Topic	**Constructing response curves: Introduction to the BODE-diagram**
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1 The BODE diagram

True ground motion is of major interest in seismology. On the other hand any measuring device will alter the incoming signal as well as any amplifier and any output device. Working in the frequency domain, the quotient of input signal and output signal is called the *response*. This response is complex, and to get a more meaningful result it can be split into two terms: amplitude response and phase response (see 5.2.3). Amplitude response means the output amplitude divided by the input amplitude at a given frequency. Phase response is the difference between output phase and input phase, or the phase shift. A graphical expression of this splitting is known as the BODE-diagram. One part shows the logarithm of amplitude *A* versus the logarithm of frequency *f* (or angular frequency ω, or period T, see Figure 1a). The other part depicts the (linear) phase ϕ versus the logarithm of frequency *f* (or ω = 2πf, or T = 1/f, see Figure 1b). For *A* the terms *amplification* or *magnification* are also used.

1.1 The signal chain

The signal passes a chain of devices. Any single element of this chain can be described by its response. It is useful to split any response into elements of first or second order. At the end the overall amplitude response of the complete chain can be constructed by multiplying all single amplitude responses, and the overall phase response by adding all single phase shifts.

1.2 First and second order elements

For the amplitude response the double logarithmic scale of the amplitude diagram facilitates an easy and fast construction. Any element can be approximated by two straight lines. One horizontal line leads to the element corner frequency, and one line drops from that point with a slope depending on the order of the element.

A first order element is completely described by its amplification A and corner frequency f_c. The slope beyond f_c is one decade in amplitude per decade in frequency. The real amplitude value at f_c is dropped to 0.707 of the maximum amplitude (see Figure 2, full line). However, for our fast construction ,we consider only a linear approximation to it (dash-dot lines).

A second order element exhibits a slope of two decades in amplitude per decade in frequency. Additionally it needs another parameter called damping D, describing the amplitude behaviour at frequencies near f_c (compare Figure 3).

2 The seismological signal chain

The seismological signal passes the chain
 - mechanical receiver
 - transducer
 - preamplifier
 - filter
 - recording unit (to be recognized separately)

Note! Below we discuss in sections 2.1 and 2.2 amplitude responses related to ground displacement. Therefore, the ordinate axis of the BODE-diagram (amplitude A) for the mechanical receiver has no unit (or the unit [m/m]), and for the transducer the unit is [V/m]. Changes to other types of movement, being proportional to ground velocity or to ground acceleration, will be described in section 3.

2.1 The mechanical receiver

The mechanical receiver is a second order system. It describes the relative movement of the pendulum (i.e., a seismic mass attached to a frame by a spring) with respect to the frame. Damping D is set mostly to 0.707 . Only at this damping value is the amplitude value at f_c also 0.707 (Figure 3, full curve). For higher values of damping one obtains a more flat curve (dashed). For lower values of D the dash-dot curve strongly exceeds the amplification level at f_c, indicating low-damped resonance oscillations of the pendulum, which can be stimulated by any signal.

The amplification of the mechanical receiver is A = 1 . This means that for frequencies $f > f_c$ the amplitude of the pendulum movement with respect to the frame is similar to the ground amplitude. For the phase shift see Figure 7b (HIGH Pass 2).

2.2 The transducer

The transducer transforms the relative movement of the pendulum into an electrical signal, i.e., in a voltage. The transducer constant G gives the value of the output voltage U depending on the relative pendulum movement z. There are three main types of transducers, distinguished by their proportionality to ground motion and its derivatives:

 - Displacement $U \sim z$ $G_d[V/m]$ (capacitance or inductance bridges)

 - Velocity $U \sim dz/dt$ $G_v[Vs/m]$ (magnet-coil systems)

 - Acceleration $U \sim d^2z/dt^2$ $G_a[Vs^2/m]$ (piezo-electric systems, $U \sim F = m\,a$)

The above proportionality of the transducer voltage to ground motion (i.e., to displacement, velocity or acceleration, respectively) is, of course, only given for frequencies $f > f_c$, i.e., for the horizontal part of the mechanical receiver response (see Figure 3).

All transducer amplitude responses can be drawn as straight lines over the full considered frequency range (Figure 4). They differ only in their slope.

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The phase responses have a constant phase shift over the whole frequency range with values of 0° (displacement), 90° (velocity), or 180° (acceleration).

