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Recurrence of large earthquakes: Bayesian inference from catalogs in the presence of magnitude uncertainties

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Abstract. We present a Bayesian method that allows to continuously update the aperiodicity of the recurrence time distribution of large earthquakes based on a catalog with magnitudes above a completeness threshold. The approach uses a recently proposed renewal model for seismicity and allows to include magnitude uncertainties in a straightforward manner. Errors accounting for grouped magnitudes and random errors are studied and discussed. The results indicate that a stable and realistic value of the aperiodicity can be predicted in an early state of seismicity evolution, even though only a small number of large earthquakes has occurred so far. Furthermore, we demonstrate that magnitude uncertainties can have drastic influence on the results and can therefore not be neglected. We show how to correct for the bias caused by magnitude errors. For the region of Parkfield we find that the aperiodicity, or the coefficient of variation, is clearly higher than in studies which are solely based on the large earthquakes.

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Introduction

The extraction of information from earthquake catalogs is a puzzle. Especially, if one focuses on the largest events, the data become sparse and thus statistical properties of only the largest events will become increasingly unstable. The emergence of enormous uncertainties makes the situation even worse. In order to overcome this problem, numerical models are useful for both, the understanding of the physics of earthquakes and the test of the feasibility of statistical methods. In particular, if a model resembles main features of observed seismicity, it may become capable for purposes of data assimilation and finally for forecasting the next big event.

The question of earthquake recurrence is one of the key problems in seismic hazard assessment. Even in fault regions that are governed by overall periodic recurrence of large events, considerable deviations from the periodicity are observed, e.g. on the Parkfield segment of the San Andreas fault, where the most recent $M6$ earthquake in 2004 came with about 16 years delay from the expected time of occurrence. From observations alone it is impossible to judge, whether such fluctuations are the rule or the exception. There are mainly two approaches to deal with this situation: the first one aims at modeling the evolution of stress and seismicity with physical models. However, the more realistic these models are, the more complicated they become and the more adjustable parameters are involved. The second approach uses statistical methods, particularly the fitting of preselected distributions to a small number of data. Although some of the distributions may have a physical background, like the Weibull distribution (*Weibull*, 1951), the applicability to the earthquake problem remains questionable. In a recent work, *Zöller et al.* (2008) have shown that the Brownian passage time distribution covers a broad range of scenarios as a consequence of the central limit theorem. Furthermore, it has been demonstrated, how stress changes from small and intermediate earthquake can cause the advanced or delayed occurrence of the next large event. Imposing empirical relations of earthquake statistics like the Gutenberg-Richter law (*Gutenberg and Richter*, 1956) for the frequency-size distribution allows to provide robust estimates of the parameters of the Brownian passage time distribution and makes the fitting of distributions to a small number of large earthquakes unnecessary. The analysis of a long catalog of synthetic earthquakes shows that the method leads to reasonable estimates of the coefficient of variation describing the degree of periodicity of large earthquakes. However, in practical applications, earthquake recordings are added in time and parameters have to be recalculated continuously. For this purpose, we derive a Bayesian approach in order to update the coefficient of variation based on the model of *Zöller et al.* (2008). Finally, we discuss the influence of emerging uncertainties on the results. Therefore, we consider the true magnitude to be a random variable drawn from a given probability distribution.

Method

The methodology is decomposed into three parts: first, the fractal oscillator; second, the calculation and maintance of the aperiodicity and third, the imposed magnitude uncertainties.

The fractal oscillator model

We give a brief review of the method to estimate the aperiodicity from an earthquake catalog. A comprehensive description is provided in *Zöller et al.* (2008).

The backbone of the method is the "fractal oscillator" (FO), a modification of the Brownian relaxation oscillator (BRO) proposed by *Matthews et al.* (2002) and extended by *Zöller and Hainzl* (2007), which serves as a model for recurrent large earthquakes in a fault region. Here we deal with stochastic models for seismicity on long time scales. Instead of simulating the whole earthquake process deterministically on all scales, we focus on the recurrence of the largest earthquakes and plug all small-scale processes into a stochastic component. This includes the occurrence of aftershocks and background seismicity, aseismic stress release and other effects. In *Zöller et al.* (2008), we have shown that the evolution of the loading and unloading characteristics in the fault region can be effectively the same as a stochastic variable with drift. In this study, we follow this idea and assume that a strictly periodic recurrence of large earthquakes according to Reid's elastic rebound theory is corrected by the loading and unloading processes between two large events. Although the stochastic modeling of the subscale processes is certainly oversimplified with respect to the details of the underlying physics, we claim that the evolution of seismic release is reasonable in a statistical sense, e.g. on long time scales. Therefore, stochastic models serve as a powerful tool when dealing with problems related to seismic hazard.

