measurement of gravitomagnetism with LARES; these are due to the uncertainties in the Earth's gravitational field, in particular in the Earth's even zonal harmonics and to the time-dependent component of the Earth's gravitational field, in particular $\dot{J}_6$ and the K_1 tide. We also briefly discuss the effect of particle drag and the error due to the uncertainties in the measurement of the orbital inclination (see Sect. 8). We finally describe the structure of the LARES satellite that is designed and built in order to minimize the non-gravitational perturbations.

5.1 Introduction

In Sect. 2 we briefly reported the LAGEOS III 1984 proposal, i.e., the use of the nodes of two laser-ranged satellites of LAGEOS type to measure the Lense-Thirring effect (Ciufolini 1984, 1986). Several papers (Ciufolini 1989), international studies (Tapley et al. 1989; Ciufolini et al. 1989, 2004), proposals (Ciufolini et al. 1998a) and Ph.D. dissertations (Ries 1989; Peterson 1997) analyzed the LAGEOS III proposal.

Unfortunately, even though such an orbit of LARES would have allowed a complete removal of the static Earth spherical harmonics secular errors in order to measure the much smaller Lense-Thirring effect, the weight of the proposed LARES satellite, of about 400 kg, and especially the high altitude of its orbit implies an expensive launch vehicle. For this reason a LARES satellite of only about 100 kg of weight was later designed (Ciufolini et al. 1998a), with a non-zero orbital eccentricity to allow some equivalence principle tests as proposed by Nordtvedt (1998).

Nevertheless, three new factors have changed the need of such a high-altitude, expensive, orbit for LARES: (a) the idea to use N laser-ranged satellites to measure the Lense-Thirring effect and to cancel the uncertainty due to the first N-1 even zonal harmonics (Ciufolini 1989; Ciufolini and Wheeler 1995; Ciufolini and Pavlis 2004) (that led to the 1995–1996 observations Ciufolini et al. 1997a, 1997b, 1998b; Ciufolini 2000 and to the 2004–2008 measurements Ciufolini and Pavlis 2004; Ciufolini et al. 2006, 2009a; Ries et al. 2008 of the Lense-Thirring effect, described in Sects. 2–4); (b) the launch of the GRACE spacecraft in 2002 and the publication of a new generation of very accurate Earth's gravity field models using the GRACE observations (Reigber et al. 2002, 2005; Tapley 2002; Watkins et al. 2002) (at the time of the first LAGEOS III proposal, Ciufolini 1989; Tapley et al. 1989; Ciufolini et al. 1989, 2004, the error due to the even zonal harmonics was much larger due to the much less accurate Earth gravity models available at that time and the LAGEOS 2 satellite was not yet launched); and (c) the possibility to launch the LARES satellite using the qualification flight of the new launcher VEGA (ESA-ELV-ASI-AVIO), however at a much lower altitude than the original proposal. Indeed in 2004, one of us (A.P.) discovered the possibility to use the qualifying flight of VEGA to orbit LARES (Paolozzi 2005). However, this launch for LARES will be at a much lower altitude than the originally planned satellite at 12270 km. The altitude achievable with this qualifying launch is of about 1450 km. Nevertheless, Ries (2005) informed us that CSR had done some simulations supporting the possibility of using a lower orbit laser-ranged satellite to measure the Lense-Thirring effect, this possibility was later on also discussed in Iorio (2005b). Precise calculations of the LARES gravitational errors were analyzed in Ciufolini (2006)

5.2 A New Laser-Ranged Satellite at a Lower Altitude than LAGEOS and LAGEOS 2

The simplest conceivable orbit in order to cancel the effect of all the even zonal harmonics on the node of a satellite would be a polar orbit (for such an orbit the effect of the even zonal harmonics on the satellite node would be zero and, however, the node of the satellite would be still perturbed by the Earth gravitomagnetic field, i.e., by the Lense-Thirring effect, Lucchesi and Paolozzi 2001, see Sects. 1 and 2).

Unfortunately, as pointed out in the 1989 LAGEOS III NASA/ASI study (Tapley et al. 1989; Ciufolini et al. 1989, 2004) and as explicitly calculated by Peterson (1997) (Chap. 5 of Peterson 1997) the uncertainty in the K_1 tide (tesseral, $m = 1$, tide) would make such an orbit unsuitable for the Lense-Thirring measurement. Indeed, a polar satellite would have a secular precession of its node whose uncertainty would introduce a large error in the Lense-Thirring measurement (in addition, it would be quite demanding to launch LARES with the requirement of a small orbital injection deviation from a polar orbit; in fact, in order to cancel the error due to the uncertainties in the static Earth gravity field, for a very accurate measurement of the Lense-Thirring effect at an altitude lower than the one planned for LAGEOS III, the deviation from a polar orbit should be less than a tenth of a degree).

Nevertheless, a non-polar orbit would have a nodal precession, due to its departure from 90 degrees of inclination, and thus one could simply fit for the effect of the K_1 tide using a periodical signal exactly at the nodal frequency. Such signal (with the periods of the

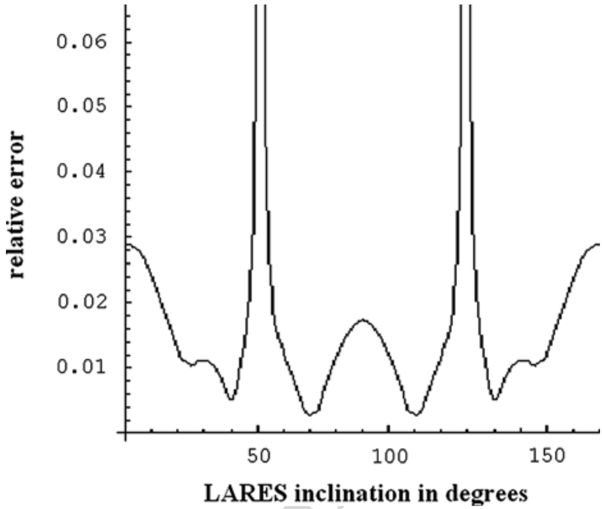


Fig. 8 Percent uncertainty in the measurement of the Lense-Thirring effect, due to the even zonal harmonics uncertainties, as a function of the inclination of LARES, using LARES, LAGEOS and LAGEOS 2. The altitude of LARES is here 1500 km and the range of the inclination between 0 and 360 degrees

LAGEOS and LAGEOS 2 satellites nodes) is indeed observed in the already mentioned LAGEOS and LAGEOS 2 analyses (Ciufolini et al. 1998b; Ciufolini and Pavlis 2004) and is the periodical signal with the largest amplitude observed in the combined residuals.

Furthermore, in regard to the effect of the static even zonal harmonics, by using the technique explained in Ciufolini (1989, 1996) and by using the nodes of the satellites LARES, LAGEOS and LAGEOS 2, we would be able to cancel the uncertainties due to the first two even zonal harmonics, J_2 and J_4, and our measurement will only be affected by the uncertainties of the even zonal harmonics with degree strictly higher than 4.

By solving the system of the three equations for the nodal precessions of LAGEOS, LAGEOS 2 and LARES in the three unknowns, J_2, J_4 and Lense-Thirring effect, we have a combination of three observables (the three nodal rates) which determines the Lense-Thirring effect independently of any uncertainty, δJ_2 and δJ_4, in the first two even zonal harmonics. This same technique was applied in Ciufolini et al. (1998b) using the nodes of LAGEOS and LAGEOS 2 and the perigee of LAGEOS 2 and in Ciufolini and Pavlis (2004) using the nodes of LAGEOS and LAGEOS 2 only.

It turns out that some values of the inclination of LARES would minimize the error in the measurement of the Lense-Thirring effect since they would minimize the error due to the uncertainty in the largest (not cancelled using the combination of the three observables) even zonal harmonic J_6.

In Fig. 8 we have plotted the relative error in the measurement of the Lense-Thirring effect as a function of the inclination by assuming an altitude of LARES of 1500 km, i.e., a LARES semimajor axis of about 7880 km.