2.3 The preamplifier

The preamplifier is a first order LOW Pass. Its corner frequency is beyond the signal range of seismology - up to several 10 kHz. Thus, only the amplification is of interest (Figure 5). The response is a horizontal line drawn at the amplification level A.

The phase shift is $\phi = 0°$, but one should keep in mind that, if using the inverted input, the phase shift will be $\phi = -180°$ over the whole frequency range.

2.4 First and second order LOW Passes

LOW Passes have constant amplifications *A* for frequencies lower than their corner frequencies f_c. For frequencies higher than f_c the amplification drops with a slope depending on the order of the filter (Figure 6a). LOW Passes cut the high frequencies, therefore, also the term High Cut is used. The phase shift for $f < f_c$ is about 0° and for $f > f_c$ it turns to - 90° (first order, LP1) or - 180° (second order, LP2; see Figure 6b), passing half of the phase shift exactly at f_c.

Of course, the given amplitude and phase values are approximations. In reality we would obtain $\phi = 0°$ only if inserting a frequency of 0 Hz, and $\phi = -90°$ (-180°) for infinite frequency values. However, the accuracy is sufficient for our fast construction.

2.5 First and second order HIGH Passes

HIGH Passes have constant amplifications A for all frequencies higher than their corner frequency f_c. For frequencies lower than f_c the amplification drops with a slope depending on their order (Figure 7a). They cut the low frequencies, so one can also find the term Low Cut.

The phase shift for $f > f_c$ is about 0°, and for $f < f_c$ it turns to +90° (first order, HP1) or +180° (second order, HP2; see Figure 7b), passing half of the phase shift exactly at f_c.

Comparable to the description of LOW Passes the given amplitude and phase values are approximations.

2.6 Second order BAND Pass

The second order BAND Pass (BP2) can be explained as a combination of a first order LOW Pass and a first order HIGH Pass. It suppresses all frequencies, except f_c, with a slope of one decade in amplitude per decade in frequency (Figure 8a). The peak at f_c can be turned into a horizontal line (symmetrical to f_c) by increasing the damping to values D > 1. Thus it is possible to construct a BAND Pass by combining a HIGH Pass with a LOW Pass.

The phase shift for $f < f_c$ is about +90°, and for $f > f_c$ it turns to -90° (see Figure 8b), passing half of the phase shift at f_c.

3 The overall response

The construction of the overall response should be divided into two steps:
- from mechanical receiver to the final filter stage; and
- adding the recorder response.

The first result, the electrical output, is useful for fitting the signal to the recorder input. It has to be fixed, meaning that changes in magnification (or signal resolution) should be done by setting up the recorder only.

3.1 From the mechanical receiver to the final filter

As defined in section 2, the amplitude response is constructed related to ground displacement. Multiplying all the units of our signal chain, we get the unit [V/m] for the ordinate axis. All elements, including mechanical receiver, transducer, and filter stages can be implemented in the same sheet with a double logarithmic grid, each element with its magnification and its corner frequency. Then the resulting amplitude response has to be constructed point by point at certain frequencies. This can be done either by multiplying the amplitudes of all elements at these frequencies, which is the more secure method, or, alternatively, by adding the distances (e.g., in millimetres) of all element amplitudes to the amplitude level line A = 1, with positive distances if above this line and negative ones if below. This method is faster. A linear addition is, in this logarithmic scale, a multiplication of the amplitude values. The final amplitude response curve can be drawn on the same sheet, together with the single elements.

3.2 Adding the recorder

In reality, at the end of our signal chain we will find a commercially available recorder, transforming the obtained voltage back into movement (drum recorder) or into computable digital values (Analogue-to-Digital Converter = ADC). Its main parameter is the input sensitivity H. In the case of a drum recorder, H is the pen deflection per Volt (in units [m/V]). For an ADC, H is the digital count per Volt (in units [digit/V]).

Thus the overall amplitude response needs a separate BODE-diagram for each recorder type. Multiplying the units we obtain the units [m/m] for the drum recorder, and [digit/m] for the ADC. You will also find derivatives of this unit, like [digit/nm] or [counts/nm].