Both models the fractal oscillator and the Brownian relaxation oscillator introduce a load state variable τ representing seismic moment release. The temporal evolution of τ is governed by two processes: first, a linear increase modeling the tectonic plate motion; second, random fluctuations accounting for small scale processes, especially small earthquakes. While the BRO imposes Gaussian random fluctuations for the latter component, the FO uses doubly-truncated fractal fluctuations mimicking Gutenberg-Richter type seismicity (*Gutenberg and Richter*, 1956). A large earthquake occurs, as soon as τ reaches a predefined static threshold τ_s. Such a threshold dynamics is widely accepted in earthquake modeling, see e.g. *Ben-Zion and Rice* (1993) and *Zöller et al.* (2004), where earthquakes of all sizes are initiated, as soon as a threshold (given by the static friction) is reached. In this study, the failure criterion only applies for the largest earthquakes, while the smaller ones are associated with the stochastic fluctuations. Thus, from the modeling perspective, our model incorporates two types of earthquakes: first a "large" earthquake, with a size depending mainly on the dimension of the fault zone, and second, intermediate and small earthquakes following Gutenberg-Richter statistics, where the magnitudes can be calculated from the seismic release (stochastic component).

Mathematically, the calculation of the time to the next large event is a first passage time problem of Gaussian or fractal fluctuations with constant drift (tectonic loading). For the BRO, the solution of the first passage time problem is the well-known Brownian passage time distribution; for a detailed description see *Redner* (2001)). However, assuming a large number of fluctuations during a seismic cycle in the FO, the sum of the fluctuations will also converge to Gaussian noise because of the central limit theorem. Consequently, the recurrence time distribution of this model is also the Brownian passage time distribution,

$$f(t;\mu_T,\alpha) = \sqrt{\frac{\mu_T^3}{2\pi\alpha^2 t^3}} \exp\left\{-\frac{(t-\mu_T)^2}{2\mu_T\alpha^2 t}\right\} \tag{1}$$

where μ_T is the mean recurrence time of large earthquakes, and the aperiodicity α is equal to the coefficient of variation

$$c_V = \alpha = \frac{\sigma_T}{\mu_T} \tag{2}$$

with the standard deviation σ_T of the recurrence times.

The recurrence time distribution for a specific fault zone may be obtained by fitting Eq. (2) to the large earthquakes from the catalog. However, the results are expected to be unstable because of the small number of large earthquakes. As an example, we refer to the Parkfield segment, which is probably the best monitored seismically active region in the world: the Parkfield catalog includes only seven $M6$ events leading to six recurrence times.

In their FO model, *Zöller et al.* (2008) have shown that c_V for large earthquakes (magnitude m_{main}) can be related to the Richter-b value in the limit case of an infinite number of intermediate and small Gutenberg-Richter distributed earthquakes with $m \in [-\infty; m_{main}]$ within a seismic cycle:

$$c_V^0 = \sqrt{\frac{b}{3-b}}. \tag{3}$$

However, deviations from the scaling behavior for high magnitudes are observed frequently in natural seismicity. Apart from statistical fluctuations, the observation of characteristic earthquake behavior is probably related to physical conditions in a specific fault region. To account for this case, a corrected value is given by

$$c_V = c_V^0 \cdot \sqrt{\frac{N_{exp}}{N_{real}}} \tag{4}$$

for $N_{real} \geq N_{exp}$. Here c_V^0 can be calculated from a maximum likelihood estimation of the b value in the scaling regime $[m_0; m_1]$ using Eq. (3). The value N_{exp} is the number of *expected* large earthquakes from the Gutenberg-Richter law $\log N_{exp} = a - bm_{main}$ in $[m_0; m_1]$, while N_{real} is the real number of large earthquakes in the catalog. Thus the relation for c_V includes both, information on the large earthquakes and on the intermediate and small earthquakes.

Bayesian inference

Bayes' theorem provides an elegant way to update a given *a priori* probability distribution $p_0(x)$ of a random variable x, if new observations $X_1, \ldots, X_n$ of x become available (see e.g. *Zöller et al.*, 2007). The conditional *a posteriori* distribution $p_1(x|X_1, \ldots, X_n)$ is then given according to Bayes' theorem by

$$p_1(x|X_1, \ldots, X_n) = \frac{L(X_1, \ldots, X_n|x)}{\int_0^\infty L(X_1, \ldots, X_n|x) p_0(x) dx} \cdot p_0(x), \tag{5}$$

where L denotes the likelihood function. Further observations $X_{n+1}, \ldots, X_{n+m}$ of x can be accounted for by using $p_1(x|X_1, \ldots, X_n)$ as a new prior and calculating the new posterior $p_2(x|X_1, \ldots, X_{n+m})$ with Eq. (5).