From Fig. 8 we can see that any inclination from 60 degrees to 86 degrees and from 94 to 120 degrees would be suitable for a measurement of the Lense-Thirring effect with an accuracy of a few percent. An inclination of LARES of about 110 degrees or 70 degrees would minimize the error. In deriving this result, we have assumed: (a) zero eccentricity for the LARES orbit, (b) we have only considered the effect of the first 5 even zonal harmonics: J_2, J_4, J_6, J_8 and J_{10} and (b) we have considered the uncertainties in the spherical harmonics J_6, J_8 and J_{10} equal to those of the EIGEN-GRACE02S Earth's gravity model (Reigber et al. 2005), i.e., we have assumed $\delta C_{60} = 0.2049 \cdot 10^{-11}$ $\delta C_{80} = 0.1479 \cdot 10^{-11}$ $\delta C_{100} = 0.2101 \cdot 10^{-11}$, where the C_{l0} are the normalized zonal harmonic coefficients related to the

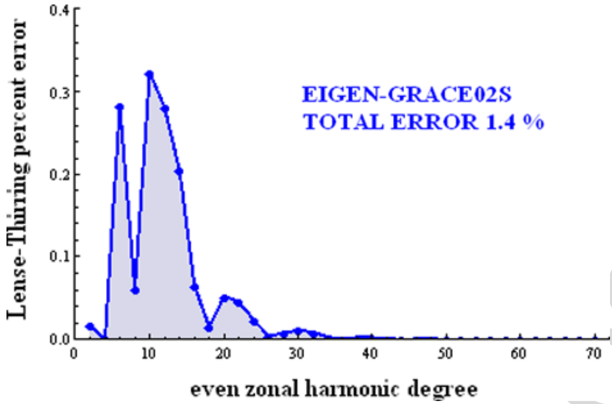


Fig. 9 Percent error in the measurement of the Lense-Thirring effect using LARES, LAGEOS and LAGEOS 2 as a function of the uncertainties of each even zonal harmonic. The model used is EIGEN-GRACE02S (GFZ Potzdam, 2004) and the uncertainties in this model include systematic errors. Using EIGEN-GRACE02S, the total error in the measurement of the Lense-Thirring effect due to the even zonal harmonics is 1.4%. An improvement by about an order of magnitude is expected at the time of the LARES data analysis

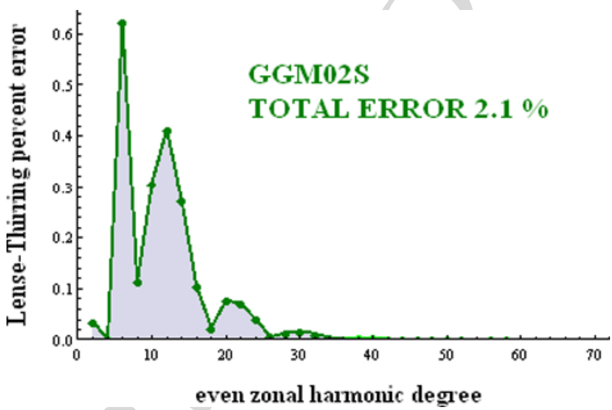


Fig. 10 Percent error in the measurement of the Lense-Thirring effect using LARES, LAGEOS and LAGEOS 2 as a function of the uncertainties of each even zonal harmonic. The model used is GGM02S (CSR, 2004) and the uncertainties in this model include systematic errors. Using GGM02S, the total error in the measurement of the Lense-Thirring effect due to the even zonal harmonics is 2.1%. An improvement by about an order of magnitude is expected at the time of the LARES data analysis

un-normalized zonal harmonic coefficients J_l by: $J_l \equiv -\sqrt{2l+1}C_{l0}$. However, by including higher degree even zonal harmonics, the results of Fig. 8 would only slightly change.

Indeed, in the next section and in Figs. 9, 10, 11, 12, we calculate and show the error in the measurement of the Lense-Thirring effect, with LARES, LAGEOS and LAGEOS 2, due to the even zonal harmonics up to degree 70, corresponding to a LARES orbit of 1450 km of altitude and 71.5 degrees of inclination. Some of the results described in this section and those shown in Fig. 8, are based on the calibrated uncertainties (i.e., including systematic errors) of the EIGEN-GRACE02S model; even though the real errors in these EIGEN-GRACE02S coefficients would probably be about two or three times larger than these published uncertainties, EIGEN-GRACE02S is a 2004 model and by the time of the

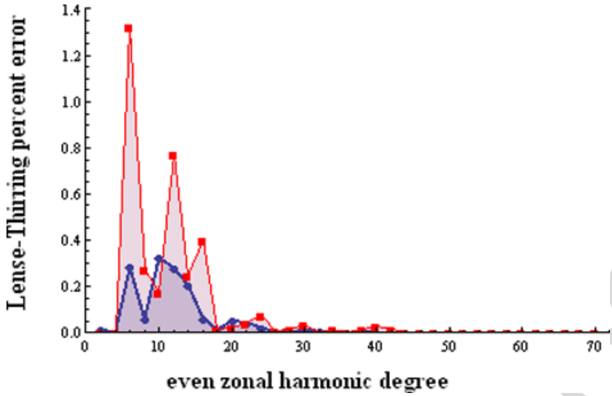


Fig. 11 Percent error in the measurement of the Lense-Thirring effect using LARES, LAGEOS and LAGEOS 2 as a function of the uncertainty due to each even zonal harmonic. The points in *blue* are the errors obtained using the model EIGEN-GRACE02S (i.e. Fig. 9 rescaled) and the points in red are the errors obtained using as uncertainty of each coefficient the difference between the value of this coefficient in the two different models EIGEN-GRACE02S and GGM02S. The total error in the measurement of the Lense-Thirring effect using EIGEN-GRACE02S is 1.4% and by using as uncertainties the differences between the coefficients of the two models is 3.4%. However, at the time of the LARES data analysis an improvement of about one order of magnitude has to be taken into account with respect with these 2004 models that were based on less than 365 days of observations of the GRACE spacecraft

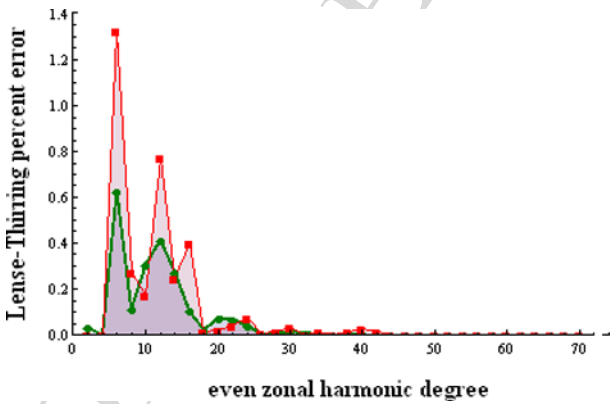


Fig. 12 Percent error in the measurement of the Lense-Thirring effect using LARES, LAGEOS and LAGEOS 2 as a function of the uncertainty due to each even zonal harmonic. The points in *green* are the errors obtained using the model GGM02S (i.e. Fig. 10 rescaled) and the points in *red* are the errors obtained using as uncertainty of each coefficient the difference between the value of this coefficient in the two different models EIGEN-GRACE02S and GGM02S. The total error in the measurement of the Lense-Thirring effect using GGM02S is 2.1% and by using as uncertainties the differences between the coefficients of the two models is 3.4%. However, at the time of the LARES data analysis an improvement of about one order of magnitude has to be taken into account with respect with these 2004 models that were based on less than 365 days of observations of the GRACE spacecraft

launch of LARES and of its data analysis (about 2010–2017), much improved Earth's gravity field models based on much longer data set of GRACE observations would be available, with true errors even smaller than the EIGEN-GRACE02S uncertainties that we used in the present analysis.

In regard to the other orbital perturbations that affect the LARES experiment, we briefly discuss here the tidal effects, particle drag and thermal drag; for a detailed treatment of other perturbations we refer to Ciufolini (1989), Tapley et al. (1989), Ciufolini et al. (1989, 2004, 2006, 2009a). In regard to the orbital perturbations on the LARES experiment due to the time-dependent Earth's gravity field, we observe that the largest tidal signals are due to the zonal tides with $l = 2$ and $m = 0$, due to the Moon node, and to the K_1 tide with $l = 2$ and $m = 1$ (tesseral tide). However, the medium and long period zonal tides ($l = 2$ and $m = 0$) will be cancelled using the combination of the three nodes together with the static J_2 uncertainty (also the uncertainty in the time-dependent secular variations $\dot{J}_2$, $\dot{J}_4$ will be cancelled using this combination of three observables). Furthermore, the tesseral tide K_1 will be fitted for over a period multiple of the LARES nodal period as explained above (see Tapley et al. 1989 and Chap. 5 of Peterson 1997) and this tide would then introduce a small uncertainty in our combination. In regard to the non-gravitational orbital perturbations, we stress here that the unmodeled thermal drag perturbations on the LARES orbit would be reduced with respect to the LAGEOS satellites thanks to the much smaller (by a factor of about 0.34) cross-sectional to mass ratio of LARES (see the Sect. 5.3 on the LARES structure); furthermore, accurate measurements of the thermal properties of LARES and of its retro-reflectors should be performed. We finally point out that the neutral and charged particle drag on the LARES node at an altitude of about 1450 km will be a small effect (of a fraction of a percent of the Lense-Thirring effect) for an orbit with very small eccentricity, even by assuming that the exosphere would be co-rotating with Earth at 1450 km of altitude and by considering the exosphere density inhomogeneities at that altitude (Ciufolini et al. 2009c). Indeed, as calculated in Ciufolini (1989), Ciufolini et al. (1990) for the LAGEOS III satellite, in the case of zero orbital eccentricity $e = 0$, the total drag effect on the LARES node would be zero; indeed the nodal rate of a satellite due to particle drag is a function of $\sin \nu \cdot \cos \nu$ (ν is the true anomaly) and the total nodal shift is then zero over one orbit; in the case of a small orbital eccentricity, the total shift would be proportional to the eccentricity and it would still be a small effect, as calculated in Ciufolini (1989). *In regard to the orbital inclination, as explained in Sects.* 2, 7.1 *and* 8, *we stress that what is critical for the measurement of the Lense-Thirring effect is that the measurement of the inclination be accurate enough but not its modeling (i.e., the prediction of its behavior on the basis of the available physical models).* In Sect. 8, we treat the accuracy in the measurement of the orbital inclination of the LAGEOS satellites and its main limitation due to atmospheric refraction.

