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Comment on “Self-similar earthquake triggering, Båth’s law, and foreshock/aftershock magnitudes: Simulations, theory, and results for southern California” by P. M. Shearer

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[1] *Shearer* [2012] finds three differences of the seismicity clustering in southern California compared to self-similar triggering models: (i) a significantly lower b -value for the aftershocks, (ii) a too large aftershock number, and (iii) a too large foreshock-aftershock ratio to be consistent with the Båth law. Based on these observations, the author concluded that the observed seismicity is not in agreement with self-similarity triggering and/or the observed clustering is not primarily caused by earthquake-to-earthquake triggering. However, I show that the observed lower b -value is likely related to incomplete recordings after mainshocks and that the apparently too large aftershock number does not disprove the self-similarity. Thus, only the enhanced foreshock-to-aftershock ratio seems to indicate some discrepancy to self-similar triggering.

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1. Consideration of Incompleteness

[2] *Shearer* [2012] assumes a completeness magnitude of 1.5 for the analyzed southern California catalog of *Lin et al.* [2007] (here called LSH catalog). However, earthquake catalogs are known to be incomplete within the first period after larger earthquakes [*Kagan*, 2004]. The duration of this incompleteness depends on the aftershock magnitudes, where the time interval is largest for the smallest events above the long-term completeness level and shortest for the largest magnitude bins. For the $M \geq 6$ mainshocks in southern California that occurred since 1985, *Helmstetter et al.* [2006] found that the incompleteness threshold varies as a function of the mainshock magnitude M and the time Δt [days] after an earthquake according to

$$m_{\text{cut}}(M, \Delta t) = M - 4.5 - 0.75 \log_{10}(\Delta t). \quad (1)$$

While this relation has been only deduced for $M \geq 6$, where the statistics are good enough for the analysis of single aftershock sequences, the same relation is likely to hold also for smaller mainshocks. While *Peng et al.* [2007] already showed incompleteness after $3 < M < 5$ mainshocks in Japan, I directly tested it for the analyzed aftershocks recorded in the LSH catalog. For this and subsequent

analysis, I adapted the spatial window as well as spatial and temporal selection criteria of *Shearer* [2012], except that I excluded for simplicity 3 months after all $M > 5.5$ events instead of some smaller values taken by *Shearer* [2012] for $5.5 < M < 7$ events. According to the assumed incompleteness (equation (1)), the slope of the frequency-magnitude distribution will steepen if the most affected first time period is excluded from the analysis. This is shown in Figure 1a, where aftershocks in the time periods 0.05–0.5 and 0.1–0.5 days follow approximately the standard b -value of 1, while b is about 0.8 for aftershocks in the whole time period. For the stacked aftershock activity following mainshocks in the three target magnitude bins 2.5–3.5, 3.5–4.5, and 4.5–5.5, I also calculated the temporal variation of the average magnitude for $m \geq 1.5$ events. For complete recordings, the average magnitude should be constant and close to $m_c + \log_{10}(e) \approx 1.93$, which is the theoretical value for Gutenberg-Richter distributed magnitudes with $b = 1$. In contrast, if smaller magnitudes are missed in the first period as suggested by equation (1), then the mean value will be significantly higher in the first period and converge to the theoretical value with time. This is exactly observed in Figure 1b, where equation (1) is found to yield a good approximation for the time period in which the catalog is significantly incomplete.

[3] In the following, I now analyze the consequences of this kind of catalog incompleteness on the results presented by *Shearer* [2012]. Firstly, I present the analytic result for Omori-type aftershock sequences ignoring secondary aftershock triggering (section 1.1). In a second step (section 1.2), the effect of incompleteness is investigated for simulated catalogs of epidemic-type self-similar aftershock triggering models.

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Figure 1. (a) The normalized frequency-magnitude distribution of aftershocks in southern California seismicity following the mainshock magnitudes between 4.5 and 5.5 as a function of the starting time. Here b -value slopes of 0.8 and 1.0 are indicated by dashed lines. (b) The average magnitude of observed aftershocks is plotted as a function of time relative to the mainshock occurrence. For each mainshock magnitude range, the points refer to subsequent 5% of $m \geq 1.5$ aftershocks observed in the first 0.5 days. The horizontal line corresponds to the theoretical value for a Gutenberg-Richter distribution with $b = 1$, and the vertical lines indicate the times after which $m \geq 1.5$ aftershocks should be completely recorded according to equation (1).

1.1. Theoretical Effect for an Omori-Utsu Decay

[4] For simplicity, we consider a simple Omori-Utsu law for the aftershock rate according to

$$R(t) = N \frac{p-1}{c^{1-p}} (c+t)^{-p} \quad (2)$$

where N is the total number of aftershocks, i.e., $\int_0^\infty R(t)dt = N$.

