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2.5D controlled-source EM modeling with general 3D source geometries

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ABSTRACT

Most 2.5D controlled-source electromagnetic (CSEM) modeling algorithms presented to date explicitly consider only sources that are point dipoles oriented parallel or perpendicular to the direction of constant conductivity. This makes simulations of complex source geometries expensive, requiring separate evaluations of many point dipole fields, and thus limits the practical applicability of such schemes for simulating and interpreting field data. We present a novel 2.5D CSEM modeling scheme that overcomes this limitation and permits efficient simulations of sources with general shape and orientation by evaluating fields for the entire source at once. We accommodate general sources by using a secondary field approach, in which primary fields are computed for the general source and a 1D back-

ground conductivity model. To carry out the required Fourier transforms between space and wavenumber domain using the same fast cosine and sine transform filters as in conventional algorithms, we split the primary and secondary fields into their symmetric and anti-symmetric parts. For complex 3D source geometries, this approach is significantly more efficient than previous 2.5D algorithms. Our finite-difference algorithm also includes novel approaches for divergence correction at low frequencies and EM field interpolation across conductivity discontinuities. We describe the modeling scheme and demonstrate its accuracy and efficiency by comparisons of 2.5D-simulated data with 1D and 3D results.

INTRODUCTION

For controlled-source electromagnetic (CSEM) modeling, various approaches have been developed, ranging from highly efficient quasi-analytic 1D solutions (e.g., Chave and Cox, 1982; Løseth and Ursin, 2007; Streich and Becken, 2011) over numerical 2.5D formulations (e.g., Everett and Edwards, 1992; Li and Key, 2007; Abubakar et al., 2008) to a wide variety of numerical full 3D solutions (e.g., Wang and Hohmann, 1993; Newman and Alumbaugh, 1995; Haber and Ascher, 2001; Streich, 2009; Schwarzbach, 2009), which provide increasingly realistic representations of the subsurface. 3D simulations are no longer prohibitively expensive, and can be fast and memory-efficient if only a limited number of forward modeling solutions is required (Druskin and Knizhnerman, 1994; Mulder, 2006; Weiss and Constable, 2006). Nevertheless, 3D modeling can become impractical if a large number of simulations is essential, e.g., for parameter testing or resolution studies in CSEM inversion. Here, 2.5D approaches, in which the model conductivity is variable in two dimensions and constant in the direction assumed to correspond to the geological strike, fill a gap. They incur significantly lower computational cost and also allow for higher-resolution and/or larger models than 3D simulations. Although the assumption of constant conductivity in one direction never fully represents reality, it can be a reasonable first-order approximation in many geological settings, and is useful for the large number of data sets containing no or limited amounts of 3D information because of 1D-line survey geometries (e.g., Weitemeyer et al., 2010).

Various 2.5D finite-difference (e.g., Stoyer and Greenfield, 1976; Abubakar et al., 2008) and finite-element (Lee and Morrison, 1985; Everett and Edwards, 1992; Unsworth et al., 1993; Li and Key, 2007) modeling schemes have been developed. For all of these codes, appli-

cations have been demonstrated exclusively for point dipole sources oriented perpendicular or parallel to the direction of constant conductivity. Such a limitation is not encountered in 1D or 3D algorithms. Sources of finite extent have also been considered in 2.5D modeling (Leppin, 1992), and efforts have been made to mitigate the source singularity by numerically distributing the source location (Mitsuhata, 2000; Kong et al., 2008); however, these approaches also require the sources to be parallel or perpendicular to the strike direction.

This limits the practical applicability of 2.5D schemes for simulating realistic field surveys, particularly on land, where long source dipoles are necessary to achieve strong EM fields, and at the same time various logistical constraints, such as the presence of inaccessible areas, buildings or infrastructure, have to be taken into account when defining source geometries. With increasing interest in EM measurements near populated areas, such problems are likely to occur frequently. As a consequence, CSEM surveys will generally have to deal with complex source geometries.