5.3 Structural Requirements for LARES Satellite

The LARES satellite has been designed in such a way to minimize all the non gravitational perturbations such as particle drag and thermal thrust (induced by the anisotropic thermal radiation from the satellite due to the anisotropic temperature distribution over the satellite surface). Indeed, the orbit of LARES is much lower than that of the two LAGEOS satellites, therefore the minimization of the non-gravitational perturbations, such as particle drag, is especially important for LARES. In the following we briefly report on the relevant aspects of the design of the LARES satellite that are different with respect to those of the LAGEOS satellites.

The minimization of the cross-sectional area-to-mass ratio of a spherical satellite yields: $\frac{A}{M} = \frac{3}{4\rho r}$, where ρ is the mean satellite density and r its radius; this is a critical parameter for the minimization of the non-gravitational perturbations, indeed these accelerations of the satellite are substantially proportional to the satellite cross-sectional area and inversely proportional to its mass. From the above simple relation one deduces that to minimize A/M

the satellite should be very large. However, due to launch cost, the size and weight of the satellite is fixed to about 400 kg. The optimal condition will then be obtained by taking the highest value for ρ. The best compromise for what concerns cost, density and mechanical properties is obtained with tungsten (density of 19350 kg/m^3). However, pure tungsten is not workable and therefore a tungsten alloy has to be considered for LARES. There are a variety of tungsten alloys some of which can reach a density of 18500 kg/m^3. To further reduce the A/M ratio, the cavities housing the CCRs should be as small as possible (Ciufolini et al. 2009b). Another issue addressed in Ciufolini et al. (2009b) is the number of CCRs to be mounted on the satellite surface, their distribution and their orientation, which must be calculated to optimize the average laser return for all attitudes. A higher number of CCRs increases the reflective area of the satellite but on the other hand it increases the A/M ratio. For what concerns the reduction of errors induced by thermal thrust, besides a small A/M ratio, one can try a better modeling of this perturbation that can be achieved by determining the satellite attitude from ground-based observations. This can be performed by photometric measurements of sun glints from the CCRs front faces or from the laser pulses that may contain a spin signature. Both techniques require a suitable CCRs arrangement on the surface (Ciufolini 2007b). However, it is important to stress that the reduced A/M ratio of LARES with respect to LAGEOS (by a factor of about 0.34) will make thermal thrust much smaller on LARES than what it is on LAGEOS.

These aspects and additional ones regarding the design of LARES are discussed in Ciufolini et al. (2009b). The satellite has been designed to minimize the non-gravitational perturbations and to be as nearly as possible a test-particle freely falling in the Earth gravitational field. In conclusion, the orbital parameters chosen for LARES are: inclination about 71.5 degrees, semimajor axis about 7830 km (i.e., altitude of about 1450 km) and eccentricity nearly zero; a relatively large orbital injection error from these values is acceptable for the measurement of gravitomagnetism. The final weight of the LARES satellite will be about 385 kg, its radius about 18 cm and it will have 92 CCRs housed in conical cavities. LARES together with the LAGEOS satellites and with the GRACE models, will allow a measurement of Earth gravitomagnetism and Lense-Thirring effect with an accuracy of a few percent.

6 Gravitational Uncertainties and Even Zonal Harmonics

In the LARES experiment, the error sources of gravitational origin, i.e., those due to the uncertainties in the Newtonian gravitational field, are by far larger than the uncertainties of non-gravitational origin, i.e. radiation pressure, both from Sun and Earth, thermal thrust and particle drag. Indeed the LAGEOS satellites and especially LARES are extremely dense spherical satellites with very small cross-sectional-to-mass ratio (Ciufolini 1989; Ciufolini et al. 2009b); in particular LARES is the densest known single object in the solar system (see the previous Sect. 5). As explained in Sects. 2 and 3, the only secular perturbations affecting the node of an Earth satellite are due to the Earth spherical harmonics of even degree and zero order, e.g., the J_2 harmonic describing the well known Earth quadrupole moment.

In this section we report the result of the precise calculation of the uncertainty of each even zonal harmonic, up to order 70, in the measurement of the Lense-Thirring effect using the satellites LAGEOS, LAGEOS 2 and LARES, and the GRACE Earth gravity models, *confirming* a total error budget of a few percent. Figures 9, 10, 11 and 12 clearly display that the uncertainties of the even zonal harmonics of degree higher than 26 are negligible in this measurement.

In Figs. 9, 10, 11 and 12 below we display the error in the LARES experiment due to each even zonal harmonic up to degree 70. Once again we stress that the large errors due to the uncertainties of the first two even zonal harmonics, i.e., of degree 2 and 4, are eliminated using the 3 observables, i.e. the 3 nodes. Figures 9, 10, 11 and 12 clearly display that the error due to each even zonal harmonic of degree higher than 4 is considerably less than 1% and in particular that the error is substantially negligible for the even zonal harmonics of degree higher than 26. In Fig. 9 we show the percent errors in the measurement of the Lense-Thirring effect with the LARES experiment due to each individual uncertainty of each even zonal harmonic corresponding to the model EIGEN-GRACE02S (Reigber et al. 2005). In Fig. 10 we show these percent errors for the model GGM02S (CSR, Austin, 2004). In Fig. 11 we display the maximum percent errors due to each even zonal harmonic obtained by considering as uncertainty for each harmonic the difference between the value of that harmonic in the EIGEN-GRACE02S model minus its value in the GGM02S model; this is a standard technique in space geodesy to estimate the reliability of the published uncertainties of a model. *Of course, in order to use this technique, one must difference models of comparable accuracy, i.e., models that are indeed comparable, or use this technique to only assess the errors of the less accurate model* (*see next Sect.* 7.2).

In conclusion, using EIGEN-GRACE02S and GGM02S, the total error in the measurement of the Lense-Thirring effect due to the even zonal harmonics is respectively 1.4% and 2.1%. However, an improvement by at least an order of magnitude is expected (with respect to EIGEN-GRACE02S) at the time of the LARES data analysis (today we already have a substantial improvement with GGM03S). Indeed, these two models, EIGEN-GRACE02S and GGM02S have been obtained with a relatively small amount of observations of the GRACE spacecraft (launched in 2002), over less than 365 days, and therefore a substantial factor of improvement of these GRACE models, of at least one order of magnitude, should be taken into account at the time of the LARES data analysis (between 2010 and 2016), thanks to longer GRACE observational periods and to the mission GOCE too.

7 A Reply to the Critical Remarks by Iorio on the Error Analysis and Error Budget of the Gravitomagnetism Measurements with LAGEOS, LAGEOS 2 and LARES

Here we demonstrate that the various claims of a single author (Iorio 2005a, 2008a, 2008b, 2008c), that the LAGEOS and LARES error analysis and error budget have been underestimated, are ill-founded. These claims are mainly based on the following four arguments: (a) the effect of the uncertainties in the measured rate of change of J_4 and of higher even zonal harmonics, (b) the unmodeled changes in the inclination due to atmospheric drag, (c) the reliability of the published uncertainties of the GRACE gravity field models and (d) in the case of the LARES experiment, the inclusion of the even zonal harmonics of degree higher than 20 in the calculation of the total uncertainty.

We show that these claims are based on, at least, four misunderstandings or miscalculations (see Sects. 2, 7.1, 7.2, 7.3, 7.4 and 8 and Ciufolini and Pavlis 2005; Ciufolini et al. 2006; Lucchesi 2005; Ries et al. 2008). In regard to the claims (a), this author is claiming in Iorio (2005a) that the uncertainties in the measured rate of change of J_4 and of higher even zonal harmonics would introduce a bias that can be as large as 45% of the Lense-Thirring effect, however several independent authors (Ciufolini and Pavlis 2005; Ciufolini et al. 2006; Lucchesi 2005; Ries et al. 2008) have proved that this type of error, in the LAGEOS and LAGEOS 2 measurements, is in fact at the level of a few percent only of the Lense-Thirring effect. In regard to the claims (b), in order to measure gravitomagnetism using the nodal rate of an Earth satellite, the orbital inclination of a satellite needs only to be accurately *measured* but *not* accurately *predicted* in its temporal behavior; this has been explained in Sect. 2. Then, in Sects. 7.1 and 8, we show that the orbital inclination of LAGEOS-type satellites is very accurately *measured* by the technique of laser ranging and thus, the corresponding error in the determination of the Lense-Thirring effect due to the *measurement* errors in the inclination of LAGEOS, LAGEOS 2 and LARES is at most at a level of a fraction of 1% of the Lense-Thirring effect. In regard to the claims (c), in Sect. 7.2, we show that different Earth gravity field models with intrinsically different accuracies cannot be compared or they *must* be compared in a proper way, i.e., the accuracy of models derived with a larger GRACE data set or with the standard, accurate, GRACE techniques to derive Earth gravity models *cannot* be estimated by taking their difference with less accurate models, derived with a smaller GRACE data set or based on less on accurate techniques or on other less accurate satellite observations. Finally, in regard to the claims (d), in Sect. 6 (Figs. 9, 10, 11 and 12) we have proven that the error due to the even zonal harmonics of degree higher than 26 is negligible in the LARES experiment, contrary to a number of miscalculations by the same author (Iorio 2008b, 2008c). Indeed, the contributions of higher degree spherical harmonics of the Earth's gravity field to a satellite orbital motion, and in particular their contributions to the satellite nodal precession, decrease as the inverse power of the semimajor axis to the degree of the even zonal harmonic, and thus quickly *decreases* with the degree and does *not increase* as claimed in various papers by the same author (Iorio 2008b, 2008c): the expansion of the Earth's gravity field in spherical harmonics is indeed an expansion with terms that decrease as the inverse power of the semimajor axis to the degree of the harmonics (Kaula 1966).