In many practical applications, the main difficulty is to find the likelihood function. However, in this study the calculation of the likelihood function becomes straightforward, if the Gutenberg-Richter law is considered to be true. With the magnitude m as the random variable (x in Eq. 5), the Gutenberg-Richter density is

$$p_\beta(m) = \beta e^{-\beta(m-m_0)} \tag{6}$$

with the abbreviation $\beta = b \ln(10)$ and the minimum magnitude m_0. Here, the Gutenberg-Richter law is only truncated for small magnitudes accounting for limited observational capacities. The additional truncation in the FO model for large magnitudes corresponds to the separation into small and large earthquakes. The likelihood function for n earthquakes with magnitudes $m_1, \ldots, m_n$ is thus

$$L(m_1, \ldots, m_n|\beta) = \prod_i p_\beta(m_i). \tag{7}$$

and can be written as

$$L(m_1, \ldots, m_n|\beta) = \beta^n \exp\left(-n \, \frac{\beta}{\hat{\beta}}\right) \tag{8}$$

with and $\hat{\beta} = (\langle m \rangle - m_0)^{-1}$, where $\langle m \rangle$ is the mean magnitude in the catalog. Maximizing $L(m_1, \ldots, m_n|\beta)$ with respect to β leads to the formula of *Aki* (1965),

$$\beta_{ML} = \hat{\beta} = \frac{1}{\langle m \rangle - m_0}. \tag{9}$$

Equations (5) and (7) allow to continuously update the Richter b value with a growing earthquake catalog. Finally, the probability density function of the b value has to be transformed to those of the coefficient of variation (Eq. 4). Since $c_V^0 = f(b) = \sqrt{b/(3-b)}$ is a monotonic function, the probability density of $c_V^0(b)$ is given by

$$p(c_V^0) = \left| \frac{1}{f'(f^{-1}(c_V^0))} \right| \cdot p(f^{-1}(c_V^0)) \tag{10}$$

leading to

$$p(c_V^0) = \frac{6c_V^0}{\left(1 + (c_V^0)^2\right)^2} \cdot p(b(c_V^0)). \tag{11}$$

In order to mimic a realistic situation, we consider c_V as a function of time t by using only catalog information for times $t' \leq t$ from the past.

Magnitude uncertainties

Information in earthquake catalogs are usually accomplished by uncertainties. In a first approach these uncertainties may be neglected. However, magnitude errors due to rounding (*Rhoades*, 1996), systematic errors, and random errors (*Tinti and Mulargia*, 1985) can have significant influence on the calculation of seismological parameters, e.g. the b value. As a simple illustration, a uniformly distributed magnitude error of $\Delta m = 0.05$ for all earthquakes in a given catalog with n events, causes a possible increase of the "true" catalog size by a factor up to 12% ($10^{b\Delta m}$), assuming the Gutenberg-Richter law with $b = 1$. Here we consider errors drawn from a uniform distribution of random numbers in a finite interval centered around the true magnitude. The interval length 0.1, 0.01 etc. account for rounding errors or grouped magnitudes.

In detail, we assume that the probability to observe an earthquake with magnitude m, given that the *true* magnitude is m', is a box function with half-width δ around m':

$$f(m - m') = \begin{cases} \frac{1}{2\delta} & \text{for } |m - m'| < \delta \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

Despite its simplicity, a box function is a reasonable choice because of the finite width. In contrast, Gaussian distributed uncertainties (*Tinti and Mulargia*, 1985) allow for the occurrence of unrealistic high values. Assuming that the true magnitudes follow a Gutenberg-Richter law, the probability density for the *observed* magnitudes is then given by the convolution of function f with the Gutenberg-Richter density $p_\beta(m)$:

$$P_\beta^\delta(m) = (p_\beta \star f)(m) = \int_{-\infty}^{\infty} p(m')f(m-m')dm' = \frac{\beta e^{-\beta\delta}}{2\delta} \int_{m-\delta}^{m+\delta} e^{-\beta(m'-m_0)}dm'. \tag{13}$$

It is important to note that the factor $e^{-\beta\delta}$ accounts for the normalization, because the magnitude can take values between $m_0 - \delta$ and ∞ (instead of $m \in [m_0; \infty]$ in the case without uncertainties). This approach assumes a Gutenberg-Richter model, where earthquakes with $m < m_0 - \delta$ are impossible, which is, at least in first order, reasonable. However, in a more refined analysis, the absence of events with $m < m_0 - \delta$ in a catalog will be expressed by a measurement function, while the Gutenberg-Richter model will be valid for all magnitudes. This study is left for future work.