By limiting the source orientations that are permitted explicitly, 2.5D algorithms gain computational efficiency. Commonly, a transform to wavenumber domain is carried out along the direction of constant conductivity, and the EM field is then computed for a set of logarithmically spaced wavenumbers. For strike-parallel or perpendicular sources, every EM field component is either symmetric or anti-symmetric along the strike direction. Accordingly, only positive wavenumbers need to be considered, and inverse transform of each field component to the space domain is accomplished by applying either a fast cosine or sine transform filter (e.g., Anderson, 1983). Using this approach, EM fields for complex source geometries can in principle be modeled by representing the source as a set of strike-parallel and perpendicular point dipoles, and separately computing and then superposing the point dipole fields.

We present a straightforward way of accommodating general 3D sources of arbitrary shape and orientation more efficiently in 2.5D simulations. Key to our approach is the use of a secondary field formulation, and the separation of primary and secondary fields into their symmetric and anti-symmetric parts. This allows the application of the same fast cosine and sine transforms as in previous 2.5D algorithms. In addition, we include recent developments for enhancing numerical accuracy and stability into the 2.5D modeling scheme (Streich, 2009; Streich et al., 2010). We first describe our strategy for fast and accurate 2.5D simulations with general sources, then verify the accuracy of our approach by comparing 2.5D solutions to 1D and 3D results for various point dipole and finite-length sources, and briefly discuss the computational efficiency.

2.5D ALGORITHM

We formulate the forward modeling problem in terms of the vector Helmholtz equation for the secondary electric field $\mathbf{E}^S$ (Ward and Hohmann, 1987; Newman and Alumbaugh, 1995; Streich, 2009):

$$\nabla \times \nabla \times \mathbf{E}^S + j\omega\mu_0\sigma^*\mathbf{E}^S = -j\omega\mu_0(\sigma^* - \sigma^{P*})\mathbf{E}^P, \quad (1)$$

where ω is the angular frequency, the complex conductivity $\sigma^* = \sigma + j\omega\varepsilon$ includes conductivity σ and permittivity ε , μ_0 is the free-space magnetic permeability, and σ^{P*} is the conductivity of a 1D layered background model. The primary electric field $\mathbf{E}^P$ is computed for conductivity σ^{P*} . General 3D sources, such as electric or magnetic point dipoles or wires of finite length and arbitrary orientation, are taken into account in the computation of $\mathbf{E}^P$ (Ward and Hohmann, 1987; Streich and Becken, 2011). Secondary sources arise according to equation 1 in regions where σ^* differs from σ^{P*} .

Assigning x to the direction in which complex conductivity σ^* is constant, we carry out the Fourier transform

$$\tilde{\mathbf{E}}^S(k_x, y, z) = \int_{-\infty}^{\infty} \mathbf{E}^S(x, y, z) e^{jk_x x} dx \quad (2)$$

to obtain the (k_x, y, z) -domain Helmholtz equation that forms the basis of 2.5D modeling,

$$\tilde{\nabla} \times \tilde{\nabla} \times \tilde{\mathbf{E}}^S + j\omega\mu_0\sigma^* \tilde{\mathbf{E}}^S = -j\omega\mu_0(\sigma^* - \sigma^{P*}) \tilde{\mathbf{E}}^P, \quad (3)$$

where the nabla operator has been transformed to $\tilde{\nabla} = (-jk_x, \partial_y, \partial_z)$. Equations 2 and 3 do not impose any restrictions on the source geometry.

Since 1D analytic solutions for the primary field $\tilde{\mathbf{E}}^P$ in (k_x, y, z) -domain and general 3D sources do not exist, we obtain $\tilde{\mathbf{E}}^P$ by quasi-analytically evaluating $\mathbf{E}^P$ in space domain (Streich and Becken, 2011), followed by Fourier transform along the x direction. For accurate transform to the wavenumber domain, $\mathbf{E}^P$ has to be sampled densely in the vicinity of the source, where it changes rapidly. In addition, the x range considered has to be sufficiently large to ensure decay of $\mathbf{E}^P$ to insignificant values. We therefore compute $\mathbf{E}^P$ on a set of logarithmically spaced x nodes. Then, exploiting the fact that any function $f(x)$ can be split into its symmetric and anti-symmetric parts $a(x)$ and $b(x)$ with respect to $x = 0$ as $a(x) = 1/2(f(x) + f(-x))$ and $b(x) = 1/2(f(x) - f(-x))$, we write the Fourier transform of $f(x)$ as

$$\tilde{f}(k_x) = \int_{-\infty}^{\infty} a(x) \cos(k_x x) dx + j \int_{-\infty}^{\infty} b(x) \sin(k_x x) dx. \quad (4)$$

We thus obtain $\tilde{\mathbf{E}}^P$ by splitting, transforming, and re-assembling each component of $\mathbf{E}^P$. The cosine and sine transforms are computed using the fast algorithms of Anderson (1983).