3.3 Introducing ground velocity and ground acceleration

If the amplitude response curve has to be constructed related to ground velocity (or ground acceleration), it is sufficient to redraw either the response of the mechanical receiver or the transducer. The simpler method is to change the transducer response. Each slope will change by one order if going from displacement to velocity, or from velocity to acceleration. The unit of the ordinate changes from [V/m] (displacement) via [Vs/m] (velocity) to [Vs²/m] (acceleration). These units will also be the units of the amplitude response from the mechanical receiver to the final filter stage. Beyond this the construction of the overall amplitude response is similar to section 3.1. The units of the recorder amplitude response will alter to [m·s/m] (velocity) or [m·s²/m] (acceleration) for the drum recorder. For the ADC we obtain [digit·s/m] (velocity) or [digit·s²/m] (acceleration).
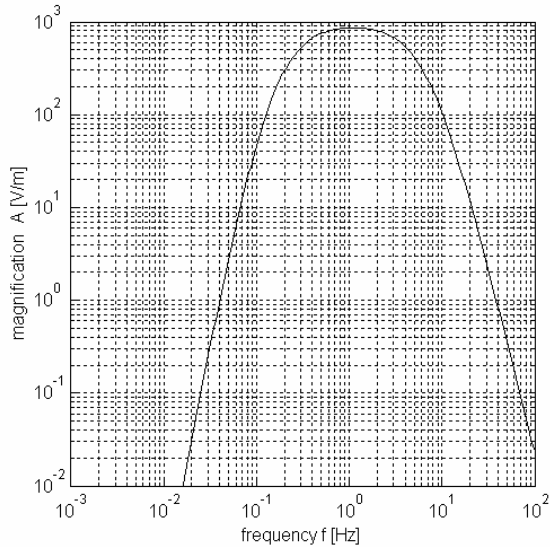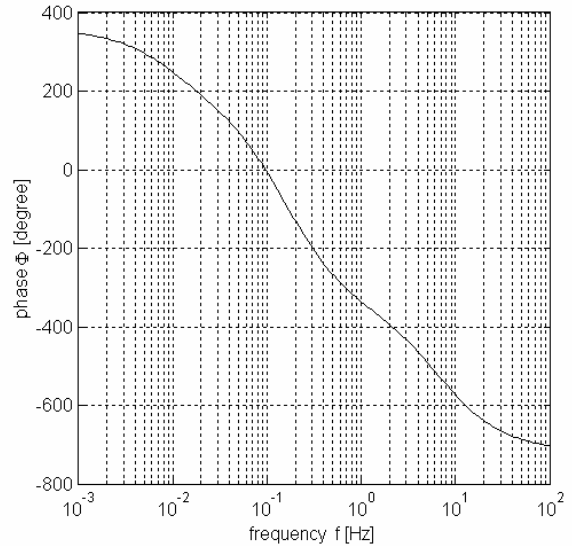
Figure 1a Amplitude Response.

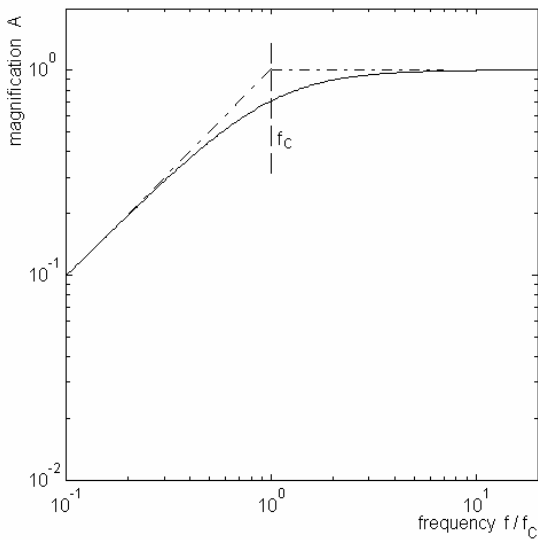**Figure 1b** Phase Response.

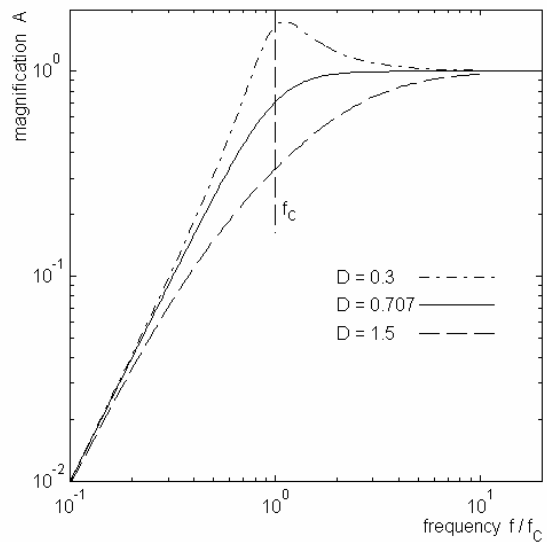**Figure 2** First Order HIGH Pass (HP1).