Calculating the integral in Eq. (13) leads to

$$P_\beta^\delta(m) = \frac{\sinh(\beta\delta)}{\delta} e^{-\beta(m-m_0+\delta)} = \mathrm{sinhc}(\beta\delta)e^{-\beta\delta} \cdot p_\beta(m). \tag{14}$$

The likelihood function is obtained by replacing the distribution p in Eq. (7):

$$L(m_1, \ldots, m_n|\beta) = \prod_i P_\beta^\delta(m_i). \tag{15}$$

In analogy with Eq. (8), the liklihood can be written as

$$L(m_1, \ldots, m_n|\beta) = \left(\frac{\sinh(\delta\beta)}{\delta}\right)^n e^{-n\beta(\langle m\rangle - m_0 + \delta)}. \tag{16}$$

Maximizing $L(m_1, \ldots, m_n|\beta)$ with respect to β leads to

$$\frac{\delta}{\tanh(\delta\beta_{ML})} = \langle m\rangle - m_0 + \delta. \tag{17}$$

A similar formula has been suggested by *Utsu* (1966) without correcting the minimum magnitude by subtracting δ. This problem has been reported by *Marzocchi and Sandri* (2003); they suggest the corrected formula for the maximum likelihood estimation $\beta_{ML} = (\langle m\rangle - m_0 + \delta)^{-1}$, which is different from Eq. (17) and has also been used by Utsu himself (*Utsu*, 1999; *Utsu*, 2002). However, for $\delta\beta \ll 1$, the tangens hyperbolicus can be approximated by the linear term of its Taylor expansion:

$$\tanh(\delta\beta_{ML}) \approx \delta\beta_{ML} = \frac{\delta}{\langle m\rangle - m_0 + \delta}; \tag{18}$$

in this approximation, the formula of *Marzocchi and Sandri* (2003) is in agreement with the Eq. (17). For most practical purposes, however, especially for grouped magnitudes with $\delta = 0.05$ or $\delta = 0.005$ and realistic b values ($b \approx 1$), the approximation in Eq. (18) is sufficient, e.g. for an uncorrected value

$b_0 = (\langle m \rangle - m_0)^{-1} = 1$ and magnitudes rounded to one decimal place ($\delta = 0.05$), the change of the corrected b value arising from the approximation in Eq. (18) is only $\Delta b_{ML} = 0.003$.

We claim that an (uncorrected) maximum likelihood estimation $\beta_0 = (\langle m \rangle - m_0)^{-1}$ calculated from a catalog with magnitude errors according to Eq. (12) is biased; the corrected value based on Eq. (17) is

$$\beta_{ML} = \frac{1}{\delta} \tanh^{-1}\left(\frac{\delta\beta_0}{1+\beta_0\delta}\right) \approx \frac{\beta_0}{1+\beta_0\delta}. \tag{19}$$

Applications

The methodology derived in the previous section is now applied to two data set: first, an observational earthquake catalog from the Parkfield segment of the San Andreas fault, and second, a simulated earthquake catalog covering about 40,000 years of seismicity evolution. The latter data are particularly useful to study the bias-correction (Eq. 19), because the "true" magnitudes are known.

The b value in the presence of magnitude uncertainty

As a first application, we consider an example for the influence of magnitude uncertainty on a maximum likelihood estimation of the Richter b value. The uncertainty is parameterized as in Eq. (12). In catalogs where magnitudes are given with one decimal place, the choice $\delta = 0.05$ in Eq. (12) is an estimate for the rounding error. Apart from rounding errors other sources of uncertainty like measurement errors, uncertainties in the equations for magnitude determination and other random errors occur. In this section, we plug all these errors into a higher value of δ in Eq. (12).

The calculation is carried out for the Parkfield segment of the San Andreas fault in California, where seven $M6$ earthquakes occurred between 1857 and 2004 (*Langbein et al.*, 2005). We use the ANSS composite catalog in the time interval between 1987 to present, including 1041 earthquakes with magnitudes $2.0 \leq M \leq 5.96$ along the Parkfield segment. As shown in *Zöller et al.* (2008), the cumulative frequency-size distribution suggests the completeness of the catalog in this range of magnitudes. A maximum likelihood fit of the Gutenberg-Richter law provides the b value without accounting for magnitude uncertainties $b = 0.89$.