After computing $\tilde{\mathbf{E}}^P$, we solve equation 3 for a set of logarithmically spaced k_x . We set the minimum wavenumber to $k_x^{min} = 2\pi/X$, where X is the total model size in the x direction. The maximum wavenumber is chosen such that the arguments in the fast sine

and cosine transforms (Anderson, 1983, 1982) decay to sufficiently small values. Following Unsworth et al. (1993) and Snyder (1976), we define a constant $C = k_x^{max} x^{min}$, with $C = 8$ for the data presented. The minimum x offset for which we evaluate the primary fields, x_{min} , is empirically chosen to be at most 1/10 of the smallest y or z cell size, honoring the fact that primary fields near the source vary much more rapidly than secondary fields.

At each k_x , we apply a finite-difference (FD) discretization to equation 3, with symmetric scaling of the system matrix analogous to Streich (2009) and Newman and Alumbaugh (1995), and solve the resulting linear system of equations using the MUMPS direct solver (Amestoy et al., 2006). We use this solver instead of more common iterative techniques, because (i) it is robust and devoid of convergence problems, even for models that are highly ill-conditioned due to high conductivity contrasts or strongly elongated cells, and (ii) it provides a large number of solutions with minor effort once an initial expensive factorization is completed; its efficiency thus increases with the number of required forward solutions, which is particularly advantageous if the algorithm is used in an inversion scheme. The treatment of sources is independent of the solution approach and will work equally well in conjunction with iterative solvers.

In addition to the parallelization included in MUMPS, we have parallelized the code over frequencies. Furthermore, we distribute the model over the available processes for parallel computation of the 1D background field and other pre- and post-processing operations. At low frequencies, numerical stabilization is required. Whereas iterative schemes can be stabilized by applying divergence corrections between solution iterations of the FD equation system (Smith, 1996), we stabilize the FD system by augmenting it with terms that explicitly enforce a divergence condition (Schwarzbach, 2009; Streich et al., 2010). For general non-symmetric primary fields, the secondary fields are non-symmetric with respect to k_x .

Therefore, we have to consider both positive and negative wavenumbers. The space-domain solution $\mathbf{E}^S$ is obtained by splitting each component of the secondary field $\tilde{\mathbf{E}}^S$ into its symmetric and anti-symmetric parts and applying inverse fast cosine and sine transforms.

Finally, accurate electric field values at given receiver locations are obtained by linear interpolation of the grid-based secondary field $\mathbf{E}^S$ or, normal to σ^* discontinuities, generalized secondary current density $\mathbf{J}^S = \mathbf{E}^S \sigma^* + \mathbf{E}^P (\sigma^* - \sigma^{P*})$ (Streich, 2009). The magnetic field is obtained from

$$\tilde{\mathbf{H}}^S = -(j\omega\mu_0)^{-1} \tilde{\nabla} \times \tilde{\mathbf{E}}^S, \quad (5)$$

followed by inverse cosine and sine transforms. This operation is less efficient, but more accurate than computing $\mathbf{H}^S$ directly in the space domain, since the application of $\tilde{\nabla} \times$ is exact in k_x , whereas the space-domain counterpart $\nabla \times$ requires a linear approximation in the x direction. All derivatives included in the operator $\tilde{\nabla} \times$ are defined and continuous throughout the model. Hence, equation 5 can be evaluated straightforwardly using finite differences. Figure 1 summarizes our 2.5D modeling strategy.