[5] Let us now consider aftershock activity with a given magnitude m , which is assumed to be above the lower magnitude threshold m_c of the catalog (e.g., $m_c = 1.5$ in southern California [Shearer, 2012]). According to equation (1),

Figure 2. The normalized aftershock number as a function of magnitude (equation (5)) considering incompleteness after the mainshock for different Omori p -values ($T = 0.5$ days, $c = 10^{-4}$ days). For comparison, the dashed black line refers to the result of the complete aftershock set with $b = 1$.

events with magnitude m are only recorded after a time lag Δt given by

$$\Delta t(m) = 10^{(M-m-4.5)/0.75}. \quad (3)$$

Thus, within the first time period T after the mainshock, only a fraction $f(m)$ of the true number of magnitude m aftershocks

$$\begin{aligned} f(m) &= \frac{\int_{\Delta t(m)}^T R(t)dt}{\int_0^T R(t)dt} \\ &= \frac{(c + \Delta t(m))^{1-p} - (c + T)^{1-p}}{c^{1-p} - (c + T)^{1-p}} \end{aligned} \quad (4)$$

is observed. Assuming that the true aftershock number is proportional to 10^{-bm} , then the observed number is

$$\begin{aligned} N_{\text{obs}}(m) &\propto f(m)10^{-bm} \\ &= \frac{(c + 10^{(M-m-4.5)/0.75})^{1-p} - (c + T)^{1-p}}{c^{1-p} - (c + T)^{1-p}} 10^{-bm} \end{aligned} \quad (5)$$

Figure 2 shows $N_{\text{obs}}(m)$ for $T = 0.5$ days, $c = 10^{-4}$ days, and different p -values. The slope (b -value) is in all cases significantly smaller than the input value $b = 1$ of the complete data set.

1.2. Effect on Results of ETAS Simulations

[6] In agreement with the analysis by Shearer [2012], I perform self-similar simulations of the epidemic type aftershock sequence (ETAS) model [Ogata, 1988] according to

$$R(t) = \mu + \sum_{i:t_i < t} Q 10^{b(m_i - m_l)} \frac{p-1}{c^{1-p}} (c + t - t_i)^{-p} \quad (6)$$

where the productivity parameter Q depends on the branching ratio r according to $Q = r/[b \ln(10)(m_u - m_l)]$. In agreement with Shearer [2012], I choose the lower and upper truncation of the Gutenberg-Richter distribution as

$m_l = 0$ and $m_u = 7$, and $b = 1$. In accordance with recent detailed analyses of early aftershocks [Enescu et al., 2009] showing that the true c -value is likely smaller than a minute, the c -value is set to 10^{-4} days (8.64 s). The p -value has been set to 1.3, and the branching ratio is chosen in a way that the overall aftershock activity matches approximately the observed activity in southern California, namely $r = 0.6$. The a -value of the background activity is set to 4, leading to $10^{2.5}$ uncorrelated events per year, i.e., approximately one per day, above the magnitude threshold $m_c = 1.5$. I also tested some other parameter combinations finding that the shape of the resulting differential magnitude curves slightly depends on the chosen ETAS parameters, but the general results remain the same. It is important to note that the purpose of this comment is to show that most of the observations of Shearer [2012] are in general agreement with self-similar triggering but not to optimize for the best parameter set. A comprehensive parameter study would go beyond the scope of this article. Furthermore, please note that I analyzed for simplicity only time-dependent ETAS simulations ignoring the spatial distribution; that is, I implicitly assumed that also distant aftershocks can be correctly identified. For more sophisticated space-time simulations as used by Shearer [2012], the application of the same spatial selection windows as for the LSH catalog reduces the number of identified aftershocks to a fraction f of that in my simulations. Consequently, the fit to the observed values would require a branching ratio that is increased by the factor $1/f$. This explains why my fit requires only a branching ratio of $r = 0.6$ instead of $r = 0.8$ used by Shearer [2012].

[7] For given parameters, I performed $N = 1000$ simulations of 36 years. In each simulated catalog, the $m < m_c = 1.5$ events were removed. Furthermore, to avoid transient effects, I also removed the first year activity. The selection of mainshocks, foreshocks, and aftershocks is done in accordance with Shearer [2012]: Target events are mainshocks in the three magnitude bins 2.5–3.5, 3.5–4.5, and 4.5–5.5. To minimize the effects of ongoing aftershock activity triggered by $M > 5.5$ mainshocks, I exclude potential target events occurring within 3 months after $M > 5.5$ events. A mainshock has to be the largest event within 3 days before and 1/2 day after it and foreshocks and aftershocks are counted within $\pm 1/2$ day. For the differential magnitude plots, I averaged the numbers of foreshocks and aftershocks in each magnitude bin over target events in all N simulations. To examine the impact of incompleteness, I created for each simulation a realistic catalog where all earthquakes are removed for which the magnitude m_i does not fulfill the condition $m_i \geq m_{\text{cut}}(m_j, t_i - t_j)$ related to any preceding earthquake $j < i$.