Figure 1 shows the logarithm of the likelihood function (Eq. 15) depending on $\beta = b \ln(10)$. The solid line is the case without uncertainties ($\delta = 0$), where Eq. (15) reduces to Eq. (7). While magnitudes are usually rounded to one decimal place the ANSS catalog provides magnitudes with two decimal places; the dashed line refers to this situation by using magnitude errors with $\delta = 0.005$. The dotted line mimics rounded magnitudes with one decimal place ($\delta = 0.05$), and the dash-dotted line the presence of additional uncertainties ($\delta = 0.1$). The curves indicate that b is strongly affected by magnitude uncertainties. Surprisingly, the estimate of b changes by approximately 9% because of rounding errors to one decimal place. The reason for this effect is the decreased lower magnitude threshold m_0 (Eq. 13). The results for the maximum likelihood estimation b_{max} and their error are provided in the caption of Fig. 1.

Simulated seismicity: Evolution of the coefficient of variation with time

Ben-Zion and Rice (1993) and *Zöller et al.* (2004; 2005) have presented a numerical model of a 2D fault embedded in a 3D elastic half-space that resembles various characteristics of Parkfield seismicity. We consider a realization of the model consisting mainly of constant tectonic loading and co-seismic stress transfer on a smooth fault in accordance with dislocation theory. The corresponding earthquake catalog covers about $40,000$ years and includes $536,697$ earthquakes with magnitude $M \geq 4.05$. Following *Zöller et al.* (2008), we define large earthquakes by the characteristic earthquake peak ($m \geq 6.75$) leading to 1279 events. The frequency-size distribution is considered in a scaling regime up to magnitudes $m = 6.3$. This data set is now used to demonstrate the Bayesian approach for updating the estimate of the coefficient of variation c_V. The calculation can be compared with the evolution of c_V calculated directly from the large events. Because this "true" value is unstable for a small number of events, we start the calculations with a learning period of 25 large events (1000 years) and then continue with time steps of five years using only data from the past. The coefficient of variation as function of time is shown as a thin dashed line in Fig. 2. Despite the considerable fluctuations, the value becomes more stable due to the increasing number of large events with time. The thick curve denotes the value of c_V calculated from the FO model (Eq. 4) with errorbars appearing as the thickness of the line. Here, the error is defined as the half-width of decay of the likelihood function to 10% of the maximum value. It is emphasized that the prediction of the aperiodicity is solely based on the small earthquakes ($4.05 \leq m \leq 6.3$) from the past entering in the a value (in N_{exp}) and the b value in Eq. (4). As an important result we observe that a stable value of c_V close to the "true" value is reached in an early state of the catalog. Recalling that the FO model is based on simple but realistic assumptions and that the large number of small earthquakes stabilizes the calculations results in a feasible estimation of the aperiodicity, even if the number of large earthquakes is small.

Now we introduce magnitude uncertainties according to Eq. (12) with $\delta = 0.05$. Since the true magnitudes in the synthetic data are known, the effect of rounding errors and the feasibility of the bias correction can be studied easily. Therefore, we round the magnitudes to one decimal place and introduce the threshold $m_{min} = 4.1$. Then, we calculate the biased estimate of c_V ($\delta = 0$) as function of time (Fig. 3, curve 1). This curve is compared with the true magnitudes above $4.1 - \delta = 4.05$ (Fig. 3, curve 2). Finally the biased values of curve 1 are corrected using Eq. (19) leading to curve 3. Figure 3 demonstrates that only the rounding of the magnitudes increases the maximum likelihood estimate of c_v by a factor of two. Additional instrumental errors will have further impact on the estimates. However, the bias can be corrected efficiently, if the assumption of the Gutenberg-Richter law is fulfilled to some degree. In the case of perfect agreement, curves 2 and 3 should be identical.

Revisiting Parkfield

The methods developed in the previous sections can now be applied to the Parkfield catalog. This data record, ranging from 1987 to present, includes only one $M6$ event, namely the 2004 earthquake. In the magnitude range $m \in [2.0; 5.0]$, the frequency-size distribution follows a power-law and includes no characteristic earthquake peak. Therefore we use the basic approach presented in *Zöller et al.* (2008), in which the coefficient of variation is given by Eq. (3); the term $\sqrt{N_{exp}/N_{real}}$ in Eq. (4) only corrects for deviations from the scaling.