VERIFICATION AND EXAMPLES

To verify the accuracy of the 2.5D algorithm, we first compare 2.5D solutions to 1D and 3D results for a 1D layered-conductivity model (Figure 2a), which contains a resistive layer grossly mimicking the geology at the carbon sequestration test site in Ketzin, Germany (Giese et al., 2009). The model includes an air layer, and a hypothetical grid of receivers is placed at the surface. The background model used for computing primary fields is a three-layer model containing air and the over- and underburden, but not the resistive layer. The (y, z) size of the model is 7600 x 1055 m. To mitigate boundary effects, we pad the model

with edges of widths ~ 5 km at the sides, 3 km at the top and 1.5 km at the bottom. To ensure comparability between the 2.5D and 3D solutions, we use the same y and z sampling with 95×43 (y, z) nodes, although a finer grid would be feasible for the 2.5D computations. The cell sizes range from 37 m to 2760 m and 9.5 m to 2530 m in the y and z directions, respectively.

In the 2.5D simulations, 179 logarithmically spaced x nodes are used for computing the background fields, with minimum and maximum cell sizes of 0.88 m and 3058 m, respectively. Correspondingly, we use 81 wavenumbers, $k_x = 0$ and eight values per decade in the range $\pm 9.78 \times 10^{-5} \text{ m}^{-1}$ to $\pm 8.43 \text{ m}^{-1}$. For the 3D simulations, to limit the computational effort, we use a subset of 87 of the x nodes and add six widely spaced edge nodes, resulting in an x extent of the model of ± 4570 m with 5-km wide edges and x -cell sizes ranging from 9.8 m to 2810 m. The same direct solver library is used for the 2.5D and 3D computations.

To illustrate the 2.5D computation of EM fields using two-sided forward and inverse transforms, we display in Figure 3 x - and k_x -domain primary and secondary fields at successive stages of the 2.5D algorithm. In this example, the source is a 1-km long horizontal grounded wire, oriented 30° to the x axis. The primary field E^P is evaluated inside the resistive layer. Along the x -line shown, E^P is larger at positive than at negative x locations (Figure 3a), since the source is closer to the line at positive x . The corresponding k_x -domain primary field $\tilde{E}_x^P$ and the secondary field $\tilde{E}_x^S$ at the surface are most asymmetric in the imaginary parts (Figure 3b and c). The resulting x -domain secondary field E_x^S (Figure 3d) is smooth, as it should be, and has symmetry properties similar to those of the primary field E_x^P .

Figure 4a–c shows simulated total-field data on a $16.2 \times 7.6 \text{ km}^2$ surface area for a point

source oriented at 30° to the x axis, Figure 4d–f for a 1-km long grounded wire source at the same orientation, and Figure 4g–k for a 700-m long, obliquely tilted borehole source. The long horizontal source is modeled by separately considering contributions from the end points and the wire (Streich and Becken, 2011), with 5-m long wire elements. The tilted source is modeled as a set of 1-m long point dipoles.

For all sources, we achieve best accuracy (total-field errors less than 0.1%) near the source. We have also verified that the errors are not larger than those obtained for sources oriented at 0 or 90° to the x axis. At larger x -distances from the source, the errors increase somewhat due to the coarsening x and wavenumber sampling. Nevertheless, low error levels, mostly less than 2%, are obtained across the entire surface, except for the regions in which field amplitudes are lowest, where larger numerical errors must be expected. Notice also from the contours in Figure 4 that, as in any secondary field approach, the choice of the primary conductivity model is important for achieving good overall accuracy.

The errors are marginally larger for the 1-km long source than for the point dipole (compare Figures 4b and 4e). This is due to the source positioning within the modeling grid. For all sources, we shift the computational grid in the x direction to the arithmetic center of the source. Accordingly, for the long transmitter, the primary field $\mathbf{E}^P$ is strongest at some distance away from the source center. Since the x sampling of the simulation grid increases logarithmically, $\mathbf{E}^P$ is sampled slightly less accurately than for the point dipole. For the tilted borehole source, the errors are slightly larger than for the surface sources (Figure 4g–i). Because one end of this source is located near the anomalous resistive layer, strong secondary sources and strong gradients in the secondary source field arise from this layer.

The accuracy of the 2.5D-modeled data can be improved further by refining the simulation grid. In Figure 5, we display results for a finer grid defined simply by halving all y and z cell sizes. The errors decrease particularly in the lowest-amplitude regions (likely, further grid optimization would improve the accuracy further). For this refined grid, 3D simulations would be nearly prohibitively expensive.

The same level of accuracy as shown in Figure 4 is obtained for different frequencies. We demonstrate this in Figure 6 for a frequency of 1 Hz. For coarser and finer wavenumber sampling, the accuracy decreases and improves as expected (Figure 7).

