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The Maximum Earthquake Magnitude in a Time Horizon: Theory and Case Studies

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Abstract

We show how the maximum magnitude within a pre-defined future time horizon may be estimated from an earthquake catalog within the context of Gutenberg-Richter statistics. The aim is to carry out a rigorous uncertainty assessment and calculate precise confidence intervals based on an imposed level of confidence α. In detail, we present a model for the estimation of the maximum magnitude to occur in a time interval T_f in the future, given a complete earthquake catalog for a time period T in the past and, if available, paleoseismic events. For this goal, we solely assume that earthquakes follow a stationary Poisson process in time with unknown productivity Λ and obey the Gutenberg-Richter law in magnitude domain with unknown b value. The random variables Λ and b are estimated by means of Bayes' theorem with non-informative prior distributions. Results based on synthetic catalogs and on retrospective calculations of historic catalogs from the highly active area of Japan and the low-seismicity, but high-risk region Lower Rhine Embayment in Germany indicate that the estimated magnitudes are close to the true values. Finally, we discuss whether the techniques can be extended in order to meet the safety requirements for critical facilities like nuclear power plants. For this aim, the maximum magnitude for all times has to be considered. In agreement with earlier work, we find that this parameter is not a useful quantity from the viewpoint of statistical inference.

Introduction

The maximum possible earthquake magnitude in a seismically active region is a key parameter of seismic hazard and risk analysis. The knowledge of this parameter is crucial for various issues like pricing strategies of insurance companies or the definition of requirements for building construction. The devastating $M9$ earthquake that occurred on 11 March 2011 in Tohoku, has demonstrated that the selection of appropriate sites for high risk industrial plants and critical infrastructure may be the most important field where quantitative constraints for catastrophic events are needed. In the framework of statistical modeling, these constraints can only be provided on the ground of data. However, regions like Germany are characterized by low seismic hazard and high seismic risk. Roughly speaken, this means that earthquakes are rare in general; however, due to the high population density as well as critical infrastructure which either exists (industry, nuclear power plants) or is planned (nuclear disposal facilities), the damage potential is enormous. In the light of sparse data, the estimation of relevant parameters is related to high uncertainties. Therefore it is an important challenge to provide a precise statement of uncertainties in terms of confidence intervals.

In a recent work, *Holschneider et al.* (2011) have shown in agreement with *Pisarenko et al.* (1996) that the maximum magnitude for all times in the framework of a doubly-truncated Gutenberg-Richter model (*Kijko* 2004) , that is the maximal possible magnitude in an infinite time horizon, is not a useful quantity for seismic hazard analysis, because the confidence interval diverges in general. This finding is independent on the type of truncation of the Gutenberg-Richter distribution. In particular, it also holds for a smoothly tapered Gutenberg-Richter distribution (*Kagan and Schoenberg*, 2001). *Holschneider et al.* (2011) argue that the maximum magnitude to occur in a finite time horizon is more useful, because large earthquakes have almost vanishing probability in finite time and thus the confidence intervals will be finite. *Pisarenko et al.* (2008) report the same finding in the context of extreme value statistics. In fact, the calculation of the maximum event, treated as a random variable, in a given time interval is a typical problem in extreme value statistics leading under certain assumptions to the General Extreme Value (GEV) distribution. *Epstein and Lomnitz*

(1966) have shown that the predictive distribution of the maximal earthquake magnitude to occur in a year is given by the Gumbel distribution, which is part of the family of GEV distributions, if a Poisson process in time and Gutenberg-Richter statistics in magnitude domain are assumed. Hazard estimates based on the Gumbel distribution require knowledge of the Poisson intensity (or the Gutenberg-Richter-a value) and the Gutenberg-Richter-b value. Of course it is possible to plug-in point estimates $\hat{a}$ for a and $\hat{b}$ for b, but in the light of the high uncertainties of these estimates, the predictive power of this method will be poor.

In the present work, we continue the study of *Holschneider et al.* (2011) and focus on the calculation of confidence intervals for the maximum magnitude in a finite time horizon. Again, we distinguish between the frequentist confidence interval and the Bayesian confidence interval. For the case that the magnitude distribution is exactly known, we show that both confidence intervals become equal in an asymptotic limit. In general, the frequentist approach becomes questionable for the study of a particular seismically active region, because probabilities are considered as relative frequencies of the outcome of a random experiment that is repeated several times. In terms of estimating parameters from an earthquake catalog, the calculation of a frequentist probability requires a large number of earthquake catalogs from the same region. This is, however, in contrast to the situation that only one catalog is available. The frequentist approach becomes useful for the risk assessment of portfolios, i.e. for insurance companies dealing with several objects like earthquake catalogs from different regions. For studies of single regions, a powerful tool is Bayesian analysis, as used by *Campbell* (1982). The goal is to account for all possible parameter values (e.g. a and b) by using Bayes' theorem. In particular, *Campbell* (1982) uses the concept of conjugate distributions after assuming that the uncertainties in the a and the b values follow a $\Gamma-$distribution.

These parameters are expressed through the "best estimates" of their mean value and standard deviation, which are then plugged into an extreme value distribution in order to calculate the seismic hazard. This approach has serious drawbacks, especially when applying it to low-seismicity regions: 1. The assumption of $\Gamma-$distributed uncertainties is questionable; 2. Plugging estimates of mean value and standard deviation into the extreme value distribution may introduce high errors depending on the goodness of the estimation. In the present work, we address these problems by using the following strategy: We solely assume a Poisson process in time with unknown productivity $\Lambda > 0$ and Gutenberg-Richter statistics with unknown $b > 0$. No further assumptions about Λ and b are made. Based on an earthquake catalog, we calculate the full Bayesian posterior distribution (or rather Bayesian predictive distribution) of the maximum magnitude in a given time period. Evaluation of this distribution allows for a comprehensive investigation of the uncertainties. In detail, we are able to provide formulas for the confidence interval of the maximum magnitude in a finite time horizon. For example, we can precisely address the question: Given a complete catalog covering 20 years, what is the magnitude that is not exceeded in the next 50 years with a given probability, say $1 - \alpha = 95\%$?

Our work is structured as follows: First, we derive the mathematical framework for the case that the magnitude distribution, i.e. the Gutenberg-Richter-b value is exactly known and only the productivity Λ of the Poisson process is unknown. Then we extend the methodology to Gutenberg-Richter statistics with unknown b value. The methods are applied first to synthetic data; second, we focus on a high-seismicity region (Japan), and on a low seismicity region, the Lower Rhine Embayment, Germany. We discuss the extension of the technique in order to calculate reference magnitudes for potential sites of critical facilities like nuclear power plants. A discussion on the conditions for the validity of the Poisson and the Gutenberg-Richter distribution as well as on the resulting requirements for data selection is provided.

Confidence Interval for the Maximum Magnitude in a Time Interval T for Known Magnitude Distribution $F_\theta(m)$

In this section, we use the unrealistic assumption that the magnitude distribution is exactly known. In the Extreme Value Distribution section, we derive the extreme values distribution in a general context. In the Terminology and Definition of Confidence Interval section, the confidence intervals of the maximum magnitude in a future time horizon are specified for a given an earthquake catalog. Then, we derive the frequentist confidence interval (Frequentist Approach section) and the Bayesian confidence interval (Bayesian Approach section) for a family of magnitude distributions F_θ depending on a parameter θ, which can be multidimensional. For both approaches, we show explicitely the result for $F_\beta(m) = 1 - \exp\left[-\beta(m - m_0)\right]$, which is the unbounded Gutenberg-Richter distribution for earthquakes with magnitude above a given threshold m_0 depending on the Gutenberg-Richter-b value or $\beta = b \log(10)$.

Gutenberg-Richter and Poisson Assumption

The following study is mainly based on two assumptions, both of which have important implications for the catalog selection and data pre-processing: first, earthquake magnitudes follow the Gutenberg-Richter distribution including the Gutenberg-Richter-b value, and second, earthquake rates are described by a stationary Poisson process in time with productivity Λ. The estimation of b requires a broad range of magnitude data, preferentially from the magnitude of the maximum observed earthquake down to the magnitude of completeness. On the other hand, the tendency for non-Poissonian earthquake clustering is strong for small magnitudes leading to errors in the estimation of Λ. Furthermore, in *Holschneider et al.* (2011) it is argued that the estimates of maximum magnitudes depend predominantly on the largest observed earthquakes. It is, therefore, desirable to study historic earthquake catalogs covering hundreds of years and spanning a range broader than one magnitude unit. Such a historic catalog including, say 20, moderate to large earthquakes, carries more information with respect to large future events than instrumental catalogs with thousands of earthquakes, but only one or two big events. In addition, the declustering of catalogs may be considered in order to better fulfill the Poisson assumption. It has to be pointed out that declustering is always a delicate issue because additional parameters and assumptions come into the game, and declustering algorithms might not be suitable for mega-earthquakes like the Sumatra mega-event. In a recent study, *Michael* (2011) has shown for the five $M \geq 8.5$ earthquakes worldwide between 2004 and 2011 that the hypothesis of a stationary Poisson process cannot be rejected, at least for the case of declustered seismicity. Adding, however, the recent M8.6 Sumatra event on 11 April 2012 the sequence becomes non-Poissonian, because the declustering algorithm is not suitable to identify this event as an aftershock of 2004 mega-earthquake (A. Michael, personal communication).

In the present work, we apply our methodology to two historic earthquake catalogs: a catalog of Japan covering 1300 years with $7 \leq m \leq 9$ and a catalog of the Lower Rhine Embayment since 1600, which is complete for $m \geq 4$ and includes a maximum earthquake with $m = 5.4$. For both cases, we will compare original seismicity with declustered seismicity using the method of *Gardner* *and Knopoff* (1974). In Japan, however, declustering may be problematic for the reasons mentioned above. In the case of the Lower Rhine Embayment, we provide the frequency-size distribution and will explicitely test the Poisson hypothesis using a Kolmogorov-Smirnov (KS) test.