The yearly update of the aperiodicity seems to saturate in a value between 0.6 and 0.65 (see circles with errorbars in Fig. 4). This value predicted by the fractal oscillator model is higher than values found in other studies, e.g. $c_V = 0.44$ in *Ellsworth et al.* (1999), or the $c_V = 0.38$ based on the empirical moments of the seven $M6$ earthquakes between 1857 and 2004. Although the real value of c_V remains unknown, we argue that the fractal oscillator model includes more information than just the occurrence time of the largest earthquakes and is thus probably more realistic.

The solid line in Fig. 4 shows the bias-corrected values of c_V calculated from Eq. (19) with $\delta = 0.005$ accounting for the fact that the ANSS catalog provides magnitudes with two decimal places and the dashed line corresponds to $\delta = 0.05$ accounting for additional error sources. Due to this relatively high precision, the bias resulting from the magnitude binning is negligible.

Discussion

In this study, we have employed the fractal oscillator (FO) model recently proposed by *Zöller et al.* (2008) in order to gain information about the recurrence time distribution of large earthquakes, especially the aperiodicity or the coefficient of variation. The empirical calculation of this quantity is usually unstable because of the small number of large events. However, the FO model estimates the occurrence time of the next large event from stress changes due to smaller earthquakes and is thus more robust.

The use of Bayes' theorem allows to continuously update the estimation of the aperiodicity, if new information (new earthquakes) becomes available. Using a synthetic earthquake catalog from a numerical fault model, the approach is demonstrated. The results indicate that a stable value of c_V is reached in an early state of seismicity evolution. The calculated value slightly underestimates the empirical value; however, the model prediction provides a reasonable close estimate becoming better with ongoing time.

Second, we have shown that magnitude uncertainties usually have significant influence on the results. Even the b value changes significantly in the presence of relatively small uncertainties, e.g. caused by grouped magnitudes (rounding errors) or observational errors. Here, we have studied magnitude uncertainties following a uniform distribution in a finite interval around the true magnitude. In the presence of these errors, the b value as well as c_V are biased systematically. We claim that estimations of the Gutenberg-Richter (GR) law (or parameters that are based on GR law) have to be corrected, if observational data are used. Using synthetic seismicity, it is demonstrated that the aperiodicity is biased significantly, if only the effect of magnitude binning is taken into account. However, the bias can be corrected almost perfectly if the rounding error is incorporated explicitely in the likelihood method. Future work will deal with other types of aleatoric uncertainties; especially the treatment of probability densities with infinite long tails is not straightforward due to the required correction of the minimum magnitude in the Gutenberg-Richter model.

In sum, the presented method has found to be powerful to extract information from sparse and noisy data. The bridge between renewal models and observational data is build by a Bayesian approach. Instead of dealing with a small number of large earthquakes, more information is gained, because more observational data are taken into account leading to an increase of robustness and reliability of the results.

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Figure captions

Figure 1. Logarithm of the likelihood function (Eq. 7 and 15) in dependence of $\beta = b \ln(10)$ calculated for Parkfield seismicity. The parameter δ refers to the magnitude uncertainty (see Eq. 12). The maximum likelihood estimations of the Richter b value are $b_{max} = 0.89 \pm 0.06$ for $\delta = 0$ (solid line), $b_{max} = 0.89 \pm 0.06$ for $\delta = 0.005$ (dashed line), $b_{max} = 0.81 \pm 0.11$ for $\delta = 0.05$ (dotted line), and $b_{max} = 0.75 \pm 0.10$ for $\delta = 0.10$ (dash-dotted line). The errors are defined as the half of the interval of b where L has decreased to $0.1 \cdot L(\beta_{max})$.

Figure 2. Thin dashed line: Coefficient of variation c_V as function of time t for earthquakes with $M \geq 6.75$ in the simulated catalog; only data from the past ($t' < t$) are used. Thick line: prediction of c_V from the FO model updated with time using the Bayesian approach without magnitude uncertainty ($\delta = 0$). The curves are calculated with a time step of five years. The thickness of the line indicates the error as described in the text.

Figure 3. Coefficient of variation calculated from synthetic earthquake catalogs. Middle curve: c_V from the original catalog (identical with thick line in Fig. 2); top curve: magnitudes rounded to one decimal place (error-corrupted); bottom curve: bias correction of curve 1 (Eq. 19 and 4).

Figure 4. Coefficient of variation calculated for Parkfield seismicity. The circles with errorbars are based on the ANSS catalog providing magnitudes with two decimal places. Errorbars are calculated as in Fig. 3. The corresponding bias-corrected values ($\delta = 0.005$ and $\delta = 0.05$) are given by the lines.