The 2.5D results are very similar to the 3D results, with relative differences mostly less than 2%. This confirms that both codes provide accurate results and indicates that the 2.5D code can be further validated by comparing 2.5D and 3D results for 2D conductivity models. Small differences likely result from the coarser x sampling of the 3D model, minor edge effects in the 3D data due to the smaller x size of the model, and slight Fourier transform noise in the 2.5D data.

The magnetic field is even more accurate than the electric field (Figure 4j–l), with relative differences between the 1D, 2.5D and 3D H_z data mostly less than 0.1%. Small differences between the 2.5D and 3D results are expected, because similar numerical evaluations of equation 5 and its space-domain counterpart are involved, which should lead to similar errors in both cases. Surprisingly small errors compared to the 1D results can be explained by noting that the secondary magnetic field is small compared to the primary field for the considered conductivity model. Therefore, errors present in the secondary field appear small in relation to the total field amplitude. Figure 8 illustrates the accuracy of the secondary magnetic field H_z^S for the same source and model as shown in Figures 4j–l.

As expected, the errors are larger than for the total field, but they remain mostly between 5 and 10%.

Figure 9 shows that results from 2.5D and 3D simulations are also very similar for the 2D conductivity model of Figure 2b and the long grounded wire surface source displayed in Figure 2c, which we used recently in a land-based CSEM field survey. Based on GPS recording of the source geometry, this source is modeled as a set of 144 wire segments (Streich and Becken, 2011). The background model again is a three-layer model. We show E_y here, because it has higher amplitudes than E_x . Numerical differences between the 2.5D and 3D simulations are on the order of ~ 0.01 –2%. These differences are significantly smaller than the amplitude changes due to the two resistive blocks (compare Figures 9b and c), demonstrating amply sufficient accuracy for modeling the effects of such resistors.

In Table 1, we display examples of computation time and memory requirements for 2.5D and 3D simulations of point dipole and horizontal wire sources. The 2.5D computations require only fractions of the 3D CPU time and memory and can conveniently be carried out on standard desktop PCs, which is not the case for the 3D simulations. Although absolute runtimes and memory consumption depend on the solver employed, similar ratios of 2.5D and 3D times and memory are expected for other solvers.

Computation times for the FD simulations include the time required for system matrix factorization, which, for a given conductivity model and discretization, is executed only once per wavenumber. Additional time is spent for every source on primary field calculation, forward and inverse Fourier transforms, solution of the FD systems and interpolation to the receiver locations. The 1D background field computations, which are carried out for 179×95 (x, y) nodes at five out of 43 z levels, and additionally for each receiver location,

take most of this additional time. Fourier transforms and FD system solutions only require ~ 1.6 and 0.9 s per source, respectively. The 1D calculations are comparatively expensive, because we evaluate the necessary Hankel integrals for the exact radii for receivers and grid points having horizontal distances shorter than a fraction, typically $1/5$ to $1/10$, of the skin depth (Anderson, 1982). This ensures optimum accuracy. For larger offsets, we switch to spline interpolation between pre-computed integral values. Elimination of the space (x, y, z) domain expressions and direct transformation from the wavenumber (k_x, k_y, z) to the (k_x, y, z) domain may further optimize these calculations significantly.

CONCLUSIONS

Three-dimensional sources of complex geometry and arbitrary orientation can be handled in 2.5D CSEM modeling equally well as in 1D and 3D modeling. We achieve this by including the sources into a 1D background model, computing primary fields in the space domain, and computing forward and inverse Fourier transforms of primary and secondary fields by splitting the field components into their symmetric and anti-symmetric parts and then applying fast cosine and sine transform filters. The results obtained are generally very accurate. Differences between 2.5D-simulated data and reference data obtained from 1D and 3D codes are small, and errors for sources with arbitrary orientations do not exceed those for strike-parallel or perpendicular sources. The small errors observed are not primarily related to the 3D source geometries, but are determined by the inherent limitations of the chosen numerical modeling approach. Our concept of accounting for general source geometries can also be used with other (e.g., finite-element) modeling schemes.