Extreme Value Distribution

We consider a random variable X drawn from a given distribution $F(x)$. Then, the maximum of n independently drawn numbers $\{x_i\}_{i=1,\ldots,n}$ is a random variable with distribution

$$\Pr(\max\{x_i\} \leq x) = [F(x)]^n. \tag{1}$$

Given a Poisson process with productivity Λ, the probability of observing n events is calculated from the Poisson distribution

$$P(n;\Lambda) = \frac{\Lambda^n}{n!} e^{-\Lambda}. \tag{2}$$

In a random experiment, where values of X are drawn from $F(x)$ and the number of events is drawn independently from a Poisson process with productivity Λ, the probability that all values x_i are $\leq x$, can be calculated using the total probability theorem. Accordingly the probabilities of all independent possible outcomes (in terms of the number n of events) have to added:

$$\Pr(\max\{x_i\} \leq x) = P(x) = \sum_{n=0}^{\infty} \frac{\Lambda^n}{n!} \exp\left(-\Lambda\right)[F(x)]^n = \exp\left[-\Lambda(1 - F(x))\right]. \tag{3}$$

The corresponding probability density function is

$$p(x) = \frac{dP(x)}{dx} = \Lambda\ f(x) \exp\left[-\Lambda(1 - F(x))\right] \tag{4}$$

with f being the probability density function with respect to the distribution function $F(x)$

$$f(x) = \frac{dF(x)}{dx}. \tag{5}$$

In the specific case, where $F(x)$ is the unbounded Gutenberg-Richter distribution $F(x) = 1 - \exp[-\beta(x - m_0)]$, $P(x)$ is the Gumbel distribution (*Gumbel*, 1958), where Λ can also be expressed by the Gutenberg-Richter a-value (*Epstein and Lomnitz*, 1966).

Terminology and Definition of Confidence Interval

In this section, we use Eq. (3) to describe earthquakes following a Poisson process with given productivity Λ and magnitude distribution $F_\theta(m)$, where θ includes one or more parameters. In detail, we specify a fixed region in space, a future time interval of length T_f, and a past time interval of length T, where an earthquake catalog is given. Supposing that the events in the past and in the future arise from the same Poisson process and magnitude distribution, we address the question: What can be inferred from a complete earthquake catalog with n events $\{m_i\}$, $i = 1, \ldots n$ during time T with respect to future predictions, in particular about the maximum earthquake to occur in time T_f? Under the assumption of constant Poisson rate we have

$$T_f = \frac{\Lambda_f}{\Lambda} T. \tag{6}$$

In Table 1, we list the terminology, which will be used in the remainder of this paper.

For exactly known θ in $F_\theta(m)$, the rate Λ remains the only unknown parameter. In general however, we have at best a (physically motivated) parameterized family F_θ of event size-distributions. Then, in addition to the randomness of the event generating process itself, we have to take into account the uncertainties of both Λ and θ.

We will consider two approaches to deal with this. In the frequentist setting, we look at confidence intervals of level $1-\alpha$, $\alpha \in [0, 1)$, for the maximum magnitude to occur in the prediction interval T_f. A frequentist confidence interval is defined as a function

$$\psi(\text{observed catalog}) = \psi(n, \{m_i\}) \to \mathbb{R}$$

or equivalently a family of functions ψ_n associated with a catalog $\{m_i\}$ of length n

$$\psi_n : \{m_i\} \to \mathbb{R}, \tag{7}$$

which assigns to every observed catalog an upper bound ψ such that on average we fail in at most a fraction of α times to give a correct upper bound for μ:

$$\mathbb{E}\left(\Pr(\psi(\text{catalog}) < \mu) | \Lambda, \theta\right) \leq \alpha, \quad \text{for all } \Lambda, \theta. \tag{8}$$

The expectation is over random catalogs in the past drawn according to the parameters Λ and θ. The probability is about the random behavior in the future of μ for these same parameters. In more concrete terms this reads

$$\sum_{n=0}^{\infty} \Pr(n, \psi_n(\{m_i\}) < \mu \mid \theta, \Lambda) \leq \alpha \quad \text{for all } \theta, \Lambda \tag{9}$$

Clearly, smaller values of ψ_n are the better in view of applications. The goal is to present optimal confidence intervals with respect to this (half) order relation. In case that the parameter θ is known with certainty, only the uncertainties with respect to Λ have to be considered. In this case the actual observed magnitudes do not contribute and the confidence intervals satisfy

$$\sum_{n=0}^{\infty} \Pr(n, \psi_n < \mu \mid \Lambda) \leq \alpha \quad \text{for all } \Lambda \tag{10}$$

The other approach uses Bayes' theorem. Plugging prior information about θ and Λ into a prior distribution $P_0(\theta, \Lambda)$, the likelihood function $L(\{m_i\}|\theta, \Lambda)$ and the resulting posterior distribution

$P(\theta, \Lambda|\{m_i\})$ can be calculated:

$$P(\theta, \Lambda|\{m_i\}) = c\, L(\{m_i\}|\theta, \Lambda)\, P_0(\theta, \Lambda), \tag{11}$$

where c is a normalization constant of the posterior distribution and L is the likelihood function

$$L(n, \{m_i\}|\Lambda, \theta) = \underbrace{\frac{\Lambda^n e^{-\Lambda}}{n!}}_{g_\Lambda(n)} \underbrace{\prod_{i=1}^{n} f_\theta(m_i)}_{h_\theta(\{m_i\})}, \tag{12}$$

which can be written as a product of two functions, $g_\Lambda(n)$ and $h_\theta(\{m_i\})$, one depending on n and the other depending on the magnitudes $\{m_i\}$. The parameter Λ refers to the time coverage T of the catalog. According to Eq. (6) the corresponding value for the future time horizon is

$$\Lambda_f = \frac{T_f}{T}\Lambda. \tag{13}$$

The probability that the maximum magnitude μ in time T_f is smaller than or equal to m is

$$P(\mu \leq m|\{m_i\}) = \int_\Theta d\theta\, d\Lambda\ P(\theta, \Lambda|\{m_i\}) P(\mu \leq m|\theta, \Lambda_f), \tag{14}$$

where Θ denotes the domain for values of θ and Λ, and $P(\mu|\theta, \Lambda_f)$ is calculated from Eq. (3):

$$\begin{aligned} P(m|\theta, \Lambda) &= \exp\left[-\Lambda_f(1 - F_\theta(m))\right] \\ &= \exp\left[-(T_f/T)\Lambda(1 - F_\theta(m))\right]. \end{aligned} \tag{15}$$

Because the Bayesian approach delivers a probability distribution, the calculation of confidence intervals becomes straightforward.

In the remainder of this section, we consider the case that the frequency-size distribution $F_\theta(m)$ is known. In this special case, the parameter θ is exactly known and does not occur anymore as an estimation parameter in Eq. (11) and (12). For example, the likelihood function in Eq. (12) reduces essentially to $g_\Lambda(n)$, while the function $h_\theta(\{m_i\})$ becomes constant and can be absorbed by the normalization constant. In the Bayesian Estimation of Maximum Magnitude and Waiting Time for an Unknown Magnitude Distribution $F_\theta(m)$ section and the Bayesian Estimation of Maximum Magnitudes and Waiting Times for the Unbounded Gutenberg-Richter Distribution section, this assumption will be abandoned and a Gutenberg-Richter distribution with unknown b values will be used.

Frequentist Approach

Since the likelihood function (Eq. 12) can be written as a product of a function $g_\Lambda(n)$ and $h_\theta(\{m_i\})$, it follows that n is a sufficient statistics for Λ (*Fisher*, 1922); consequently, the estimated value of Λ depends solely on the number n of earthquakes rather than on all the details of the catalog. Assuming a Poisson process with productivity Λ and a magnitude distribution $F_\theta(m)$ with known θ, the probability for the maximum magnitude in time T_f to be smaller than or equal to m and to have n events in the catalog during time T, is

$$Pr(n, \mu \leq m|\Lambda, \theta) = \frac{\Lambda^n}{n!} e^{-\Lambda} \exp\left\{-\frac{T_f}{T}\Lambda[1 - F_\theta(m)]\right\} = \frac{\Lambda^n}{n!} e^{-\Lambda} \exp\{-(T_f/T)\Lambda[1 - F_\theta(m)]\}. \quad (16)$$

We first suppose that the distribution of magnitudes is known (i.e. θ is known). Then the only uncertainty comes from Λ, the productivity. For the calculation of the frequentist confidence interval defined by Eq. (7), each possible earthquake catalog can be considered as a possible outcome of a random experiment. Therefore, all possible catalogs arising from a Poisson process with productivity Λ have to be taken into account in terms of event numbers n. Applying the total probability theorem, we have to search for numbers of the form ψ_n with

$$\underbrace{\sum_{n=0}^{\infty} \frac{\Lambda^n}{n!} e^{-\Lambda} \exp\{-\Lambda(T_f/T)[1 - F_\theta(\psi_n)]\}}_{:=S(\Lambda,\theta,\psi_n)} \geq 1 - \alpha \qquad \text{for all } \Lambda > 0. \tag{17}$$

Without further assumptions, it is not possible to provide analytical expressions for ψ_n from Eq. (17). In the following, we show that an asymptotic expression for small values of α and high values of Λ can be derived, given that the distribution F_θ is exactly known. For small values $\alpha \ll 1$, we can expand the exponential expression in $S(\Lambda, \theta, \psi_n)$ into a series in $\varepsilon_n = (T_f/T)(1 - F_\theta(\psi_n))$ leading to

$$S(\Lambda, \theta, \psi_n) = \sum_{n=0}^{\infty} \frac{\Lambda^n}{n!} e^{-(1+\varepsilon_n)\Lambda} = 1 - \sum_{n=0}^{\infty} \frac{\Lambda^{n+1}}{n!} \varepsilon_n e^{-\Lambda} + \ldots \tag{18}$$