7.1 Orbital Inclination Determination

In Sect. 2 we have discussed and stressed that what is *critical for the measurement of the Lense-Thirring effect* is that the *modeling* of the classical *node* precession (2) (i.e., the prediction of its behavior on the basis of the available physical models) must be accurate, at the level of a few milliarcsec or less (Ciufolini 1986). What is critical is *not* that all the quantities entering this equation, i.e., the Earth parameters and the orbital parameters, and in particular the Earth spherical harmonic coefficients and the semimajor axis and the inclination, must be *predicted* in their variations, but instead what is critical is that they must be *determined* with sufficient accuracy via satellite laser ranging and other techniques (such as the GRACE spacecraft to determine the Earth gravity field). For example if the variations of the *inclination* and of the *semimajor axis* of LAGEOS are not well modeled because the effect of particle drag (i.e., "atmospheric drag") is not known with sufficient accuracy, this is *not* critical for our measurement of the Lense-Thirring effect because we are able to *measure* the variations of inclination and semimajor axis accurately enough with satellite laser ranging (see next Sect. 8 on the uncertainty in the determination of the orbital inclination due to the atmospheric refraction modeling errors) and thus we are able to precisely quantify the effect of these variations on the nodal rate, (2) (with the orbital estimators GEODYN, EPOS-OC and UTOPIA). Indeed, in the next Sect. 8 we estimate that the average measurement error in the inclinations of LAGEOS and LAGEOS 2 (that are roughly constant) is respectively about 30 µarcsec (i.e., 0.03 milliarcsec) for LAGEOS and about 10 µarcsec for LAGEOS 2, that, when propagated in the nodal rate, (2), corresponds to respectively about 0.6% and 0.36% of the Lense-Thirring effect.

The difference between *modeling* and *determining* an orbital element may seem trivial but is critical to understand one of the misunderstandings published in Iorio (2008a) about the error induced in our measurement of the Lense-Thirring effect by the unmodeled inclination variations; indeed this paper (Iorio 2008a) concludes: "The atmospheric drag, both in its neutral and charged components, will induce a non-negligible secular decrease of the inclination of the new spacecraft yielding a correction to the node precession of degree $l = 2$ which amounts to 3–9% yr^{-1} of the total gravitomagnetic signal. Such a corrupting bias would be very difficult to be modeled". However, as stressed above, these variations in the inclination are very accurately determined with satellite laser ranging (see Sect. 8) even tough they are not modeled (i.e., predictable) at a comparable level of accuracy; nevertheless, the measurement of the inclination variations is what is only needed to accurately model the nodal rate, (2).

7.2 Comparing Different Earth Gravity Field Models

In this section we discuss how to correctly compare different Earth gravity field models, i.e., the relative accuracy assessment for GRACE gravity models.

The GRACE mission provided a leap in the accurate determination of models for the gravitational field and its temporal variations. Nonetheless, not all models derived from GRACE data are of equal or even nearly equal quality, simply because they are based on the highly accurate GRACE observations.

There are several reasons behind this fact which is easily asserted from an examination of the error spectra for nearly every such model available, at the site of the Int. Center for Global Earth Models (ICGEM) at: http://icgem.gfz-potsdam.de/ICGEM. The first and most obvious reason is that most of these models are based on an entirely different set of data, from as little as one month to as much as several years. Some of them are derived rigorously from standard reduction techniques and others use innovative approximation techniques that rely on the accurate position and velocity determination of the orbits from the GPS tracking data. When one additionally considers the fact that GRACE's primary mission is to observe the temporal change of the field, one can realize that all of these models are de facto associated with a mean epoch as far as the representation of the static part of the field and since they span quite diverse time periods, an additional source of diversity is the way that temporal change was handled during their creation, i.e. whether it was accounted for and if so, what rates and for which part of the model they were applied. These models are not referenced to a fixed epoch, and this implies that especially for what concerns the very low degrees, users must apply very carefully the re-referencing (mapping) of these coefficients to a fixed epoch for all of them, prior to attempting any comparisons. This task is nearly impossible to accomplish rigorously for all fields, since there are several other background models used during the data reduction process that affect these rates, each of which could be handled in various ways in the different software and by the different groups that generated these fields. A particular example is the consideration or not of the effect of the oceanic pole tide, which directly and severely modifies the magnitude and temporal evolution of the second degree and order one terms. Similarly, the zonals are affected by numerous corrections that can also modify the results from different reduction approaches, even if the underlying data are identical. Even for the tightly controlled models produced by the three groups that comprise the GRACE project office, UT/CSR, GFZ/Potsdam and NASA/JPL, the direct comparison at the single coefficient level is nearly impossible, although comparison of the RMS coefficient differences by degree or by order, do seem to provide much more reliable comparison statistics and give a more robust way of assessing the relative accuracy of these models (see Fig. 13).

In these special cases, the three groups have gone through strict intercomparison of their software, as well as conventions and standards, yet there are enough differences in their

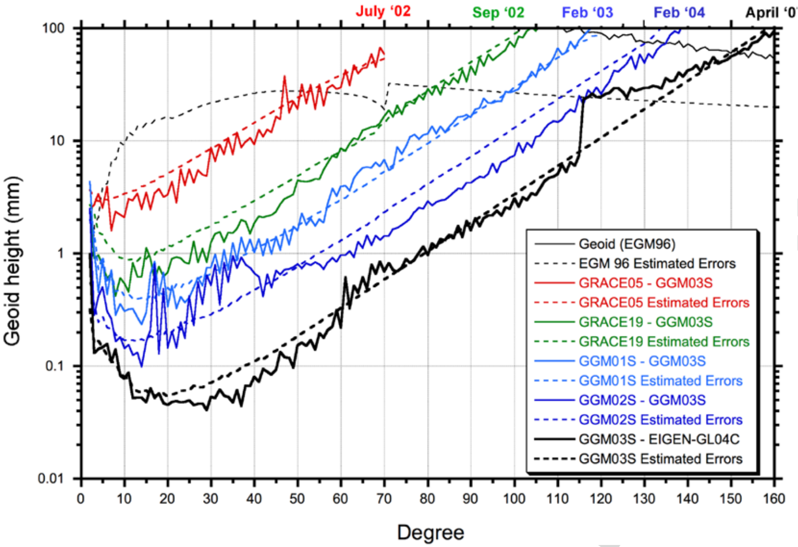


Fig. 13 Successive static gravity model improvement from GRACE data as a function of the data span used in the solution and correlation of the model errors and the assumed calibrated model errors (Tapley et al. 2007). In this figure different gravity field models are compared by comparing the difference of the corresponding coefficients for each degree (by including the sum over every corresponding *order*, see Sect. 7.2) with their published uncertainties. For example the gravity field model GGM01S, a preliminary GRACE model, less accurate and based on a much short period of observation of GRACE, is compared with GGM03S, one of the best models today available. Clearly the difference between GMM01S and GGM03S, the *solid line* in *light blue*, is of comparable size with the accuracy of the less accurate model, i.e., GGM01S, the *dashed line* in *light blue*, however the same difference is about one order of magnitude larger than the estimated accuracy of the more accurate model, i.e., GGM03S, the *dashed black line*

processing, and in some of the underlying models and constants used, that a mere differencing of the coefficients will most certainly lead the uninformed user to the wrong conclusion if these differences are used to characterize the accuracy of the models. This situation becomes much more complicated when one compares models derived by independent groups that are free to choose models and procedure to their liking. Most of these cases make no use of "forward" modeling of atmospheric and oceanic circulation induced variations in the gravitational field, a standard practice with the models produced by the project office. The aforementioned and variations in the handling of tides are some of the areas where differences between individual coefficients can arise, and mislead the user to the erroneous assumption that one of the compared models is less accurate than the other. A far better approach to compare these diverse models is to examine their "by degree" and "per degree" spectral differences with respect to a single benchmark model and after a careful mapping of the coefficients to the same epoch. Such a comparison is available already online at the ICGEM site. What we can surmise from an inspection of those spectra is that the models are very diverse, even when they come from the same group (e.g. GGM02 and GGM03). In conclusion, we must stress that in no case should one use each coefficient differences between diverse models as a substitute for the accuracy estimates of any GRACE gravity model. The proper calibration of their errors is accomplished with the use of ground truth data sets as for example a global set of GPS-derived geoid undulations over the solid Earth surface and oceanographic observations over the oceans.

Let us further discuss how to correctly compare different Earth gravity field models and some of the previous misleading comparisons. In Iorio (2007), the author is comparing different models, some of which already obsolete in 2007 and obtained with the use of less accurate space missions before GRACE, such as CHAMP. For example the author is comparing two state of the art models in 2004, EIGEN-GRACE02S and GGM02S, with the preliminary, older and obsolete models EIGEN-GRACE01S and GGM01S. Of course, the author is getting a large uncertainty as a result, indeed in Iorio (2007) he concludes "it turns out that the systematic error $\delta\mu$ in the Lense-Thirring measurement is quite larger than in the evaluations so far published based on the use of the sigmas of one model at a time separately, amounting up to 37–43% for the pairs GGM01S/GGM02S and EIGEN-GRACE01S/EIGEN-GRACE02S". However, in order to assess the uncertainty of the newer, much more accurate models, this is simply wrong; for example whereas the uncertainty associated with the measurement using EIGENGRACE02S is about 10% of the Lense-Thirring effect, the uncertainty using the older models is much larger, e.g., for GGM01S was already published (Ciufolini et al. 2006) to be as large as 24%. This explains why the author gets a large percent "error" in comparing EIGEN-GRACE02S and GGM02S with EIGEN-GRACE01S and GGM01S.