For sources that are reasonably well approximated by point dipoles, there is no gain in computational efficiency in our approach compared to standard 2.5D schemes, because data

for point dipole sources with arbitrary orientations can be produced by two runs of a conventional 2.5D algorithm with sources at 0 and 90°, and superposition of the results. When using a direct solver, this standard approach would outperform our technique. In marked contrast, for complex source geometries, our new algorithm is far superior in efficiency, since it accommodates complex sources by a single FD computation at each wavenumber; separate FD computations for each dipole element are not required. With the direct solution approach, we can also model multiple sources efficiently by reusing the Cholesky decompositions of the system matrices, which makes the algorithm a well-suited forward engine for CSEM inversion. However, depending on the considered source geometry, the 1D background computations can require significant computational time, leaving a potential for further optimization.

The inclusion of general source geometries significantly enhances the applicability of 2.5D CSEM modeling schemes for field-data simulation and interpretation. In marine surveys, our approach will be useful in special situations that require considering the true source geometry, such as shallow, short-offset surveying of gas hydrates. The new scheme will be most beneficial for simulations of land sources, which commonly are large and exhibit complex 3D geometries that cannot easily be split into strike-parallel and perpendicular dipoles.

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1 Computation times (seconds), averaged for 9 different point dipole and 9 long wire sources, and memory usage for 2.5D and 3D simulations for the (non-refined) model of Figure 2a and the corresponding smaller 3D model. Factorization and additional costs per source are shown in terms of total runtimes for 2.5D computations carried out on four and 3D computations carried out on 48 2.4GHz CPUs. Total CPU times are *(factorization time + 9*(time per point dipole + time per long wire source))*NCPUs*.

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- 2 (a) 1D and (b) 2D conductivity models used for verifying the 2.5D modeling code. Black dots indicate receivers. (c) Geometry of the source used in the second example.
- 3 Intermediate results of 2.5D EM field computation; (a–d) correspond to the stages indicated in Figure 1. Space- and wavenumber-domain electric fields at negative (left panels) and positive (right panels) x and k_x coordinates are shown. The source is a 1-km long horizontal wire source oriented 30° to the x axis, with end points at $(x, y, z) = (-433, 2750, 0.1)$ m and $(+433, 3250, 0.1)$ m. We display the real and imaginary parts of (a) the space-domain primary field E_x^P at $(y, z) = (3280, 645)$ m (i.e., near the y -coordinate of the source end point and inside the anomalous resistor; see Figure 2a), (b) the k_x transform $\tilde{E}_x^P$ of the field shown in (a), (c) the k_x -domain secondary field $\tilde{E}_x^S$ at the same y location, but at the surface, and (d) the x -domain transform E_x^S of the field shown in (c).
- 4 (a–i) Electric field E_x and (j–l) magnetic field H_z at the surface, normalized by the source dipole moment, for the 1D model of Figure 2a and a frequency of 0.1 Hz. The source was: (a–c) a horizontal point dipole located at $(x, y, z) = (2000, 3000, 0.1)$ m, oriented 30° to the x axis; (d–f) a 1-km long horizontal grounded wire having the same center point and orientation as the point dipole from (a–c); and (g–l) a long tilted borehole source extending from $(x, y, z) = (1000, 3000, 0.1)$ m to $(700, 2800, 600.1)$ m. The left column shows 2.5D modeling results, the center column shows relative errors to 1D results, computed as $|(F^{2.5D} - F^{1D})/F^{1D}| * 100$, and the right column shows relative errors to 3D results (for the x range covered by the 3D model). Contours indicate the ratio of secondary to primary

fields as $\log_{10}(|F^S/F^P|)$.

5 Errors of 2.5D modeling results relative to 1D results, as for Figure 4b, e and h, but for a refined FD simulation grid with all y and z cell sizes halved.

6 Comparison of 2.5D, 1D and 3D-simulated data as for Figure 4d–f, but for a frequency of 1 Hz.

7 Errors of 2.5D data relative to 1D data as for Figure 4e, but for (a) six and (b) ten wavenumbers per decade.

8 Comparison of 2.5D, 1D and 3D-simulated magnetic field data as for Figure 4j–l, but showing the secondary field H_z^S .