The function $s_n = \frac{\Lambda^{n+1}}{n!} \varepsilon_n e^{-\Lambda}$ has a local maximum at $\Lambda_n = n + 1$; the other terms in the sum will change the location of the local maxima of the order of magnitude $O(\varepsilon)$. The value of the sum at the local maximum is to first order in ε equal to the value at the unchanged points, $k \in \mathbb{N}$, leading to

$$\sum_{n=0}^{\infty} \frac{k^{n+1}}{(n+1)!} e^{-k} (T_f/T)(n + 1)[1 - F_\theta(\psi_n)] = \alpha, \tag{19}$$

which can also be written as a matrix equation

$$\sum_{n=0}^{\infty} A_{kn} \eta_n = A\eta = \alpha \tag{20}$$

with

$$A_{kn} = \frac{k^{n+1}}{(n+1)!} e^{-k} \text{ and } \eta_n = (T_f/T)(n + 1)[1 - F_\theta(\psi_n)]. \tag{21}$$

This infinite system of equations can only be solved numerically. However, Eq. (19) is fulfilled, if we formally set

$$(T_f/T)(n + 1)[1 - F_\theta(\psi_n)] = \frac{\alpha}{1 - e^{-k}}, \tag{22}$$

which becomes in the asymptotic limit $k \to \infty$ and expression that does not involve k anymore,

$$(T_f/T)(n+1)[1 - F_\theta(\psi_n)] = \alpha, \tag{23}$$

and which therefore gives an asymptotic solution for large n

$$\psi_n = F_\theta^{-1}\left[1 - \frac{\alpha}{(T_f/T)(n+1)}\right]. \tag{24}$$

Inserting the Gutenberg-Richter distribution $F_\beta(\psi_n) = 1 - \exp\left[-\beta(\psi_n - m_0)\right]$, which depends only on β $(= b\log(10))$, we get

$$\psi_n = m_0 - \frac{1}{\beta}\log\left[\frac{\alpha}{(T_f/T)(n+1)}\right]. \tag{25}$$

Unknown Parameter θ

Finally, for the sake of completeness, we consider the case of unknown θ. In this case the observed magnitudes carry information about the unknown parameter θ and we need to satisfy

$$\sum_{n=0}^{\infty}\left(\prod_{i=1}^{n}\int dF_\theta(m_i)\right)\frac{\Lambda^n}{n!}e^{-\Lambda}\exp\{-\Lambda(T_f/T)[1 - F_\theta(\psi_n(m_1,\ldots,m_n))]\} \geq 1 - \alpha \qquad \text{for all } \Lambda > 0,\ \theta. \tag{26}$$

This relation can only be solved numerically for ψ_n. In this paper however we will not pursue the numerical analysis of this expression.

Bayesian Approach

In the first step of the Bayesian approach, we consider again the case that $F_\theta(m)$ is exactly known and the productivity Λ is the only unknown parameter which is estimated by means of Bayes' theorem (Eq. 11). As discussed earlier, the likelihood function (Eq. 12) reduces to

$$L(n, \{m_i\}|\Lambda) \propto g_\Lambda(n). \tag{27}$$

The Bayesian posterior in Eq. (11) becomes

$$P(\Lambda|\{m_i\}) \propto g_\Lambda(n)\, P_0(\Lambda), \tag{28}$$

with the prior distribution $P_0(\Lambda)$. Finally, we use Eq. (14) and (15):

$$P_\theta(\mu \leq m|\{m_i\}) = \int_0^\infty \frac{\Lambda^n}{n!} \exp(-\Lambda) \exp\left[-(T_f/T)\Lambda(1-F_\theta(m))\right] P_0(\Lambda) d\Lambda. \tag{29}$$

Here $P(\mu \leq m|\{m_i\})$ denotes the posterior distribution for the maximum magnitude in the future time interval T_f being smaller than or equal to m given n earthquakes in the past time interval of length T. The distribution $P_0(\Lambda)$ includes possible prior information about Λ. In the common situation that no such information is available, a non-informative prior is chosen, in the simplest case a flat prior $P_0(\Lambda) = const$. We note that the flat prior is improper in the sense that $P_0(\Lambda)$ is not normalized. This is, however, not a problem as long as the posterior distribution can be normalized.

Substituting $z = \Lambda[1 + (T_f/T)(1 - F_\theta(m))]$, the integral in Eq. (29) can be transformed to a $\Gamma-$function with respect to z. Using the flat prior $P_0(\Lambda) = const.$ the value becomes

$$P(\mu \leq m|\{m_i\}) = \frac{1}{\{1 + (T_f/T)[1 - F_\theta(m)]\}^{n+1}}. \tag{30}$$

We note that for magnitudes m with $F_\theta(m) = 0$, $P(\mu \leq m|\{m_i\}) > 0$ becomes constant. In particular, for a distribution F with a lower magnitude cutoff m_0, the case $m < m_0$ has finite probability, namely the probability that no earthquake occurs within T_f $(n = 0)$.

The corresponding probability density function of the maximum event in the future μ is

$$p(\mu) = \frac{(T_f/T)(n+1) f_\theta(\mu)}{[1 + (T_f/T)(1 - F_\theta(\mu))]^{n+2}}. \tag{31}$$

From Eq. (30), Bayesian confidence intervals can be calculated: Given a confidence level $\alpha \in [0;1)$, having observed n events in the past, the probability that during time T_f an earthquake with $m > \psi_n$ occurs, is at most α, for ψ_n given by

$$\psi_n = F_\theta^{-1}\left\{1 - \frac{T}{T_f}\left[\left(\frac{1}{1-\alpha}\right)^{\frac{1}{n+1}} - 1\right]\right\}. \tag{32}$$

As in the previous section, we insert the Gutenberg-Richter distribution with known β for F_θ and get

$$\psi_n = m_0 - \frac{1}{\beta}\log\left\{\frac{T}{T_f}\left[\left(\frac{1}{1-\alpha}\right)^{\frac{1}{n+1}} - 1\right]\right\}. \tag{33}$$

Finally, we show that the frequentist and the Bayesian confidence interval (Eq. 25 and 33) become identical in the limit $\alpha \ll 1$. The term $(1-\alpha)^{-1/(n+1)}$ in Eq. (32) and (33) can be expanded for $\alpha \ll 1$ as

$$(1-\alpha)^{-\frac{1}{n+1}} \approx 1 + \frac{1}{n+1}\alpha \tag{34}$$

leading to the same formula as in Eq. (24) and (25):

$$\psi_n \approx F_\theta^{-1}\left\{1 - \frac{T}{T_f}\left[1 + \frac{\alpha}{n+1} - 1\right]\right\} = F_\theta^{-1}\left[1 - \frac{\alpha}{(n+1)(T_f/T)}\right]. \tag{35}$$

In the Bayesian Estimation of Maximum Magnitude and Waiting Time for an Unknown Magnitude Distribution $F_\theta(m)$ section and the Bayesian Estimation of Maximum Magnitudes and Waiting Times for the Unbounded Gutenberg-Richter Distribution section, this approach will be extended in order to take into account the uncertainties of b (or β).

Bayesian Estimation of Maximum Magnitude and Waiting Time for an Unknown Magnitude Distribution $F_\theta(m)$

In the previous section, the exact knowledge of the magnitude distribution $F_\theta(m)$ has been assumed. However, uncertainties of the parameters contained in θ, cannot be neglected. Therefore, we have to take into account the full Bayesian posterior function $P(\theta, \Lambda|\{m_i\})$ from Eq. (11) including uncertainties of θ. Because the likelihood function can be written as a product (see Eq. 12)

$$L(n, \{m_i\}|\Lambda, \theta) = g_\Lambda(n) h_\theta(\{m_i\}), \tag{36}$$

we can calculate the posterior with respect to θ

$$P(\theta|\{m_i\}) \propto h_\theta(\{m_i\}) P_0(\theta) \qquad \text{with} \qquad h_\theta(\{m_i\}) = \prod_{i=1}^{n} f_\theta(m_i), \tag{37}$$

where $P_0(\theta)$ is a (flat) prior for θ.

Combining this result with Eq. (31), the Bayesian posterior density of the maximum magnitude in a finite time horizon given by T_f becomes

$$p_{T_f}(\mu|\{m_i\}) \propto \int_\Theta \frac{f_\theta(\mu)}{[1 + (T_f/T)(1 - F_\theta(\mu))]^{n+2}} P(\theta|\{m_i\}) \; d\theta, \tag{38}$$

where Θ denotes the domain for the (multidimensional) parameter θ; for example, in the case of the unbounded Gutenberg-Richter distribution, θ reduces to a single value, namely β, and the integration will be carried out from 0 to ∞.