One cannot evaluate the accuracy of the latest GRACE gravity models by comparing them with previous obsolete models obtained using shorter periods of GRACE observations or CHAMP or obtained with different and less accurate techniques. For example, in a more recent paper (Iorio 2008c) the author is comparing the model ITG-Grace03s with the latest EIGENGRACE and GGM models. However, the ITG models (Univ. of Bonn), such as ITG-Grace03s, and TUM (Tech. Univ. of Munich) models are based on similar techniques that do *not* use precise modeling of the satellites orbits and extraction of the gravitational signal from the observed perturbations, but rather make use of the precise satellites orbits computed from GPS data and accelerometry, and then use the energy balance equation. Although they are very useful as experimental approaches, there is no comparison with the accuracy of precise models such as the GGM03S and EIGEN-GRACE03 models. One can see the error spectra for the various models following the links under each model in the tables available at: http://www.gfz-potsdam.de/, there one can see how the accuracy of the ITG models compares to that of a fairly high accuracy model.

Furthermore, these models cover different periods of time (thus they refer to a different mean epoch). For example, when any of these models state that they have a "zero tide" J_2, this means that their quadrupole coefficient J_2 is different from the J_2 of most of the other models by 4.173×10^{-9} in fully normalized coefficient space. If one does not remove that "kind of reference frame bias", taking the difference of J_2 for two such models, this would make them look as extremely different and it will generate a bias in the error calibration of the accuracy estimates of the coefficients. Similarly, in regard to other even zonal harmonics such as J_4, J_6, etc.

In a more recent paper (Iorio 2008c) by the same author, the claims (Iorio 2008b) of an error of 1000% in the measurement the Lense-Thirring effect was resized down by almost two orders of magnitude, with a new claim of an error that, in the worst possible case, can be as large as 33% (by comparing the models AIUB-GRACE01S and ITG-Grace03s with EIGEN-GRACE02S). However, even this more recent comparison is affected by at least four misunderstandings and miscalculations: (a) as just explained, one cannot simply take the difference between the corresponding even zonal harmonics coefficient of two different models, because, *among other things*, they refer to different mean epochs and they have been obtained with different $\dot{C}_{lm}$ corrections; (b) in Iorio (2008c) the author is comparing more recent models, however, the author is still taking the difference between models with different intrinsic accuracies, for example in Table 2 he is comparing the model GGM02S with ITG-Grace03s, however, ITG-Grace03s has a lower intrinsic accuracy than GGM02S (see above and Fig. 13); (c) the author is often comparing the difference of each even zonal coefficient of two different models with their *formal* error to conclude that the differences "ΔC_{l0} are always larger than the linearly added sigmas, apart from l = 12 and l = 18", however, it is well known and rather trivial that the formal error, i.e., the error that does *not* include the *systematic* errors, is usually much smaller than the true error (which includes systematic errors); (d) one cannot take the difference between the even zonal coefficients of different models and then sum the *absolute* values of each difference, called "SAV" in Iorio (2008c), (or similarly take the root sum square of these differences) and then compare this "SAV" sum with the Lense-Thirring effect (48.2 milliarcsec/yr) in order to more or less implicitly imply that the results of the measurement of the Lense-Thirring effect with these two models would differ by this "SAV" sum. In fact, one *must* perform a *real* data reduction to compare the results with two different models and in the real data reduction with a model, some even zonal coefficients will be larger and some smaller with respect to the other model, in other words in order to compare the results with two different models, the differences with their plus or minus signs have to be considered. For example, in the more recent paper (Iorio 2008c), in Tables 3 and 10, the author calculates a difference between GGM02S and GGM03S of 24%, however by doing the real data reduction, Ries et al. (2008) obtained a difference between the two models of 13%.

The *proper*, or improper, way of comparing different Earth gravity models is manifest from Fig. 13. In this figure different gravity field models are compared by comparing the difference of the corresponding coefficients for each degree (by including the sum over every corresponding *order*, see above) with their published uncertainties. For example the gravity field model GGM01S, a preliminary GRACE model, less accurate and based on a much short period of observation of GRACE, is compared with GGM03S, one of the best models today available. Clearly the difference between GMM01S and GGM03S, the solid line in light blue, is of comparable size with the accuracy of the less accurate model, i.e., GGM01S, the dashed line in light blue, however the same difference is about one order of magnitude larger than the estimated accuracy of the more accurate model, i.e., GGM03S, the dashed black line.

We further elucidate this point with a very simple example. Suppose we wish to discuss the accuracy in the measurement of the gravitational constant G, we then take one of its latest measured values, let us say, 6.674215 $\times 10^{-11}$ with a relative uncertainty of 0.0015%, whereas the G value measured in the eighteen century was, let us say, about 6.74 $\times 10^{-11}$ with a relative uncertainty of a few percent. Now, in order to evaluate the accuracy of the latest value of G (and this is exactly what the author of Iorio (2008b, 2008c) is doing by comparing in Iorio (2008b, 2008c) even zonal coefficients of different gravity field models with different intrinsic accuracies), we take the difference between these two values. In this way we obtain a relative difference of about 1%. What can one reasonably conclude from this? One may conclude that the older estimate of the uncertainty in G was of the correct order of magnitude but certainly one *cannot* conclude that the 0.0015% uncertainty of the modern measurement of G is wrong by a factor 1000! One cannot seriously difference the older value of the gravitational constant with its newer value to assess the accuracy of its recent measurement! Similarly one cannot seriously compare some obsolete and preliminary Earth gravity models, such as GGM01 and EIGEN-GRACE01S (as this author is doing in Iorio 2008b) with some of the most accurate models obtained with GRACE,[1] such as the latest EIGEN-GRACE and GGM models, used for the Lense-Thirring measurement! Furthermore one cannot seriously compare models obtained with GRACE only with models that use GRACE together other less accurate observations; nevertheless, this is exactly what this author is doing in Iorio (2008b, 2008c).

7.3 Uncertainties in the Higher Degree Even Zonal Harmonics

In Sect. 6, Figs. 9, 10, 11 and 12, we have shown that the total error in the LARES experiment, due to the uncertainties in the even zonal harmonics of degree strictly higher than four, is of the order of one percent only.

Figures 9, 10, 11 and 12 prove indeed that the claims (published in Iorio 2008b, 2008c) that by considering the uncertainties in the even zonal harmonics higher than degree 20, i.e., from degree 20 to degree 70, the error increases by as much as 1000%, are obviously wrong and misleading by at least three orders of magnitude. Indeed, in Iorio (2008b), the author concludes "it turns out that, by using the sigmas of the covariance matrices of some of the latest global Earth's gravity solutions based on long data sets of the dedicated GRACE mission, the systematic bias due to the mismodeled even zonal harmonics up to $l = 70$ will amount to ≈100–1000%", nevertheless, in a more recent paper (Iorio 2008c), these claims have been quite weakened and the author now claims an error that can be as large as 26% using the combination LAGEOS, LAGEOS 2 and LARES: "Straightforward calculations up to degree $l = 60$ with the standard Kaula approach yield errors as large as some tens percent".

Indeed, the results shown in Figs. 9, 10, 11 and 12 have been obtained both by precise analytical propagation of the error of each coefficient in the nodal equation (2) (using Mathematica) and confirmed by orbital propagation (with GEODYN). However, these results can also be easily understood and derived, in order of magnitude, in the following simple way: (a) the uncertainties of each even zonal harmonic published with a GRACE gravity field model are very roughly constant, from degree 12 to degree 60; they may at most change by a factor 5,[2] so it is the difference between different Earth gravity models of comparable accuracy (see Fig. 13 that displays the difference between a number of gravity field models as a function of the degree of each harmonic; see also the above discussion about the proper way of comparing different Earth gravity field model); furthermore, (b) in the nodal rate (2), the size of each even zonal coefficient of degree l roughly decreases as the inverse, $l + 1.5$, power of the satellite semimajor axis, a (or better of the ratio between the Earth radius, $R_\oplus$, and the satellite semimajor axis, a), where the 1.5 power comes from the common coefficient $\frac{n}{a} = \frac{2\pi}{Pa}$ in (2) and P is the satellite orbital period. For example, for $l = 2$ the error corresponding to the term $l = 2$ is proportional to $(R_\oplus/a)^{3.5}$, however for $l = 60$ is proportional to $(R_\oplus/a)^{61.5}$. Now, since the error corresponding to each even zonal harmonic uncertainty can be calculated by multiplying each even zonal uncertainty (very roughly constant from $l = 12$ to $l = 60$) for the corresponding even zonal coefficient in the nodal rate equation (2) (that goes as the inverse, $l + 1.5$, power), the total error due to each even zonal harmonic uncertainty is roughly proportional to the inverse, $l + 1.5$, power of the satellite semimajor axis a. This simply explains the results displayed in Figs. 9, 10, 11 and 12.

[1] The error spectra of most GRACE Earth's gravity models are available at: http://icgem.gfz-potsdam.de/ICGEM.