9 (a) E_y obtained from the 2.5D code for the 2D model and 3D source shown in Figures 2b and c, for a frequency of 0.1 Hz. (b) Relative difference to the 3D result, computed as $|(E_y^{2.5D} - E_y^{3D})/E_y^{3D}| * 100$. (c) Relative difference to E_y for a 1D background model not containing the two resistive blocks. Dashed lines indicate the horizontal extent of the conductivity anomalies (see also Figure 2b).

Figure 1: Flow chart showing the 2.5D modeling algorithm. Annotations (a), (b), (c) and (d) indicate the points at which data are shown in Figure 3a–d.

Figure 2: (a) 1D and (b) 2D conductivity models used for verifying the 2.5D modeling code.

Black dots indicate receivers. (c) Geometry of the source used in the second example.

Figure 3: Intermediate results of 2.5D EM field computation; (a–d) correspond to the stages indicated in Figure 1. Space- and wavenumber-domain electric fields at negative (left panels) and positive (right panels) x and k_x coordinates are shown. The source is a 1-km long horizontal wire source oriented 30° to the x axis, with end points at $(x, y, z) = (-433, 2750, 0.1)$ m and $(+433, 3250, 0.1)$ m. We display the real and imaginary parts of (a) the space-domain primary field E_x^P at $(y, z) = (3280, 645)$ m (i.e., near the y -coordinate of the source end point and inside the anomalous resistor; see Figure 2a), (b) the k_x transform $\tilde{E}_x^P$ of the field shown in (a), (c) the k_x -domain secondary field $\tilde{E}_x^S$ at the same y location, but at the surface, and (d) the x -domain transform E_x^S of the field shown in (c).

Figure 4: (a–i) Electric field E_x and (j–l) magnetic field H_z at the surface, normalized by the source dipole moment, for the 1D model of Figure 2a and a frequency of 0.1 Hz. The source was: (a–c) a horizontal point dipole located at $(x, y, z) = (2000, 3000, 0.1)$ m, oriented 30° to the x axis; (d–f) a 1-km long horizontal grounded wire having the same center point and orientation as the point dipole from (a–c); and (g–i) a long tilted borehole source extending from $(x, y, z) = (1000, 3000, 0.1)$ m to $(700, 2800, 600.1)$ m. The left column shows 2.5D modeling results, the center column shows relative errors to 1D results, computed as $| (F^{2.5D} - F^{1D}) / F^{1D} | * 100$, and the right column shows relative errors to 3D results (for the x range covered by the 3D model). Contours indicate the ratio of secondary to primary fields as $\log_{10}(|F^S/F^P|)$.

Figure 5: Errors of 2.5D modeling results relative to 1D results, as for Figure 4b, e and h, but for a refined FD simulation grid with all y and z cell sizes halved.

Figure 6: Comparison of 2.5D, 1D and 3D-simulated data as for Figure 4d–f, but for a frequency of 1 Hz.

Figure 7: Errors of 2.5D data relative to 1D data as for Figure 4e, but for (a) six and (b) ten wavenumbers per decade.

Figure 8: Comparison of 2.5D, 1D and 3D-simulated magnetic field data as for Figure 4j–l, but showing the secondary field H_z^S .

Figure 9: (a) E_y obtained from the 2.5D code for the 2D model and 3D source shown in Figures 2b and c, for a frequency of 0.1 Hz. (b) Relative difference to the 3D result, computed as $|(E_y^{2.5D} - E_y^{3D})/E_y^{3D}| * 100$. (c) Relative difference to E_y for a 1D background model not containing the two resistive blocks. Dashed lines indicate the horizontal extent of the conductivity anomalies (see also Figure 2b).

	One-time cost (fac- torizations)	Additional cost per point dipole	Additional cost per horiz. wire	Total CPU time	Total memory usage (GB)
2.5D	7.7	6.7	45.3	1903	1.64
3D	591	3.3	43.4	48495	79.2

Table 1: Computation times (seconds), averaged for 9 different point dipole and 9 long wire sources, and memory usage for 2.5D and 3D simulations for the (non-refined) model of Figure 2a and the corresponding smaller 3D model. Factorization and additional costs per source are shown in terms of total runtimes for 2.5D computations carried out on four and 3D computations carried out on 48 2.4GHz CPUs. Total CPU times are *(factorization time + 9*(time per point dipole + time per long wire source))*NCPU*s.