On the other hand, we can fix a target magnitude μ_T and consider the waiting time T_f to the next earthquake with magnitude $\geq \mu_T$ as a random variable that is estimated:

$$\begin{aligned} P_{\mu_T}(T_f|n, \theta) &= 1 - \int_0^\infty \frac{\Lambda^n}{n!} \exp(-\Lambda) \exp[-(T_f/T)\Lambda(1 - F_\theta(\mu_T))] d\Lambda \\ &= 1 - \frac{1}{[1 + (T_f/T)(1 - F_\theta(\mu_T))]^{n+1}}. \end{aligned} \tag{39}$$

The density of Eq. (39) with respect to T_f becomes

$$p_{\mu_T}(T_f|n,\theta) = \frac{(n+1)[1-F_\theta(\mu_T)]}{[1+(T_f/T)(1-F_\theta(\mu_T))]^{n+2}}. \tag{40}$$

Combining now Eq. (37) with a flat prior density $P_0(\theta)$ and Eq. (40), we get the posterior density of T_f for n observed events as a function of μ_T:

$$p_{\mu_T}(T_f|\{m_i\},n) \propto \int_\Theta \frac{1-F_\theta(\mu_T)}{[1+(T_f/T)(1-F_\theta(\mu_T))]^{n+2}} P(\theta|\{m_i\})\ d\theta. \tag{41}$$

Bayesian Estimation of Maximum Magnitudes and Waiting Times for the Unbounded Gutenberg-Richter Distribution

For observed seismicity, it is widely accepted that the frequency-magnitude statistics of earthquakes follows the Gutenberg-Richter law (*Gutenberg and Richter*, 1956). Therefore, we apply Equations (37) to (41) for the Gutenberg-Richter law (Estimation Based on Catalog Data section). In the Estimation Based on Catalog Data and a Paleoearthquake section, we also show, how additional paleoseismological knowledge can easily be taken into account.

Bounded Versus Unbounded Gutenberg-Richter Distribution

The Gutenberg-Richter (GR) distribution is commonly used in two versions: The unbounded GR distribution allows magnitudes to have infinite size:

$$F_\beta(m) = 1 - \exp\left[-\beta(m-m_0)\right]; \quad m \geq m_0. \tag{42}$$

Although the probability of occurrence of very large earthquakes approaches zero, this version violates, in principle, the law of energy conservation. Technically this flaw can be overcome by truncating the unbounded GR distribution at a finite value M:

$$F_{\beta M}(m) = \frac{\exp(-\beta m_0) - \exp(-\beta m)}{\exp(-\beta m_0) - \exp(-\beta M)}; \quad m_0 \le m \le M. \tag{43}$$

The value of M is, however, unknown. A comprehensive analysis on the estimation of M is given by *Holschneider et al.* (2011); in this work, it is shown that a rigorous statistical estimation of M from an earthquake catalog including precise confidence intervals is not possible. For the present study, we argue that the question of truncation plays no role for the estimation of the maximum magnitude in a time horizon, as long as the length of the time horizon is short or moderate (< 1000 years or so). The reason is that the probability of occurrence of a very large event is negligible on such time intervals. This can be illustrated in the following way: Assume that Λ and the b-value were exactly known: In a setup with $m_0 = 4$, $b = 1$, and, on average, one earthquake with magnitude $\ge m_0$ in ten years, we estimate the maximum magnitude m_{95} in the next 50 years with confidence level $\alpha = 0.05$ using Eq. (3) for the truncated GR distribution (Eq. 43) $F_M(m)$ with three values $M = 7.5, 8.5, 12$. For each value of M, we calculate the relative error of m_{95} in relation to the corresponding estimate from the unbounded GR distribution $F_{M=\infty}(m)$ (Eq. 42). In the case $M = 7.5$, the use of the unbounded GR distribution leads to an overestimation of 0.2%, for $M = 8.5$, this value becomes 0.02%, and for $M = 12$ it is $< 10^{-6}$. Since the overestimation of m_{95} for the unbounded GR distribution compared to the truncated GR distributions is negligible and the exact value of M is unknown, it is justified to use the unbounded GR distribution. The unrealistic high magnitudes that violate energy conservation, do not contribute to the estimation of maximum magnitudes for small and moderate time horizons. The picture changes, however, if the time horizons become large, e.g. several thousands of years. For this case, the unbounded GR distribution has to be replaced by the truncated one, and M becomes an unknown parameters similar to β and Λ. The corresponding methods will be developed in the Application IV: Critical Facilities section.

Estimation Based on Catalog Data

Now, we focus on the case of small and moderate time horizons. As discussed in the Bounded Versus Unbounded Gutenberg-Richter Distribution section, we will use the unbounded Gutenberg-Richter distribution. After observing n earthquakes with magnitudes m_i, the Bayesian posterior distribution (Eq. 37) using a flat prior distribution of β becomes

$$P(\beta|\{m_i\}) \propto \beta^n \exp[-\beta n(\langle m\rangle - m_0)], \tag{44}$$

where n and the sample mean of the magnitudes $\langle m\rangle$ are sufficient statistics for β. Finally, Eq. (37), (38) and (44) can be combined in order to compute the posterior probability density $p_{T_f}(m)$ accounting for uncertainties of both Λ and β:

$$p_{T_f}(\mu|\{m_i\}, n) \propto \int_0^\infty d\beta\ \beta^{n+1} \frac{\exp[-\beta(\mu - m_0)]\exp[-\beta n(\langle m\rangle - m_0)]}{\{1 + (T_f/T)[1 - F_\beta(\mu)]\}^{n+2}}. \tag{45}$$

Analogously, we can derive the formula for the Bayesian posterior density with respect to the waiting time T_f to an earthquake with given target magnitude μ_T. From Eq. (37), (41) and (42), we get

$$p_{\mu_T}(T_f|\{m_i\}, n) \propto \int_0^\infty d\beta\beta^n \frac{\exp[-n\beta(\langle m\rangle - m_0)]\exp[-\beta(\mu_T - m_0)]}{\{1 + (T_f/T)\exp[-\beta(\mu_T - m_0)]\}^{n+2}}. \tag{46}$$

Estimation Based on Catalog Data and a Paleoearthquake

In this section we take into account the knowledge of a single paleoearthquake with magnitude $m_P \pm \Delta m$ occurring as the largest event during a time interval T_P, which covers typically some thousand years. The generalization to more than one paleoevent is straightforward and is therefore not presented in this study. Assuming a Poisson process with intensity Λ/T and the unbounded Gutenberg-Richter distribution $F_\beta(m)$ (Eq. 42), the probability that at least one earthquake with magnitude between $m_P - \Delta m$ and $m_P + \Delta m$ occurs during the interval of length T_P is

$$P_{\beta\Lambda}(m_P) = \sum_{n=1}^{\infty} \frac{(\gamma\Lambda)^n}{n!} \exp(-\gamma\Lambda) \underbrace{[F_\beta(m_P + \Delta m) - F_\beta(m_P - \Delta m)]}_{:=\Delta F_\beta(m_P)}; \tag{47}$$

here $\gamma = T_p/T$ is the time interval in units of the catalog length T. As long as no other information is available, the true magnitude of the paleoearthquake is assumed to arise from a uniform distribution within the interval $[m_0 - \Delta m; m_0 + \Delta m]$. Evaluating the sum in Eq. (47) gives

$$P_{\beta\Lambda}(m_P) = \Delta F_\beta(m_P)[1 - \exp(-\gamma\Lambda)] \tag{48}$$

Using this result, Eq. (29) becomes

$$P_{T_f,\beta,m_P,\Delta m,T_p}(\mu \leq m|\{m_i\}, n) =$$

$$\int_0^\infty \frac{\Lambda^n}{n!} \exp(-\Lambda) \Delta F_\beta(m_P)[1 - \exp(-\gamma\Lambda)] \exp[-(T_f/T)\Lambda(1 - F_\beta(m))] P_0(\Lambda) d\Lambda. \tag{49}$$

Inserting the flat prior for $P_0(\Lambda)$ and evaluating the integral with respect to Λ leads to the density

$$p_{T_f,\beta,m_P,\Delta m,T_p}(\mu|\{m_i\}, n) \propto$$

$$\int_0^\infty d\beta \left[\frac{f_\beta(\mu)\Delta F_\beta(m_P)}{[1+(T_f/T)(1-F_\beta(\mu))]^{n+2}} + \frac{f_\beta(\mu)\Delta F_\beta(m_P)}{[1+\gamma+(T_f/T)(1-F_\beta(\mu))]^{n+2}}\right] \beta^n \exp[-\beta n(\langle m\rangle - m_0)] \tag{50}$$

in analogy to Eq. (45).

The equation corresponding to Eq. (46) for the waiting time T_f to the next earthquake exceeding a target magnitude μ_T becomes

$$p_{\mu_T,m_P,\Delta m,T_p}(T_f|\{m_i\},n) \propto$$

$$\int_0^\infty d\beta \left[\frac{(1-F_\beta(\mu_T))\Delta F_\beta(m_P)}{[1+(T_f/T)(1-F_\beta(\mu_T))]^{n+2}} + \frac{(1-F_\beta(\mu_T))\Delta F_\beta(m_P)}{[1+\gamma+(T_f/T)(1-F_\beta(\mu_T))]^{n+2}}\right] \beta^n \exp\left[-\beta n(\langle m\rangle - m_0)\right]. \tag{51}$$

Application I: Synthetic Earthquake Data

Before applying the methods developed in the previous sections to real earthquake catalogs, we perform tests using synthetic data which have been created under controlled conditions; in particular, we simulate earthquake catalogs with known values of Λ and b addressing two questions: 1. Is the estimated maximum magnitude consistent with the outcome of a frequently repeated simulation of future seismicity? 2. Does the method proposed in this study improve the results achieved from the Campbell-method based on the Bayesian extreme-value distribution (*Campbell*, 1982)?

When we compare magnitude values extracted from the Bayesian posterior distribution with results from repeated Monte-Carlo simulations, we essentially compare them with values from frequentist confidence intervals. Therefore, the calculations in this section do not reflect the full Bayesian point of view. However, for practical purposes, we feel that it is important to provide a test on the performance of the method; because the Bayesian posterior distribution is not testable in a rigorous sense, we assume the results coming from a "black box" and test them in a frequentist context.

Our testing includes the following steps:

1. For given Λ, b, and m_0, we generate 10,000 earthquake catalogs with time coverage T, each

catalog based on a Poisson process with intensity Λ/T and a Gutenberg-Richter distribution $F_b(m) = 1 - 10^{-b(m-m_0)}$.

2. For each catalog, we estimate the maximum magnitude $\tilde{m}_{95}$ in a future time horizon T_f (50 years) using the confidence level $\alpha = 0.05$.