Figures

Figure 1

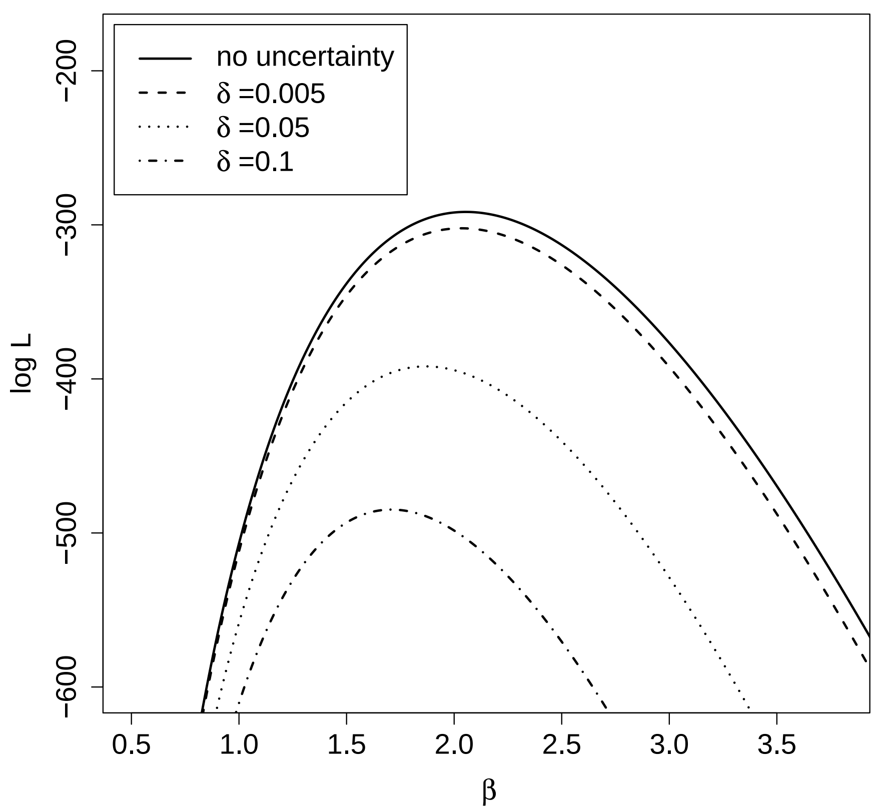


Figure 2

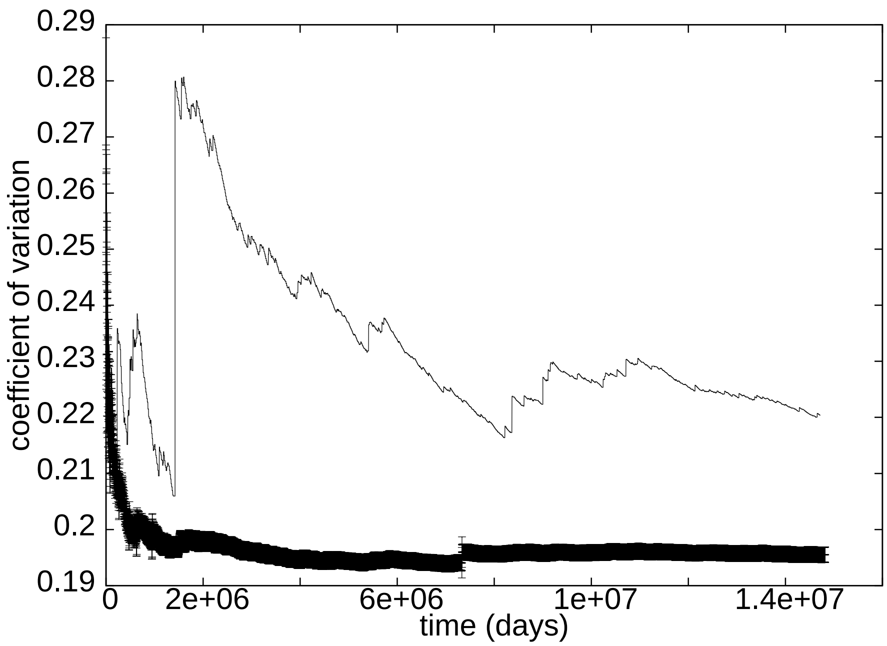

0.29
0.28
0.27
0.26
0.25
0.24
0.23
0.22
0.21
0.2
0.19
coefficient of variation
0
2e+06
6e+06
1e+07
1.4e+07
time (days)