[2] See, e.g., the EIGEN-GRACE02S calibrated errors at: http://www-app2.gfz-potsdam.de/pb1/op/grace//results/.

7.4 Uncertainties in the Rate of Change of J_4 and of Higher Even Zonal Harmonics

The error due to the uncertainties in the measured rate of change of J_4 and of higher even zonal harmonics, claimed by the same author to be as large as 45%, has been proved by several independent authors (Ciufolini and Pavlis 2005; Ries et al. 2008) to be at the level of a few percent only in the measurement of the Lense-Thirring effect with the satellites LAGEOS and LAGEOS 2. Here, we simply refer to these papers.

8 Measurement of the Orbital Inclination of LAGEOS-type Satellites and Atmospheric Delay Modeling Errors in SLR

In this section we analyze the uncertainty in the measurement of the inclination of the LAGEOS satellites. This measurement uncertainty is mainly due to the atmospheric refraction. As discussed in Sects. 2 and 7.1, the corresponding error in the determination of the Lense-Thirring effect using the LAGEOS satellites is induced by the uncertainty in the *measurement* of the inclination and *not* by the uncertainty in the *modeling* of the inclination (i.e., the uncertainty in the prediction of its behavior).

Atmospheric refraction is an important accuracy-limiting factor in the use of Satellite Laser Ranging (SLR) for high-accuracy science applications. In most of these applications, and particularly for the establishment and monitoring of the Terrestrial Reference Frame, of great interest is the stability of its scale and its implied height system. The modeling of atmospheric refraction in the analysis of SLR data is based on the determination of the delay in the zenith direction and on the subsequent projection to a given elevation angle using a mapping function. Mendes et al. (2002) pointed out some limitations in the Marini–Murray model used in SLR since its introduction in 1973, namely, the modeling of the elevation dependence (the mapping function component of the model), and $a < 1$ mm bias in the computation of the zenith delay. The mapping functions developed by Mendes et al. (2002) represent a significant improvement over the built-in mapping function of the Marini–Murray model and other known mapping functions.

The new mapping functions can be used in combination with any zenith delay model. Mendes et al. (2002) concluded that current zenith delay models have errors at the millimeter level, which increase significantly at 0.355 micrometers, reflecting inadequacy in the dispersion formulae incorporated in these models. A more accurate zenith delay model was developed, applicable to the range of wavelengths used in modern SLR instrumentation (0.355 to 1.064 micrometers) (Mendes and Pavlis 2004). Using a three-dimensional ray-tracing procedure based on globally distributed satellite data from the Atmospheric Infrared Sounder (AIRS) instrument on NASA's AQUA platform, as well as three-dimensional analysis fields from the European Center for Medium Weather Forecasting (ECMWF), Mendes et al. assessed the new zenith delay models and mapping functions both spatially and temporally. They also looked at the magnitude of the horizontal gradient contribution to the total delay by ray-tracing and using a parametric model. Ray-tracing does not depend on any models or mapping functions, it uses a three-dimensional spherical grid that covers Earth from its surface to the top of the atmosphere and generates the atmospheric delay value by following the (non-planar) path of a light ray from the tracking station to the satellite and back. The path of the ray is governed by the local refractive index computed on the basis of the conditions within each three-dimensional cell of the grid (with horizontal and vertical resolution ~50 km).

Mendes et al. used meteorological data sets from NASA's AIRS in order to validate the new zenith delay and mapping function models, and to develop new models that include variations in horizontal refractive indices. The AIRS Level-2 support product gives profiles of temperature, pressure and water vapor from the surface to the top of the atmosphere in 100 standard pressure levels. The pressure levels extend from 1100 mb up to 0.1 mb.

Mendes et al. developed a ray-tracing algorithm specifically tailored for AIRS and ECMWF data in order to calculate the atmospheric delay by directly integrating all the values through which the ray traverses, independent of any mapping function. They used a new formulation for the group refractivity based on formulas by Ciddor (1996) that include both hydrostatic and non-hydrostatic components of the group refractivity. In order to perform the ray-tracing, the data are first processed and grouped into 10×10 degree latitude/longitude grids up to 0.1 mb in order to build three-dimensional atmospheric profiles around each operational ILRS-SLR tracking station. AIRS-based Ray-Tracing (ART) and ECMWF-based Ray-Tracing (ERT) were used to calculate both the total atmospheric delay as well as the delay due to horizontal gradients in refractivity. Given the independent accuracy assessment of the simple, closed-formula models in use today, (e.g. Marini–Murray), the effect of the horizontal gradients (which is not accounted for in these models), is the largest remaining error in refraction delay modeling today. If we can quantify the level of that unaccounted error induced on the orbit of LAGEOS due to neglecting the horizontal gradients, we can then place an upper bound on the maximum error in our Lense-Thirring estimate. Using the three-dimensional ray-tracing approach which *fully* accounts for the *total* atmospheric delay including the gradients, we analyzed a few years of LAGEOS and LAGEOS 2 data as a test case. Comparing the results of these reductions to those obtained using the standard model (Mendes–Pavlis without gradients), we showed that the most significant result was a reduction in the variance of the observation residuals up to 25%, with only random and insignificant differences in the orbits. This was intuitively expected since atmospheric refraction does not enter the dynamical model and it is the most significant outcome of implementing the new approach in calculating the atmospheric delay using the three-dimensional ray-tracing methodology. With insignificant, random and no secular differences in the orbits, the effect on the estimated Lense-Thirring parameter is also equally insignificant and of no further concern for the Lense-Thirring experiment.

Since refraction does not enter the dynamical model being a media propagation effect on the tracking data, one would expect a priori that the effects on the orbit would be very small, much smaller than the actual variations seen in the tracking data residuals that absorb the majority of refraction errors. This is indeed the case and it can be seen very clearly in the statistics displayed in Figs. 14, 15, typical examples of the many cases of LAGEOS and LAGEOS 2 arcs that we examined.

From Fig. 14 we can estimate that on LAGEOS, on the average, the effect on inclination is at the 30 μarcseconds (0.03 milliarcseconds) level with a comparable standard deviation. For comparison purposes notice that 30 μarcseconds at LAGEOS altitude are only <1.5 mm. On the LAGEOS orbit, a 30 μarcseconds measurement uncertainty in the inclination corresponds (from the nodal rate (2) of LAGEOS due to the even zonal harmonics) to a nodal rate of 0.6% of the Lense-Thirring effect.

Very similar conclusions are reached from inspection of Fig. 15, displaying the results of the analysis of several years of older data taken on LAGEOS 2. We notice here that from the analysis of three years of data we again see a very minor effect at the level of about 10 μarcseconds on the average in either the inclination or the node of LAGEOS 2, with an even smaller standard deviation than the one for LAGEOS. This is explained by the fact that in recent years (after 2004) the network has allowed tracking at lower elevations below the original 20° minimum and all the way down to 10°. Refraction errors drop significantly at higher elevations and since the preponderance of the LAGEOS 2 data in these early years is

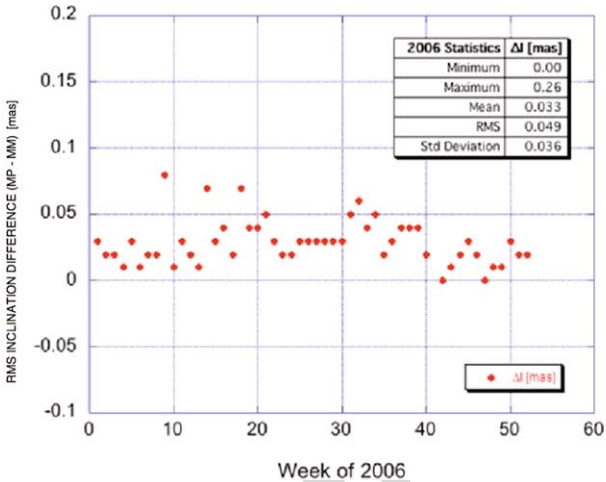


Fig. 14 Weekly arc RMS variation (RMS Difference, Marini–Murray minus Mendes–Pavlis, in mas (milliarcsec)) in the inclination fits of LAGEOS data reduced with two different atmospheric delay models, the Marini–Murray and the Mendes–Pavlis

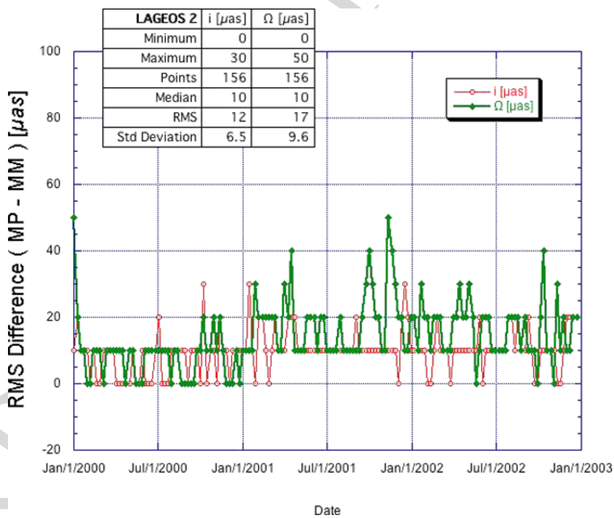


Fig. 15 Weekly arc RMS variation (RMS Difference, Marini–Murray minus Mendes–Pavlis, in μarcsec (microarcsec)) in the inclination and node fits of LAGEOS 2 data reduced with two different atmospheric delay models, the Marini–Murray and the Mendes–Pavlis

at elevations above 20°, the discrepancy between the two orbits is less affected by the much smaller refraction errors. On the LAGEOS 2 orbit, a 10 μarcsecond measurement uncertainty in the inclination corresponds (from the nodal rate (2) of LAGEOS 2 due to the even zonal harmonics) to a nodal rate of 0.36% of the Lense-Thirring effect.