3. We generate 10,000 future catalogs with time coverage T_f. From each catalog we extract the maximum magnitude and calculate m_{95}, the 95% quantile of the distribution of maximum magnitudes.

4. We count the fraction of cases, where m_{95} underestimates the maximum magnitude in the future catalog. Due to the imposed confidence level of $\alpha = 0.05$, this fraction should be close to 5%.

5. We compare the estimated values $\tilde{m}_{95}$ (from step 2) with the "true" value m_{95} from step 3.

For our tests, we use the following values for the parameters: $\Lambda/T = 0.1yr^{-1}$, $b = 1$, $m_0 = 4$, $T_f = 50yr$ and $\alpha = 5\%$. First, we focus on step 4 in order to test the overall performance of our method. Using $T = 1000$ years catalog length to estimate the maximum magnitude in a future time interval of $T_f = 50$ years, the maximum magnitude is underestimated in 6.2% of the 10,000 cases. For shorter catalogs ($T = 100$ years) this fraction is 5.5%. These numbers are in overall good agreement with the imposed confidence level $\alpha = 5\%$.

Performing step 5, the length of the (past) catalog T is also a crucial parameter: For a high value of T corresponding to a large number of events, we expect that the method in this study as well as the Campbell-method provide good estimates of the true value in terms of the future scenarios (step 3). This is because the uncertainties of Λ and b will be small for long catalogs. A corresponding simulation result is given in Fig. 1(a) for $T = 1000$ years. The "true" value of m_{95} is found to be $m_{95} = 6.09$ (black vertical line); the estimation based on the method of *Campbell* (1982) results on average in $\langle \tilde{m}_{95}^{C} \rangle = 6.06$, while the distribution in Eq. (45) leads to $\langle \tilde{m}_{95}^{ZHH} \rangle = 6.03$.

In the more relevant case of a short catalog, the growing uncertainties in the estimation of Λ and b will clearly influence the posterior distribution of the maximum magnitude in the time horizon T_f. Results for $T_f = 100$ years are shown in Fig. 1(b). The corresponding values for the upper bound of the 95% confidence interval are $m_{95} = 6.11$ (black), $\langle \tilde{m}_{95}^{ZHH} \rangle = 6.44$ (solid), and $\langle \tilde{m}_{95}^{C} \rangle = 6.79$ (dashed). We find that both methods overestimate the value m_{95}. The bias in our method is, however, smaller than that in the Campbell-method. Furthermore, for $T = 100yr$ the fraction of cases with magnitude estimates that are unrealistically high ($m_{95} > 10$) is still 1.4% for the Campbell method, while the corresponding fraction is only 0.3% for our method.

We conclude that for small number of data our technique improves the uncertainty assessment compared to the method of *Campbell* (1982). However, both methods use Bayes' theorem and depend, therefore, on a prior distribution which is, strictly speaking, subjective and arbitrary. Even if no seismological prior information is given, different options for prior distributions are available. Apart from the flat prior used in this study, we mention the Jeffreys prior (*Jeffreys*, 1946) that is invariant with respect to reparametrization of the parameter to be estimated. For example, it makes no difference whether m or 10^m is estimated. Having in mind that the size of an earthquake can be expressed with different measures (e.g. magnitude, moment, energy), the Jeffreys prior might be a suitable alternative, although there is no compelling reason for this choice. A detailed investigation of this question is left for future work.

Application II: Retrospective Estimations of the Tohoku Earthquake

The M9 Tohoku earthquake in Japan (*Peng et al.*, 2012), which occured on 11 March 2011, offers excellent opportunities for retrospective testing of seismicity models, because high quality data are available for both, instrumental and historic seismicity. We emphasize, however, that our methods to estimate maximum magnitudes are not suitable for rigorous retrospective testing for the following reasons: First, we consider maximum magnitudes in a pre-defind time interval, where the length T_f depends on the type of application. Second, the outcome of our estimations is not a single value of μ, but a full probability distribution, which is eventually evaluated with respect to a pre-defined confidence level α expressing the probability of error one is willing to accept. Using a catalog that ends before 2011, both parameters can be adjusted in order to "forecast" the Tohoku earthquake, which is, of course, not a reasonable mission. Instead, we test the plausibility of our method by performing the following experiments:

1. We use a catalog of historic earthquake in Japan, beginning in 684 and 30 years (option 1), and 50 years (option 2) before the Tohoku earthquake.

2. Based on a confidence level of $1-\alpha = 0.95$, we calculate μ for a future time interval of $T_f = 30$ years (option 1), and $T_f = 50$ years (option 2).

3. For both options, we also provide the probability that the magnitude $\mu_t = 9.0$ will be exceeded within 30 years, and within 50 years.

The earthquake catalog is combined from the JMA catlog from 1926 to 2005, and the NOAA catalog from 684 to 1925. Visual inspection of the frequency-size distribution provides catalog completeness for magnitudes $m \geq 7.0$. The combined catalog from 684 to 2005 contains 234 earthquakes; a maximum-likelihood estimate of the b value results in $b = 0.98 \pm 0.06$. Despite the large time coverage and the high value of the magnitude of completeness, we provide results not only for the original catalog, but also for the declustered seismicity applying the method of *Gardner and Knopoff* (1974) The declustered catalog contains 211 earthquakes with $b = 0.93 \pm 0.06$.

Results are provided in Table 2. The values are in overall reasonable in relation to the Tohoku earthquake. The results show that a M9 event was not unexpected. The probability for such an earthquake or an even larger one is 7% and 12% for the original catalog, respectively. The declustering leads to small changes; results are given in parentheses. Finally, we provide the value of μ based on $1 - \alpha = 0.95$ and $T_f = 50$ years for the catalog until today including the M9 Tohoku earthquake. For the original catalog with 243 earthquakes, we find $\mu = 9.10$; for the declustered catalog including 217 events the value is $\mu = 9.17$. It is conspicuous that these values are smaller in comparison to the truncated catalogs without the recent mega-event. However, we emphasize that despite the occurrence of the Tohoku earthquake in 2011, the average magnitude $\langle m \rangle$ (see Eq. 45) slightly decreased in the last 50 years.

Application III: Lower Rhine Embayment, Germany

In this section, we focus on a low-seismicity region, the Lower Rhine Embayment, Germany. Compared to Japan, the most conspicuous difference is the sparseness of data and, of course, the absence of mega-earthquakes. In the first part, we introduce the earthquake catalog of the Lower Rhine Embayment. In the second part, we present results for the case of known magnitude distribution and, in the third part, for the general case. Finally, we discuss a retrospective estimation as in the previous section with focus on the 1992 Roermond earthquake.

Data

We investigate earthquakes in the Lower Rhine Embayment (LRE), Germany. In this section, we briefly describe the data; for more details, we refer to *Reamer and Hinzen* (2004), *Hinzen and Reamer* (2007) and *Schmedes et al.* (2005). The LRE is of particular interest for seismic hazard assessment, because it it characterized by a large damage potential in the regions around Cologne. The strongest earthquakes include the 1756 Düren earthquake with $m_w = 5.4$ and the 1992 Roermond event with $m_w = 5.2$, while paleoseismic studies suggest an event with magnitude 6.7 (*Camelbeeck et al.*, 2000). The earthquake catalog of the Northern Rhine area can be accessed online (see Data and Resources section). In this work, we generate a subcatalog for the LRE in the same spatial region as shown in Fig. 1 in *Schmedes et al.* (2005). This catalog covers the time from 1600 until 2011 and has been partly published in *Reamer and Hinzen* (2004). Local magnitudes have been transformed to moment magnitudes according to the relation

$$m_w = 0.722 m_l + 0.743 \tag{52}$$

suggested in *Reamer and Hinzen* (2004). In the present study, we analyze two catalogs: First, the catalog from 1600 to 2011 with a magnitude of completeness of $m_0 = 4.0$, containing 26 earthquakes during the time interval of length $T = 411$ years in the study area. Figure 2(a) shows the magnitude distribution. This catalog will be labeled hereinafter as LRE. Second, the catalog after removing six aftershocks, which have been identified with the algorithm of *Gardner and Knopoff* (1974).

The frequency-size distribution of the declustered catalog, called LRE-DEC, is given in Fig. 2(b).

The maximum likelihood estimation $\hat{b}$ of the Gutenberg-Richter b value is

$$\begin{aligned} \hat{b} &= 1.08 \pm 0.21 \qquad \text{for catalog LRE;} \\ \hat{b} &= 1.00 \pm 0.22 \qquad \text{for catalog LRE-DEC.} \end{aligned} \tag{53}$$

In Table 3 we summarize parameters of the two catalogs, which are relevant for the present study.

Testing the Poisson Model

The methods to estimate maximum magnitudes in a time horizon are based on the first-order approximation that earthquake occurrence follows a Poisson process with constant rate λ corresponding to exponentially distributed waiting times. Therefore we first test the hypothesis that the catalogs LRE respectively LRE-DEC are realizations of a Poisson process.

For both catalogs, we perform a one-sample Kolmogorov-Smirnov (KS) test with an exponential distribution of waiting times Δt as a null hypothesis. For the rate, we simply use $\lambda = 1/\langle \Delta t \rangle$ corresponding to the ratio of the event number and the total catalog time. To account for uncertainties of this point estimate, we also consider deviations ($\langle \Delta t \rangle \pm$ 1 year) from this value. The KS test will return a p-value which is smaller then 0.05, if deviations with imposed significance level $\alpha = 0.05$ from the Poisson model are present (rejection of the null hypothesis). On the other hand, for $p > 0.05$, the Poisson model is not rejected. The p-values are calculated with the R code "ks.test()" included in the R package "stats" (see Data and Resources section) based on *Marsaglia et al.* (2003). The results provided in Table 4 indicate that the Poisson model is rejected for the catalog LRE, while it is not rejected for the declustered catalog LRE-DEC. This result favors the catalog LRE-DEC for the further analysis. In general, two effects have to be balanced in this context: first, declustering leads to Poissonian earthquake occurrence, and second, reducing the number of events will also reduce the accuracy of the b-value estimation (see Data section). To find the influence of the declustering, we will perform all calculations for both catalogs.