Figure 3

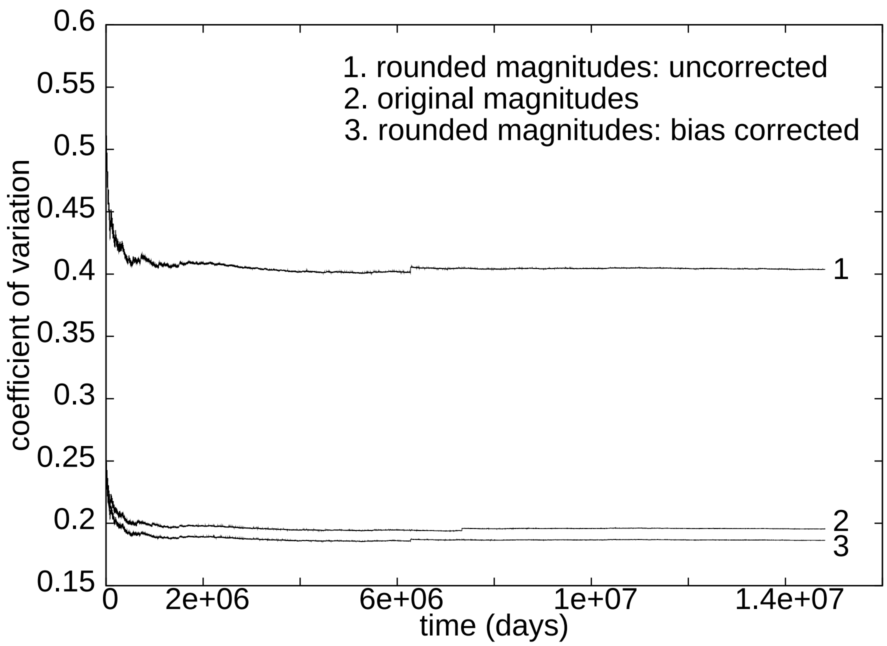
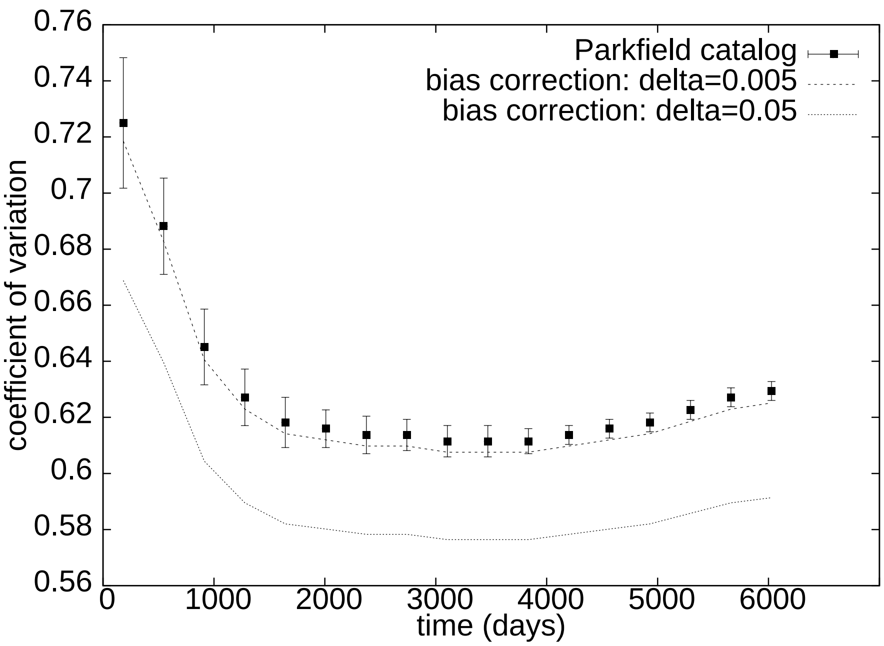

1. rounded magnitudes: uncorrected
2. original magnitudes
3. rounded magnitudes: bias corrected
coefficient of variation
0.6
0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0
2e+06
6e+06
1e+07
1.4e+07
time (days)
1
2
3

Figure 4


Parkfield catalog
bias correction: delta=0.005
bias correction: delta=0.05
coefficient of variation
time (days)
0.76
0.74
0.72
0.7
0.68
0.66
0.64
0.62
0.6
0.58
0.56
0
1000
2000
3000
4000
5000
6000