Figure 3 Second Order HIGH Pass (HP2) or
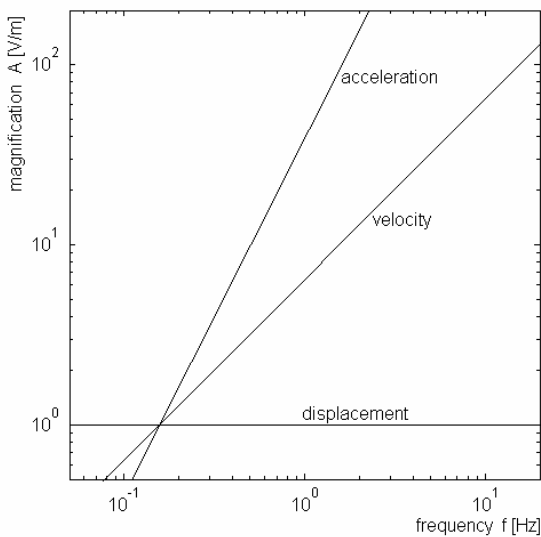Mechanical Receiver.

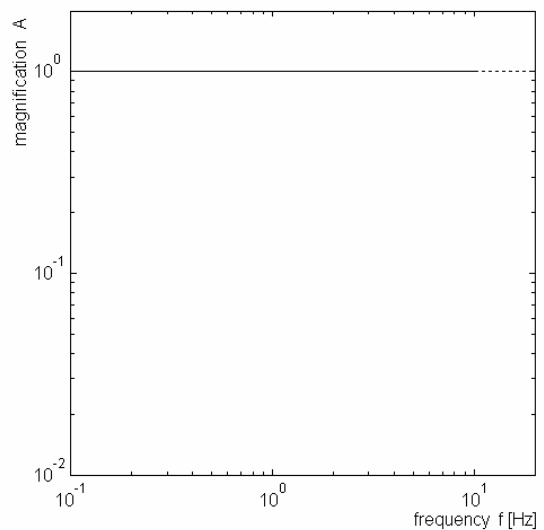**Figure 4** Transducer Amplitude Response.

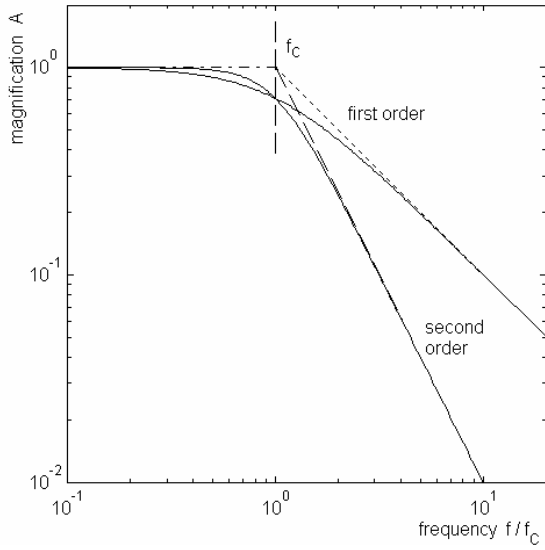**Figure 5** Preamplifier Amplitude Response.

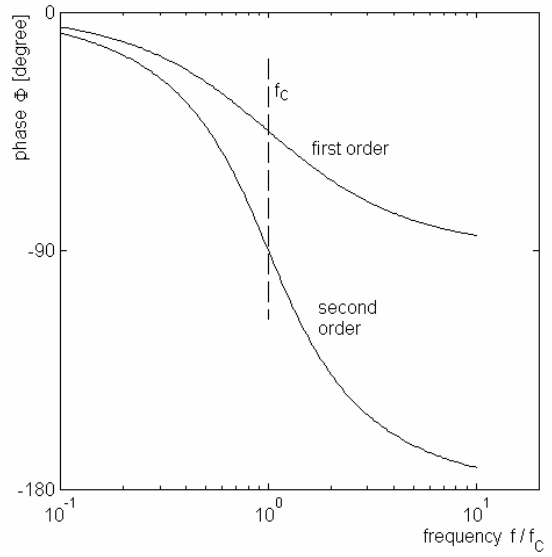**Figure 6a** LOW Pass Amplitude Response.