Lower Rhine Embayment: Results for Known Magnitude Distribution

In the Frequentist Approach section and the Bayesian Approach section, formulas for the frequentist and the Bayesian confidence interval for known magnitude distributions have been derived. Now, we calculate the upper limit of the confidence interval for the two earthquake catalogs from the Lower Rhine Embayment described in the Data section with the estimated $\hat{b}$ values in Eq. (53). For the time horizon of 50 years we get the values listed in Table 5. We note that Eq. (25) for the frequentist confidence interval is an asymptotic approximation ($\alpha \ll 1, \Lambda \gg 1$); therefore, no values are provided for $\alpha = 0.5$ in the frequentist case. The values in Table 5 show that both confidence intervals are nearly identical. This is an expected result, because in the Bayesian Approach section, we have explicitly shown that for $\alpha \ll 1$ the Bayesian confidence interval becomes identical with the asymptotic formula of the frequentist confidence interval.

The Lower Rhine Embayment: Results for the General Case

Now, we apply the techniques developed on the previous sections to the earthquake catalog of the Lower Rhine Embayment (LRE) in Germany described above. Based on a given confidence level α, we calculate for the two catalogs LRE and LRE-DEC (1) posterior distributions of the maximum magnitude to occur in a given time horizon T_f (Eq. 45), and (2) posterior distributions of the waiting time T_f to the next earthquake with a given target magnitude μ_T (Eq. 46). Both types of distribution are calculated on the one hand for the situation where only the earthquake catalog is given and on the other hand for the situation, where the information of paleoearthquakes is taken into account (Eq. 50 and 51). Figures 3 and 4 show the full posterior distributions. In both figures, the plot on the left hand side does not take into account paleoseismicity, while the plot on the right hand side is based on the catalog and the paleoearthquake with $m = 6.7 \pm 0.3$ at 47.5 kyr BP (*Camelbeeck et al.*, 2000). Table 6 provides results for the upper bound of the Bayesian confidence interval for three values of α (0.50, 0.05, and 0.01). Table 7 includes results for the waiting times for $\alpha = 0.5, 0.05, 0.01$ and the scenario earthquakes $\mu_T = 5$ and $\mu_T = 6$. For illustration, we note that the time of 98 years for $\mu_T = 6$ (Table 7) and $\alpha = 0.05$ without including paleoseismicity means: With a probability of $1 - \alpha = 95\%$ the next $M6$ earthquake will occur at earliest in 89 years from now.

As a first observation, we find that single paleoearthquakes have an influence on the posterior distribution, but the picture does not change completely: The maximum magnitude increases and the waiting time to an earthquake with given target magnitude is reduced. It is, therefore important to gather more information from paleoseismology in order to reduce uncertainties in the calculation of the seismic hazard and the seismic risk.

The selection of the time horizon depends on the practical requirements; building codes for private residences may be defined on time scales of decades, while nuclear waste deposits require 1 million years. In the latter case the assumption of constant Poisson intensity Λ/T will become controversial. In this section, we have chosen a time horizon of 50 years which might be relevant for many individuals. Furthermore, we have carried out the calculations for both catalogs, LRE, and

LRE-DEC; a comparison of the results indicates that earthquake clusters have only small influence on the estimated values.

As an example, we consider the case with $\alpha = 0.05$, the catalog LRE-DEC, and the $m = 6.7$ paleoearthquake. Here, the maximum magnitude in the next 50 years is estimated to $m = 6.06$ (Table 6). An earthquake of this size occurring in the area of Cologne with a focal depth of 10km leads to an estimated loss of $US 14.5 billion (*Allmann et al.*, 1998). Increasing the confidence level to $\alpha = 0.01$ would lead to an estimated loss of more than $US 100 billion (*Allmann et al.*, 1998). For a moderate confidence level ($\alpha = 0.5$) the magnitude decreases to $m = 4.69$, which can still lead to serious damage, if the earthquake occurs in a populated area. On the other hand, if we aim at estimating the time where we can sure with $1 - \alpha = 0.95$ confidence that no $M6$ earthquake occurs, we get 49 years for the catalog LRE-DEC (Table 7) in agreement with the result in Table 6 ($\alpha = 0.05$, catalog LRE-DEC and m=6.7 paleoearthquake).

We note that our study does not allow to calculate probabilities of earthquake occurrence in specific spatial regions within the study area. However, as an example for a worst case scenario, we conclude that the possibility of an $M6$ earthquake in the area of Cologne in the next 50 years has to be considered from a statistical point of view.

The Lower Rhine Embayment: Retrospective Estimation of μ

As in the Application II: Retrospective Estimations of the Tohoku Earthquake section, we carry out retrospective estimations by cutting the catalog and comparing the estimations of μ based on an imposed confidence level with the observed seismicity after the cut. In particular, we truncate the LRE catalogs 30 years (50 years) before the $m = 5.15$ Roermond earthquake on 13 April 1992 and calculate μ using $1-\alpha=0.95$ confidence and the future time horizon $T_f=30$ years ($T_f=50$ years). As in the Application II: Retrospective Estimations of the Tohoku Earthquake section, we list magnitude estimations and probabilities for $\mu=5.15$ corresponding to the Roermond event in Table 8. Again, we consider clustered and declustered seismicity and the occurrence of a paleoearthquake.

The calculated values overestimate the magnitude of the Roermond event. However, the longer catalog ending 30 years before the Roermond earthquake provides magnitudes that are closer to the true value. In general, we have to note that small number of events in the LRE results in higher uncertainties, e.g. in comparison with the Japan. This observation is most conspicuous for high levels of confidence, because the forecasted magnitude are only poorly supported by data. Using $\alpha=0.5$, the estimated magnitudes for the different options are closely located around $\mu\approx 4.6$. Again, it has to be emphasized that magnitude estimations depend not only on the future time horizon, but also on level of confidence that one is willing to accept.

Application IV: Critical Facilities

The future horizon T_f is constrained by individual requirements. For the definition of building codes, return periods of tens to hundreds of years are typical. In *Holschneider et al.* (2011) it is argued that for such relatively short intervals the maximum magnitude for all times has almost no influence on the estimation of the maximum magnitude in the finite future interval. This will change, however, if long time horizons, e.g. 10^3 to 10^6 years, are considered. For this aim, we derive equations that consider the maximum magnitude M for all times in the truncated Gutenberg-

Richter law (Eq.43) as an unknown parameter that has to be estimated in addition to β and Λ.

In particular, we estimate the maximum magnitude using the doubly truncated Gutenberg-Richter distribution $F_{\beta M}(m)$ (see Eq. 43) and a flat prior $P_0(M)$ for M.

Using solely catalog data, we get from Eq. (37)

$$p_{T_f}(\mu|\{m_i\},n) \propto \int\limits_0^\infty d\beta \int\limits_{m_{\max,\text{obs}}}^\infty dM \times \\ \times \left[\tfrac{\beta}{\exp(-\beta m_0)-\exp(-\beta M)}\right]^{n+1} \tfrac{\exp(-\beta\mu)\exp(-\beta n\langle m\rangle)}{[1+(T_f/T)(1-F_{\beta M}(\mu))]^{n+2}} P_0(M). \tag{54}$$

Taking a paleoearthquake into account the formula becomes with Eq. (50)

$$p_{T_f,m_P,\Delta m,T_p}(\mu|\{m_i\},n) \propto \int\limits_0^\infty d\beta \int\limits_{m_{\max,\text{obs}}}^\infty dM \left[\tfrac{f_{\beta M}(\mu)\Delta F_{\beta M}(m_P)}{[1+(T_f/T)(1-F_{\beta M}(\mu))]^{n+2}} + \tfrac{f_{\beta M}(\mu)\Delta F_{\beta M}(m_P)}{[1+\gamma+(T_f/T)(1-F_{\beta M}(\mu))]^{n+2}}\right] \times \\ \times \left[\tfrac{\beta}{\exp(-\beta m_0)-\exp(-\beta M)}\right]^n \exp[-\beta n\langle m\rangle] P_0(M), \tag{55}$$

where $m_{\max,\text{obs}}$ is the maximal observed magnitude of an earthquake; in Eq. (55) this will be general the lower limit of the paleoearthquake: $m_{\max,\text{obs}} = m_P - \Delta m$.

Equation (54) and (55) have two important implications: First, the integral is divergent, since $1+(T_f/T)(1-F_{\beta M}(\mu))$ goes to a constant for $M \to \infty$. Second, if the integration with respect to M is carried out from $m_{\max,\text{obs}}$ to a final values $\widetilde{M}$, and $\widetilde{M} \to \infty$ afterwards, the results become identical to the estimations with the unlimited Gutenberg-Richter distribution. In agreement with *Holschneider et al.* (2011), we find that M, the maximum magnitude for all times is not a useful parameter from the viewpoint of statistical inference.

A simple way to overcome this problem is to truncate the Gutenberg-Richter law at a magnitude which cannot be exceeded in a given tectonic setting for physical reasons. For the Lower Rhine Embayment a threshold of $M = 8$ is certainly realistic in terms of a worst case scenario. Using a time horizon of 10^5 years and 50% quantiles as in (*Bundesamt für Strahlenschutz*, 2010), the maximum magnitude for declustered seismicity becomes $\mu = 7.54$ without the paleoearthquake, and $\mu = 7.80$ including the paleoearthquake. The results for the catalog including clusters are $\mu = 7.44$ (no paleoearthquake) and $\mu = 7.79$ (including the paleoearthquake). We point out that the values of μ depend on the choice of M as already discussed in *Holschneider et al.* (2011). Choosing M small, will result in $\mu \approx M$ for long time horizons.