Final tests were performed with the direct application of three-dimensional atmospheric ray-tracing as detailed in Hulley and Pavlis (2007), where two years of data were reduced with atmospheric delay corrections that are obtained with this approach and are thus free of any error in the zenith delay or the mapping function used in the model. Furthermore, three-dimensional AIRS-based Ray-Tracing includes automatically the effect of the horizontal gradients as explained ibidem, and what these tests demonstrated is that indeed, as expected, the errors in the atmospheric delay modeling are absorbed by the residuals of the individual stations. The use of the superior 3D ART approach results in a significant variance reduction in the station residuals, whether the meteorological information comes from the AIRS global fields or the ECMWF assimilation fields, which in recent years begun using the AIRS data as part of their standard input for their assimilation scheme.

In conclusion, on the combination of the nodes of LAGEOS and LAGEOS 2 (to measure the Lense-Thirring effect) the error due to inclination measurement uncertainties is of the order of 0.5% and similarly for the LARES + LAGEOS + LAGEOS 2 experiment.

9 Conclusions

We have described the measurements of frame-dragging by the Earth spin with the satellites LAGEOS and LAGEOS 2 using the Earth's gravity models obtained by the GRACE spacecraft. These measurements of Earth gravitomagnetism have confirmed the general relativistic prediction of the Lense-Thirring effect with an accuracy of about 10% and have been recently *independently* repeated by the group of the Center for Space Research of University of Texas at Austin that reported an accuracy of about 12%. We have then introduced the LARES satellite to be launched in 2010 by the Italian Space Agency for a measurement of frame-dragging with an accuracy of a few percent.

Our detailed analyses and the agreement of the independent results of the four groups: University of Salento and University of Rome, University of Maryland, GFZ of Potsdam and Center for Space Research of the University of Texas at Austin, confirm our error budget of the LAGEOS-LAGEOS 2 and LARES frame-dragging measurements.

These independent results demonstrate that the claims a single individual, that the LAGEOS-LAGEOS 2 and LARES frame-dragging error budget has been underestimated, are based on erroneous analyses; we have then pointed out the misunderstandings and miscalculations on which these claims are based. We have indeed analyzed and precisely quantified a number of conceivable error sources, e.g., the uncertainties in higher Earth's even zonal harmonics, up to degree 70, and the uncertainties in the orbital elements of the LAGEOS satellites, including the orbital inclination. We have also shown the correct way to compare different Earth's gravity field models obtained with GRACE and discussed the reliability of the published uncertainties of the GRACE models.

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References

J.I. Andrès et al., J. Geophys. Res. **109**, B06403 (2004)
N. Ashby, B. Shahid-Saless, Geodetic precession or dragging of inertial frames? Phys. Rev. D **42**, 1118–1122 (1990)
B.M. Barker, R.F. O'Connel, Gen. Relativ. Gravit. **11**, 149–175 (1979)
P.L. Bender et al., The lunar laser ranging experiment. Science **182**, 229–238 (1973)
B. Bertotti, I. Ciufolini, P.L. Bender, New test of general relativity: measurement of de Sitter geodetic precession rate for lunar perigee. Phys. Rev. Lett. **58**, 1062–1065 (1987)
P.E. Ciddor, Refractive index of air: New equations for the visible and near infrared. Appl. Opt. **35**(9), 1566–1573 (1996)
I. Ciufolini, Theory and experiments in General Relativity and other metric theories, Ph.D. Dissertation, Univ. of Texas, Austin, Pub. Ann Arbor, Michigan, 1984
I. Ciufolini, Measurement of the Lense-Thirring drag on high-altitude laser-ranged artificial satellites. Phys. Rev. Lett. **56**, 278 (1986)
I. Ciufolini, A comprehensive introduction to the Lageos gravitomagnetic experiment: from the importance of the gravitomagnetic field in physics to preliminary error analysis and error budget. Int. J. Mod. Phys. A **4**, 3083 (1989); see also Tapley et al. (1989)

I. Ciufolini, Gravitomagnetism and status of the LAGEOS III experiment. Class. Quantum Gravity **11**, A73–A81 (1994)
I. Ciufolini, On a new method to measure the gravitomagnetic field using two orbiting satellites. Nuovo Cim. A **109**, 1709 (1996)
I. Ciufolini, The 1995–99 measurements of the Lense-Thirring effect using laser-ranged satellites. Class. Quantum Gravity **17**, 2369 (2000)
I. Ciufolini, in *Proceedings of the I SIGRAV School on General Relativity and Gravitation*, World Scientific, Frascati (Rome), September 2002
I. Ciufolini, On the orbit of the LARES satellite (2006). arXiv:gr-qc/0609081v1
I. Ciufolini, Dragging of inertial frames. Nature **449**, 41–48 (2007a)
I. Ciufolini, The design of LARES: a satellite for testing General Relativity, in *Proc. 58th International Astronautical Congress*, Hyderabad, India, 24–28 September 2007 (2007b)
I. Ciufolini, Frame-dragging, gravitomagnetism and lunar laser ranging. New Astron. (2009). doi:10.1016/j.newast.2009.08.004
I. Ciufolini, E.C. Pavlis, A confirmation of the general relativistic prediction of the Lense-Thirring effect. Nature **431**, 958–960 (2004)
I. Ciufolini, E.C. Pavlis, On the measurement of the Lense-Thirring effect using the nodes of the LAGEOS satellites, in reply to "On the reliability of the so-far performed tests for measuring the Lense-Thirring effect with the LAGEOS satellites" by L. Iorio. New Astron. **10**(8), 636–651 (2005)
I. Ciufolini, J.A. Wheeler, *Gravitation and Inertia* (Princeton Univ. Press, Princeton, 1995)
I. Ciufolini et al., ASI-NASA Study on LAGEOS III, CNR, Rome, Italy, 1989
I. Ciufolini et al., Effect of particle drag on the LAGEOS node and measurement of the gravitomagnetic field. Nuovo Cim. B **105**, 573 (1990)
I. Ciufolini, F. Chieppa, D. Lucchesi, F. Vespe, Test of Lense-Thirring orbital shift due to spin. Class. Quantum Gravity **14**, 2701 (1997a)
I. Ciufolini, D. Lucchesi, F. Vespe, F. Chieppa, Measurement of gravitomagnetism. Europhys. Lett. **39**, 359 (1997b)
I. Ciufolini, A. Paolozzi et al., LARES phase. A study for the Italian Space Agency, 1998a
I. Ciufolini, E.C. Pavlis, F. Chieppa, E. Fernandes-Vieira, J. Perez-Mercader, Test of general relativity and measurement of the Lense-Thirring effect with two Earth satellites. Science **279**, 2100 (1998b)
I. Ciufolini et al., Italian Space Agency Phase A Report on LARES, Italian Space Agency, ASI, Rome, 1998c
I. Ciufolini et al., INFN study on LARES/WEBER-SAT, 2004
I. Ciufolini, E.C. Pavlis, R. Peron, Determination of frame-dragging using Earth gravity models from CHAMP and GRACE. New Astron. 527–550 (2006)
I. Ciufolini et al. (eds.), Gravitomagnetism and its measurement with laser ranging to the LAGEOS satellites and GRACE Earth gravity models, in *Proc. of the First International School of Astrophysical Relativity "John Archibald Wheeler": Frame-Dragging, Gravitational-Waves and Gravitational Tests*, Erice, Italy, 2006 (Springer, Berlin, 2009a)
I. Ciufolini et al. (ed.), The LARES space experiment: LARES orbit, error analysis and satellite structure, in *Proc. of the First International School of Astrophysical Relativity "John Archibald Wheeler": Frame-Dragging, Gravitational-Waves and Gravitational Tests*, ed. by I. Ciufolini, R. Matzner, Erice, Italy, 2006 (Springer, Berlin, 2009b)
I. Ciufolini, A. Anselmo et al., Effect of particle drag in the LARES experiment (2009c, to be published)
S.C. Cohen, P.J. Dunn, LAGEOS scientific results. J. Geophys. Res. B **90**, 9215–9438 (1985)
L. Cugusi, E. Proverbio, Relativistic effects on the motion of Earth's artificial satellites. Astron. Astrophys. **69**, 321–325 (1978)
W. Cui et al., Astrophys. J. **492**, L53–L58 (1998)
G. Felici, The meaning of systematic errors, a comment to "Reply to On the systematic errors in the Detection of the Lense-Thirring effect with a Mars orbiter", by L. Iorio (2007). arXiv:gr-qc/0703020v1
R.S. Gross, J. Geophys. Res. **101**(B4), 8729 (1996)
G.C. Hulley, E.C. Pavlis, A ray-tracing technique for improving Satellite Laser Ranging atmospheric delay corrections, including the efects of horizontal refractivity gradients. J. Geophys. Res. **112**, B06417 (2007). doi:10.1029/2006JB004834
International Earth Rotation Service (IERS) Annual Report, 1996. Observatoire de Paris, Paris, July 1997
L. Iorio, On the reliability of the so-far performed tests for measuring the Lense-Thirring effect with the LAGEOS satellites. New Astron. **10**, 603–615 (2005a)
L. Iorio, The impact of the new Earth gravity models on the measurement of the Lense-Thirring effect with a new satellite. New Astron. **10**, 616–635 (2005b)
L. Iorio, Evidence of the gravitomagnetic field of Mars. Class. Quantum Gravity **23**, 5451–5454 (2006)
L. Iorio, On some critical issues of the LAGEOS/LAGEOS II Lense-Thirring experiment (2007). arXiv:0710.1022v1 [gr-qc]