Figure 6b LOW Pass Phase Response.
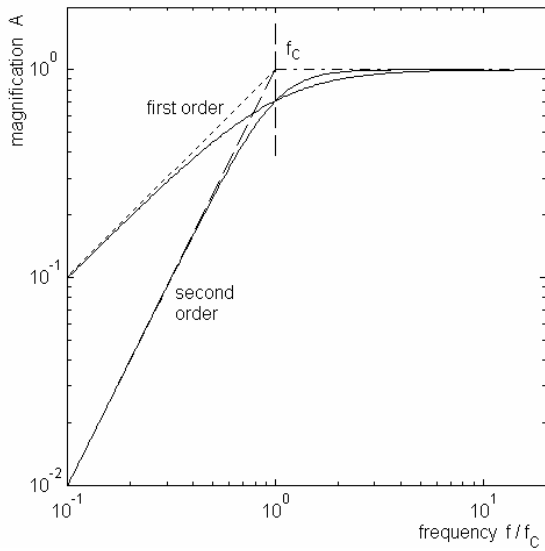
Figure 7a HIGH Pass Amplitude Response.
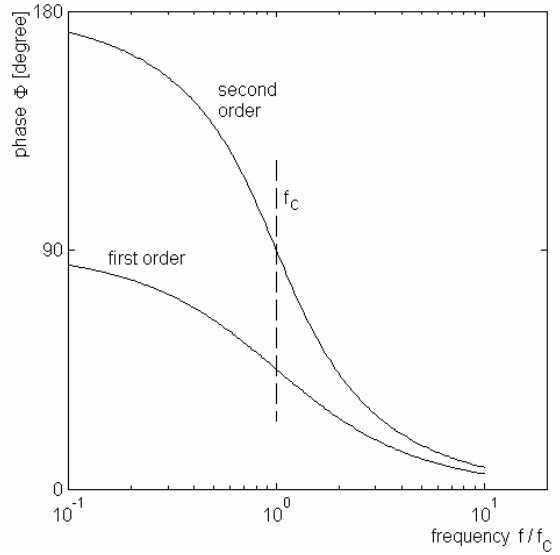
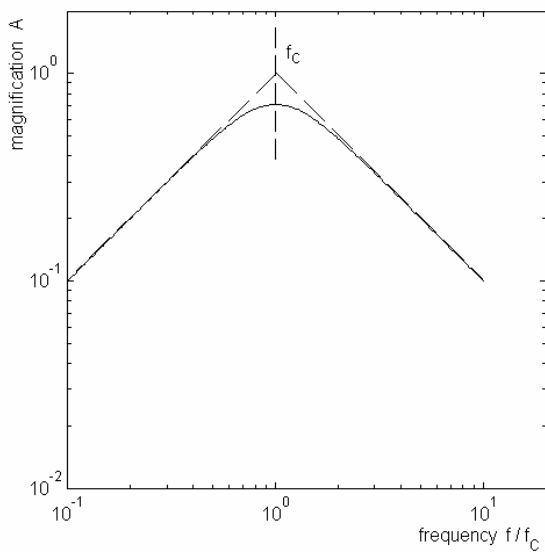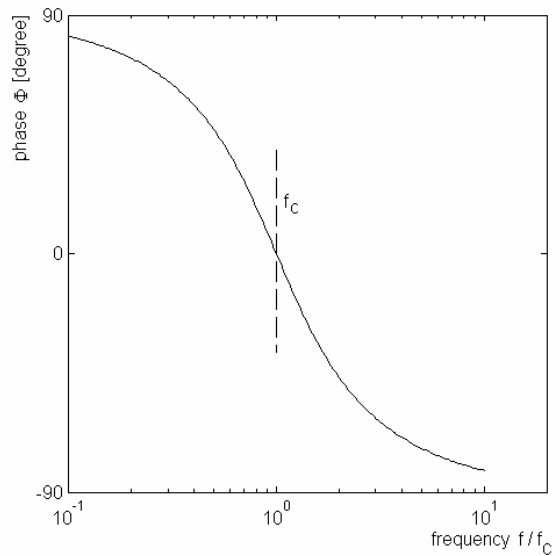**Figure 7b** HIGH Pass Phase Response.

Figure 8a BAND Pass Amplitude Response.

Figure 8b BAND Pass Phase Response.

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