A more sophisticated way to deal with the problem discussed above is to calculate first the bivariate posterior distribution with respect to b and Λ based on the catalog, and second the predictive distribution of the magnitudes. Evaluating the latter at a pre-defined level of confidence allows to estimate the magnitude of a design earthquake. The detailed study of the predictive distribution is left for further work.

Conclusions

The knowledge of the maximum magnitude in a specific region is one of the holy grails in seismic hazard assessment. Apart from pure scientific interest, this parameter is of direct relevance for pricing strategies of insurance companies. For this aim it is not sufficient to provide point estimates of the maximum magnitude in terms of expectation value and variance; it is rather important to calculate magnitude ranges on the basis of a pre-defined confidence level, which is subject to the agreement of insurance company and customer. In mathematical terms, the knowledge of confidence intervals for the maximum magnitude for given confidence level is required. *Holschneider et al.* (2011) have shown that confidence intervals diverge in most cases, if the maximum magnitude

M for all times in a specific region is studied in the framework of a doubly-truncated Gutenberg-Richter model. In the present study, we show that confidence intervals can be precisely calculated in a Bayesian framework, the maximum magnitude in a finite time horizon is considered. In general, computing maximum values in time intervals is subject to extreme value statistics: For example, annual maximum values from a data set may be used to calibrate a General Extreme Value Distribution (*Coles*, 2001). This approach neglects, however, information from smaller events and becomes unstable for sparse data. For earthquake occurrence, empirical findings like the Gutenberg-Richter (GR) law can be used to overcome this drawback to some extent. On the other hand, the GR law includes two parameters, the a and the b value which are not known a priori. Inserting point estimates or distributions for a and b introduces new errors of unknown size.

In the present work, we estimate the maximum magnitude in a finite time horizon with the following strategy: First, we use minimum assumptions, namely a Poisson process with unknown productivity Λ for earthquake rates and a GR distribution with unknown b value for earthquake magnitudes. Second, in order to provide a rigorous uncertainty assessment, we calculate the full Bayesian posterior distribution of both the maximum magnitude for a given time horizon and the waiting time to a target earthquake with given magnitude. The approximate validity of the Poisson model and the Gutenberg-Richter model has been achieved by using historic earthquake catalogs, which can be considered as complete in a specific magnitude range. If declustering techniques are applicable, results are compared for catalogs with and without clusters. We note, however, that for catalogs including mega-earthquakes ($m > 8$), declustering becomes questionable and probably unstable. The Bayesian posterior distribution is evaluated with respect to precise confidence intervals for both quantities. The required data include a complete earthquake catalog and, if available, paleoearthquakes.

As a first result, we find that retrospective estimations of the maximum magnitude prior to the Tohoku earthquake in the high-seismicity area of Japan, and the Roermond earthquake in the low-seismicity region of Germany are reasonably close to the observed magnitudes. We emphasize, however, that this is not a rigorous retrospective testing because the results depend on the imposed time horizon and the confidence level α. For the Lower Rhine Embayment, Germany, we present results for various scenarios. Using a declustered earthquake catalog from 1600 until 2011 with magnitudes $m \geq 4$ and a paleoearthquake with $m = 6.7$, we conclude that in the next 50 years an earthquake with $m \geq 6$ occurs with 5% probability. This same estimation for the catalog with clusters differs only by 0.3%. In general the results indicate that declustering has only minor influence on the estimated values, as long as the total number of events is not reduced too drastically.

The extrapolation of the method to very long time horizons can be carried out technically by considering the maximum magnitude M for all times as an unknown parameter within the Bayesian framework. In agreement with earlier work (*Holschneider et al.*, 2011), we find that M is a useless parameter from the viewpoint of statistical inference. Design earthquakes for critical facilities may be estimated by assuming a fixed upper bound M that cannot be exceeded for physical reasons in a specific tectonic environment. Using a time horizon of 10^5 years and $M = 8$, the magnitude of the design earthquake for the Lower Rhine Embayment is around $\mu = 7.5$.

Crucial issues in estimating maximum magnitudes, are data quality and selection. Since historic earthquake catalogs include more information on high magnitude events than instrumental catalogs, the balancing of catalog length, completeness and realization of Poissonian earthquake occurrence may become a complicated task and needs more detailed exploration in the future. While we use a constant magnitude of completeness in this work, the next step will be the generalization of the methods towards episodes with different completeness levels. Although, the Bayesian approach is, in principle, applicable to arbitrarily sparse data, the limit of credibility will be reached, if high confidence levels are required for estimations that are based on poor data.

Finally, we note that the approach of the present work can be extended, if new information, for example earthquake on earthquake slip, become available. The balancing of earthquake slip with average slip rates on the fault can provide a further important constraint for the largest earthquakes and can thus reduce the uncertainties in the calculation of the seismic hazard and the seismic risk.

Data and Resources

Earthquake catalog of the Northern Rhine area available via http://www.seismo.uni-koeln.de/catalog/index.htm, last accessed 11 September 2012.

JMA earthquake catalog of Japan (1926-2005) available via http://wwweic.eri.u-tokyo.ac.jp/db/jma/index.html, last accessed 11 September 2012.

NOAA earthquake catalog (684-1925) of Japan available via http://www.ngdc.noaa.gov, last accessed 11 September 2012.

For the Kolmogorov-Smirnov test, we used R software available via http://www.r-project.org, last accessed 11 September 2012.

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Tables

Table 1

Table 1: Terminology used from the Terminology and Definition of Confidence Interval section to the end of the paper

parameter	explanation
T	Duration of catalog
T_f	Future time horizon
Λ	productivity of Poisson process in the past
Λ_f	$(= \Lambda T_f / T)$ productivity of Poisson process in the future
$m_{\mathrm{max,obs}}$	magnitude of maximum observed earthquake
μ	Maximum magnitude during T_f
M	maximum magnitude for all times
$\{m_i\}$	Set of magnitudes in the catalog
n	Number of events in the catalog

Table 2

Table 2: Results for the Tohoku earthquake

catalog end (years before Tohoku)	nr of events	μ	probability ($\mu_T = 9.0$)
30	191 (174)	9.15 (9.19)	0.07 (0.07)
50	163 (146)	9.48 (9.28)	0.12 (0.08)

μ is the magnitude that is not exceeded within 30 or 50 years based on a confidence level of $1 - \alpha = 0.95$; the last column includes the probability of having an earthquake with magnitude 9.0 in the respective time interval; values in parentheses refer to the declustered catalog. See text for more explanation.

Table 3

Table 3: Parameters of the catalogs LRE and LRE-DEC

catalog	n	$\langle m \rangle$
LRE	26	4.39
LRE-DEC	20	4.43

n : number of events; $\langle m \rangle$: sample mean of magnitudes. Common parameters include the magnitude of completeness $m_0 = 4$, the total duration of the catalog $T = 411$ years, and the maximum observed event: $m_{\mathrm{max,obs}} = 5.36$.

Table 4

Table 4: Kolmogorov-Smirnov test for catalogs LRE and LRE-DEC and the Poisson model as null hypothesis

catalog	n	$\lambda =$ $1/\langle\Delta t\rangle$	p	$\lambda =$ $1/(\langle\Delta t\rangle - 1\text{yr})$	p	$\lambda =$ $1/(\langle\Delta t\rangle + 1\text{yr})$	p
LRE	26	14.49yr	0.0058	13.49yr	0.0069	15.49yr	0.0050
LRE-DEC	20	19.07yr	0.3111	15.47yr	0.3542	17.47yr	0.2752

Table 5

Table 5: Values for the upper limit of the confidence interval ψ_n for the frequentist and the Bayesian approach and the catalogs LRE and LRE-DEC

	Frequentist			Bayes		
catalog	$\alpha = 0.5$	$\alpha = 0.05$	$\alpha = 0.01$	$\alpha = 0.5$	$\alpha = 0.05$	$\alpha = 0.01$
LRE	-	5.68	6.33	4.62	5.67	6.33
LRE-DEC	-	5.71	6.41	4.56	5.70	6.41

The calculations are based on Eq. (25) and (33) using a future time horizon of $T_f = 50$ years. Because Eq. (25) is an approximation for $\alpha \ll 1$, no values for $\alpha = 0.5$ are provided in the frequentist case.

Table 6

Table 6: Results for the catalogs LRE and LRE-DEC excluding and including a paleoearthquake with magnitude m_P

catalog	$m_P \pm \Delta m(T_p)$	$\alpha = 0.5$	$\alpha = 0.05$	$\alpha = 0.01$
LRE	-	4.59	5.70	6.45
	6.7±0.3 (47.5 kyrs BP)	4.71	6.05	6.95
LRE-DEC	-	4.53	5.74	6.58
	6.7±0.3 (47.5 kyrs BP)	4.62	6.01	6.95

Upper bounds of the Bayesian confidence intervals for the maximum magnitude within a time horizon of 50 years including uncertainties of Λ and b.

Table 7

Table 7: Results for the catalogs LRE and LRE-DEC excluding and including a paleoearthquake with magnitude m_P

catalog	$m_P \pm \Delta m(T_p)$	μ_T	$\alpha = 0.5$	$\alpha = 0.05$	$\alpha = 0.01$
LRE	-	5.0	143	10	2
	6.7±0.3 (47.5 kyrs BP)	5.0	92	6	1
	-	6.0	1848	98	19
	6.7±0.3 (47.5 kyrs BP)	6.0	758	45	9
LRE-DEC	-	5.0	153	10	2
	6.7±0.3 (47.5 kyrs BP)	5.0	107	7	1
	-	6.0	1637	85	16
	6.7±0.3 (47.5 kyrs BP)	6.0	812	49	9

Estimated time interval, where no earthquake with magnitude $\geq \mu_T$ is expected based on the confidence level α.