L. Iorio, Will the recently approved LARES mission be able to measure the Lense-Thirring effect at 1%? Gen. Relativ. Gravit. (2008b). doi:10.1007/s10714-008-0742-1; see also: arXiv:0803.3278v5 [gr-qc]
L. Iorio, On the impact of the atmospheric drag on the LARES mission. arXiv:0809.3564v2 (2008a); see also: arXiv:0809.3564v1
L. Iorio, An assessment of the systematic uncertainty in present and future tests of the Lense-Thirring effect with satellite laser ranging (2008c). arXiv:0809.1373v2 [gr-qc]
W.M. Kaula, *Theory of Satellite Geodesy* (Blaisdell, Waltham, 1966)
A.R. Khan, R.F. O'Connell, Nature **261**, 480 (1976)
S.M. Kopeikin, Phys. Rev. Lett. **98**, 229001 (2007)
K. Krogh, Iorio's "high-precision measurement" of frame-dragging with the Mars Global Surveyor. Class. Quantum Gravity **24**, 5709 (2007)
D.M. Lucchesi, Reassessment of the error modelling of non-gravitational perturbations on LAGEOS 2 and their impact in the Lense-Thirring determination. Part I. Planet. Space Sci. **49**, 447 (2001)
D.M. Lucchesi, Reassessment of the error modelling of non-gravitational perturbations on LAGEOS II and their impact in the Lense-Thirring determination. Part II. Planet. Space Sci. **50**, 1067–1100 (2002)
D.M. Lucchesi, The impact of the even zonal harmonics secular variations on the Lense-Thirring effect measurement with the two Lageos satellites. Int. J. Mod. Phys. D **14**, 1989–2023 (2005)
D.M. Lucchesi, A. Paolozzi, A cost effective approach for LARES satellite, in *XVI AIDAA*, Palermo, September 2001, pp. 1–14. Paper no. 111
C.F. Martin, D.P. Rubincam, J. Geophys. Res. **101**(B2), 3215 (1996)
V.B. Mendes, E.C. Pavlis, High-accuracy zenith delay prediction at optical wavelengths. Geophys. Res. Lett. **31**, L14602 (2004). doi:10.1029/2004GL020308
V.B. Mendes, G. Prates, E.C. Pavlis, D.E. Pavlis, R.B. Langley, Improved mapping functions for atmospheric re-fraction correction in SLR. Geophys. Res. Lett. **29**(10), 1414 (2002). doi:10.1029/2001GL014394
C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973)
T.W. Murphy Jr., K. Nordtvedt, S.G. Turyshev, Phys. Rev. Lett. **98**, 071102–1-4 (2007a)
T.W. Murphy Jr., K. Nordtvedt, S.G. Turyshev, Phys. Rev. Lett. **98**, 229002 (2007b)
R. Noomen, S. Klosko, C. Noll, M. Pearlman (eds.), Toward millimeter accuracy in NASA CP 2003-212248, in *Proc. 13th Int. Laser Ranging Workshop* (NASA Goddard, Greenbelt, 2003)
K. Nordtvedt, Int. J. Theor. Phys. **27**, 1395–1404 (1988)
K. Nordtvedt, LARES and tests on new long ranges forces, in *LARES phase. A study for the Italian Space Agency* (1998), pp. 34–38
R.F. O'Connell, A note on frame dragging. Class. Quantum Gravity **22**, 3815–3816 (2005)
A. Paolozzi, private communication (2005)
E.C. Pavlis, Geodetic contributions to gravitational experiments in space, in *Recent Developments in General Relativity*, ed. by R. Cianci et al., Genoa, 2000 (Springer, Berlin, 2000), pp. 217–233
D.E. Pavlis et al., GEODYN Operations Manuals, Contractor Report, Raytheon, ITSS, Landover, MD, 1998
G.E. Peterson, Estimation of the Lense-Thirring precession using laser-ranged satellites, Ph.D. Dissertation, Univ. of Texas, Austin, 1997
Ch. Reigber, F. Flechtner, R. Koenig, U. Meyer, K. Neumayer, R. Schmidt, P. Schwintzer, S. Zhu, GRACE orbit and gravity field recovery at GFZ Potsdam—-first experiences and perspectives. Eos. Trans. AGU **83**(47) (2002). Fall Meet. Suppl., Abstract G12B-03
C. Reigber, R. Schmidt, F. Flechtner, R. Konig, U. Meyer, K.H. Neumayer, P. Schwintzer, S.Y. Zhu, J. Geodyn. **39**, 1–10 (2005)
J. Ries, private communication (2005)
J.C. Ries, Simulation of an experiment to measure the Lense-Thirring precession using a second LAGEOS satellite, Ph.D. dissertation, The University of Texas, Austin, 1989
J.C. Ries, R.J. Eanes, B.D. Tapley, Lense-Thirring precession determination from laser ranging to artificial satellites, in *Proc. III William Fairbank Meeting* (World Scientific, Singapore, 2003a), p. 201
J.C. Ries, R.J. Eanes, B.D. Tapley, G.E. Peterson (eds.), Prospects for an improved Lense-Thirring test with SLR and the GRACE gravity mission, in toward millimeter accuracy, in *Proc. 13th Int. Laser Ranging Workshop*, Report NASA CP 2003-212248, NASA Goddard, Greenbelt, Maryland, 2003b
J.C. Ries, R.J. Eanes, M.M. Watkins, Confirming the frame-dragging effect with satellite laser ranging, in *16th International Workshop on Laser Ranging*, Poznan, Poland, October 2008, pp. 13–17
D.P. Rubincam, General relativity and satellite orbits: the motion of a test particle in the Schwarzschild metric. Celest. Mech. **15**, 21–33 (1977)
D.P. Rubincam, J. Geophys. Res. **93**(B11), 13803 (1988)
D.P. Rubincam, Drag on the LAGEOS satellite. J. Geophys. Res. B **95**, 4881–4886 (1990)
D.P. Rubincam, A. Mallama, J. Geophys. Res. **100**(B10), 20285 (1995)
C. Schmid, Phys. Rev. D **74**, 044031 (2006)

G. Sindoni, C. Paris, P. Ialongo, On the systematic errors in the detection of the Lense-Thirring effect with a Mars orbiter (2007). arXiv:gr-qc/0701141
I.H. Stairs, S.E. Thorsett, Z. Arzoumanian, Measurement of gravitational spin-orbit coupling in a binary-pulsar system. Phys. Rev. Lett. **93**, 141101 (2004)
B.D. Tapley, The GRACE mission: status and performance assessment. Eos. Trans. AGU **83**(47) (2002). Fall Meet. Suppl., Abstract G12B-01
B. Tapley, J.C. Ries, R.J. Eanes, M.M. Watkins, *NASA-ASI Study on LAGEOS III*, CSR-UT publication n. CSR-89-3, Austin, Texas, 1989
B. Tapley, J. Ries, S. Bettadpur, D. Chambers, M. Cheng, F. Condi, S. Poole, The GGM03 mean Earth gravity model from GRACE, Eos Trans. AGU **88**(52) (2007). Fall Meet.Suppl., Abstract G42A-03
K.S. Thorne, R.H. Price, D.A. Macdonald, *The Membrane Paradigm* (Yale Univ. Press, New Haven, 1986)
R.A. Van Patten, C.W.F. Everitt, Possible experiment with two counter-orbiting drag-free satellites to obtain a new test of Einstein's General Theory of Relativity and improved measurements in geodesy. Phys. Rev. Lett. **36**, 629–632 (1976)
M. Watkins, D. Yuan, W. Bertiger, G. Kruizinga, L. Romans, S. Wu, GRACE gravity field results from JPL. Eos. Trans. AGU **83**(47) (2002). Fall Meet. Suppl., Abstract G12B-02
S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (Wiley, New York, 1972)
J.M. Weisberg, J.H. Taylor, General relativistic geodetic spin precession in binary pulsar B1913+16: mapping the emission beam in two dimensions. Astrophys. J. **576**, 942–949 (2002)
J.G. Williams, X.X. Newhall, J.O. Dickey, Relativity parameters determined from lunar laser ranging. Phys. Rev. D **53**, 6730–6739 (1996)
J.G. Williams, S.G. Turyshev, D.H. Boggs, Phys. Rev. Lett. **93**, 261101-1-4 (2004a)
J.G. Williams, S.G. Turyshev, T.W. Jr. Murphy, Improving LLR tests of gravitational theory. Int. J. Mod. Phys. D **13**, 567–582 (2004b)
H. Yilmaz, Proposed test of the nature of gravitational interaction. Bull. Am. Phys. Soc. **4**, 65 (1959)