[8] The comparison of the results for (a) self-similar simulations, (b) incomplete catalog versions of those simulations, and (c) seismicity in southern California is shown in Figure 3. While the b -value is found to be significantly smaller for southern California aftershocks in comparison to case (a) in agreement with Shearer [2012], the results for case (b) show a similar low b -value as a consequence of catalog incompleteness.

[9] Besides the b -value, also the estimated ratio between the number of foreshocks and aftershocks is affected by the catalog incompleteness. On average, the ratio is found to be significantly increased in the incomplete catalogs with

Figure 3. Differential magnitude curves for (a) complete and (b) incomplete synthetic catalogs in comparison with the results for southern California seismicity. Lower and upper curves refer to the mean number of foreshocks and aftershocks, while the bold to thin lines refer to mainshocks in the range 2.5–3.5, 3.5–4.5, and 4.5–5.5, respectively. The parameters of the simulations are $b = 1$, $\mu = 10^{2.5}$ [$m \geq 1.5$ events/year], $r = 0.6$, $c = 10^{-4}$ [days], and $p = 1.3$.

respect to the values found for the complete simulations. In particular, the values increase from 0.40 to 0.43 for mainshocks in the range of $M = [2.5, 3.5]$, from 0.09 to 0.13 for $M = [3.5, 4.5]$, and from 0.03 to 0.05 for $M = [4.5, 5.5]$. However, within the 1000 simulations, the ratio exceeded in only 7.9% of the synthetic catalogs the observed value of 0.09 for $M = [4.5, 5.5]$ and never the observed values of 0.61 and 0.24 for $M = [2.5, 3.5]$ and $[3.5, 4.5]$, respectively. Thus, the foreshock-aftershock ratio remains at least for the smaller mainshocks significantly higher in California than in simulations. However, a final proof of the significance requires a comprehensive study for systematically varied parameters ($\mu, r, m_l, m_u, b, c, p$), which is kept for future studies.

2. Båth Law

[10] Shearer [2012] argues that the observed aftershock number is too large to be consistent with the

Båth law, because an average magnitude difference of 1.2 is reproduced in self-similar simulations only if the branching ratio is around 0.5 as shown in Figure 1 of *Shearer* [2012]. In contrast, he needs a branching ratio of 0.8 [*Shearer*, 2012, Figure 10] to reproduce the observed level of aftershock activity. Similarly, a branching ratio of $r = 0.6$ is needed in my simulations to realize comparable aftershock numbers (see Figure 3). However, this comparison is misleading because the theoretical result related to Figure 1 of *Shearer* [2012] is derived under the assumption that all aftershocks, including late and distant events, are considered, while for the derivation of the empirical Båth law only aftershocks which are selected within space-time windows are taken into account. The latter does not take late or distant aftershocks into account, because they cannot be easily discriminated from background activity. Thus, the empirical Båth law is only valid for aftershocks clustered closely in space and time, while Figure 1 of *Shearer* [2012] is related to all aftershocks, and consequently, the discrepancy between the branching ratios does not disprove self-similarity triggering. In contrast, the almost perfect agreement between the differential magnitude curves of the simulated and observed aftershock activity in southern California (see Figure 3b) indicates that the mean maximum magnitude of closely clustered aftershocks is the same in the model and observations.

[11] In sum, the theoretical results presented in Figure 1 of *Shearer* [2012] could only be compared to observed aftershocks, if also the late and distant aftershocks are taken into account. However, this would certainly lead to a smaller empirical value of the mean magnitude difference between the mainshock and its largest aftershock and thus to a revised version of the Båth law, which would in return indicate a larger branching ratio in Figure 1 of *Shearer* [2012]. Anyway, due to the discrimination problems, the true branching ratio cannot be directly deduced from the observed magnitude differences for aftershocks selected in limited space-time windows.

3. Summary

[12] As shown in this comment, out of the three differences found by *Shearer* [2012] between observed and self-similar seismicity, two of them apparently do not contradict self-similarity triggering: the apparently lower b -value for aftershock activity seems to be related to catalog incompleteness, while the apparently too large aftershock number can be explained by the fact that the empirical Båth law is derived from closely clustered events and not for all aftershocks. However, the third difference, the larger foreshock-to-aftershock ratio observed in southern California, might be significant and related—as *Shearer* [2012] discusses—to embedded swarm activity.

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