Table 8

Table 8: Results for the Roermond earthquake; see text for explanation.

catalog end (years before Roermond)	declustered	paleo-earthquake	nr of events	μ	probability ($\mu_T = 5.15$)
30	-	-	24	5.45	0.10
	-	$\times$	24	5.80	0.26
	$\times$	-	21	5.47	0.10
	$\times$	$\times$	21	5.75	0.24
50	-	-	22	5.69	0.17
	-	$\times$	22	6.13	0.28
	$\times$	-	19	5.74	0.17
	$\times$	$\times$	19	6.07	0.25

Figure captions

Figure 1: Synthetic earthquake catalogs: Distribution of the estimated upper bound m_{95} of the confidence interval of the maximum magnitude for a time horizon of 50 years. Solid: Eq. (45) of this study; dashed: Campbell method based on the Bayesian extreme-value distribution (*Campbell*, 1982); vertical line: "true" value calculated from 10,000 future earthquake catalogs, each covering 50 years. Plot (a) is based on 10,000 earthquake catalogs with time coverage $T = 1000yr$, while for (b) $T = 100yr$ has been used.

Figure 2: Frequency-magnitude distributions of the earthquake catalogs for the Lower Rhine Embayment; solid: catalog LRE from 1600 to 2011 (26 earthquakes); dashed: declustered catalog LRE-DEC from 1600 to 2011 (20 earthquakes).

Figure 3: Probability density function for the magnitude, that is not exceeded within a time interval of 50 years, given the LRE catalog. The solid line refers to the catalog LRE, while the dashed line refers to the declustered catalog LRE-DEC; (a) only the LRE catalog is taken into account (Eq. 45); (b) results including also a paleoearthquake with moment magnitude 6.7 ± 0.3 (Eq. 50).

Figure 4: Probability density function for the time interval, where no earthquake with magnitude ≥ 6 is expected using the catalogs LRE (solid line) and LRE-DEC (dashed line); (a) result without taking into account a paleoearthquake; (b) result including the $m = 6.7$ paleoearthquake.

Figures

Figure 1

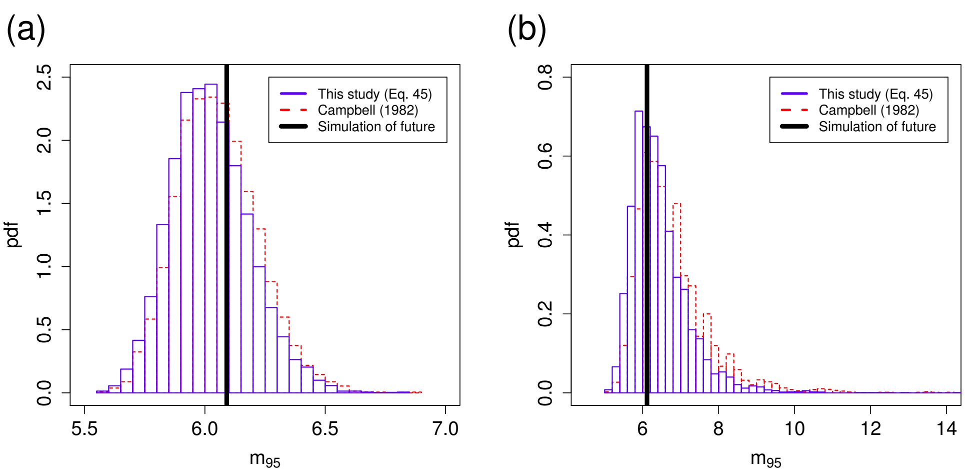


Figure 2

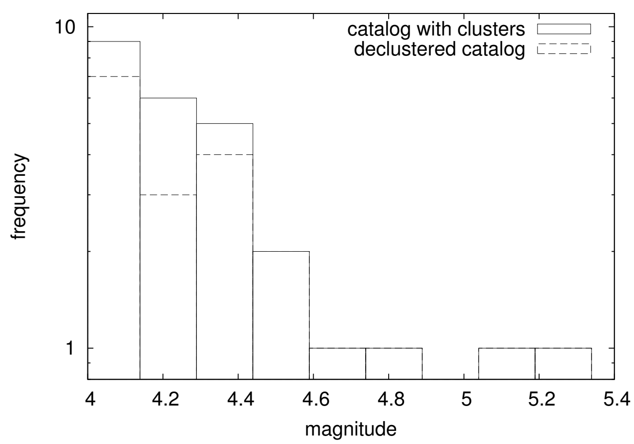
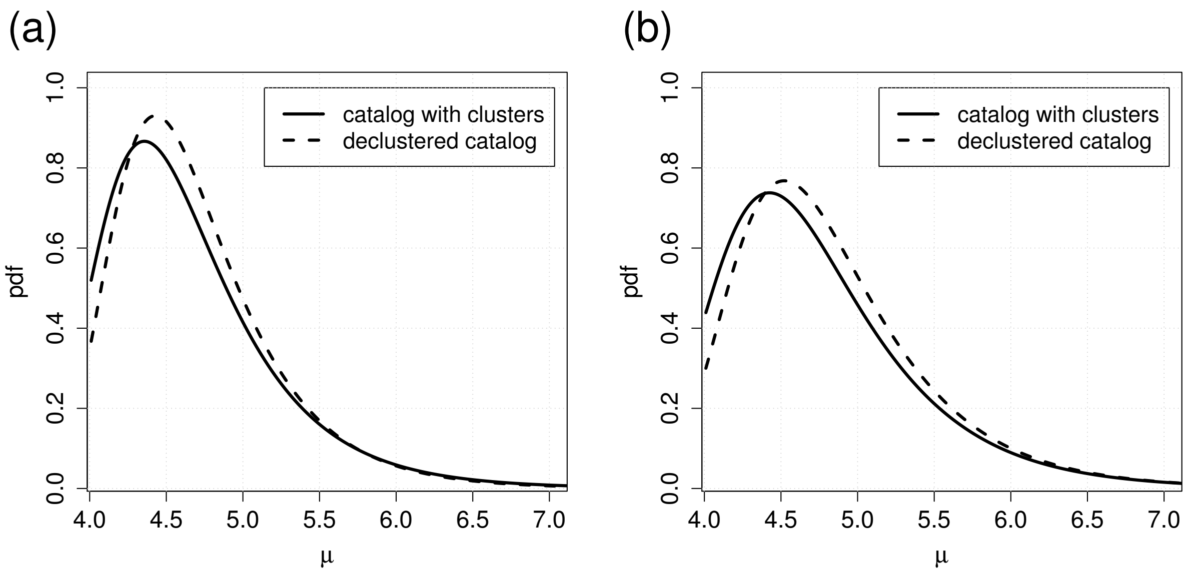

catalog with clusters
declustered catalog
10
1
frequency
4
4.2
4.4
4.6
4.8
5
5.2
5.4
magnitude

Figure 3



(a)
catalog with clusters
declustered catalog
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0.0
0.2
0.4
0.6
0.8
1.0
4.0
4.5
5.0
5.5
6.0
6.5
7.0
μ
(b)
catalog with clusters
declustered catalog
pdf
0.0
0.2
0.4
0.6
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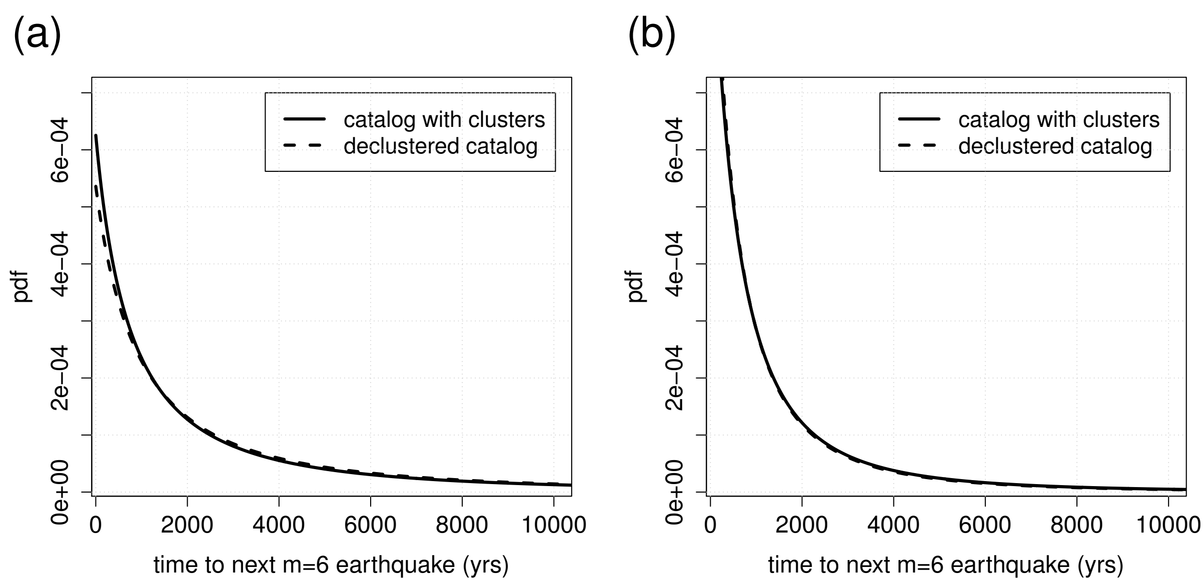
4.5
5.0
5.5
6.0
6.5
7.0
μ

Figure 4


(a)
(b)
catalog with clusters
declustered catalog
pdf
0e+00
2e−04
4e−04
6e−04
0
2000
4000
6000
8000
10000
time to next m=6 earthquake (yrs)