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Zusammenfassung

Das vorliegende Heft enthält die während der Tagung „Statistische und tektonophysikalische Aspekte der Seismizität" am Zentralinstitut Physik der Erde der DAW zu Berlin, Institutsteil Jena, vom 16. bis 18. Mai 1972 gehaltenen Vorträge, die als Grundlage für die Beratungen der Arbeitsgruppe „Statistische Methoden" innerhalb der Subkommission „Seismizität von Europa" der Europäischen Seismologischen Kommission dienten.

In der Eröffnungsansprache wies der Direktor des Instituts, Herr Prof. STILLER, auf die Bedeutung der quantitativen Seismizitätsdarstellung für die komplexen Fragestellungen des Geodynamik-Projektes hin. Der Präsident der Europäischen Seismologischen Kommission, Herr Prof. SAVARENSKY, Moskau, betonte die statistischen Aspekte der Seismizität und erläuterte die Möglichkeiten zur physikalischen Interpretation der gewonnenen Gesetzmäßigkeiten sowie die Anwendung auf die Abschätzung des Erdbebenrisikos unter verschiedenen Gesichtspunkten.

SCHENKOVÁ unterzog die Erdbeben des nordöstlichen Mittelmeers im Zeitraum (1901, 1967) einer Analyse ihrer zeitlichen Folge anhand mehrerer Modelle, aus denen durch den χ^2-Test ein optimales ermittelt wurde.

PURCARU untersuchte Modelle der Häufigkeits-Magnituden-Abhängigkeit, z. B. die von UTSU und von OKADA, vom Standpunkt der Informationstheorie.

In zwei von PANZA vorgetragenen Arbeiten ist die Seismizität von Italien einschließlich der Darstellungsgrundlagen besprochen worden, wobei die GUTENBERG-RICHTERsche Magnituden-Häufigkeits-Relation zugrunde liegt. Ihre Koeffizienten wurden für die verschiedenen seismotektonischen Einheiten Italiens ermittelt. Außerdem sind für diese Gebiete Modelle der Intensitätsverteilung um die Epizentren aufgestellt worden. Mit seismischen Daten, die bis zur Zeitenwende reichen, wurden sodann Karten des seismischen Risikos und der aktiven Verwerfungen hergestellt. Außerdem demonstrierte PANZA die starke Abhängigkeit der Magnitudenangabe, wie sie aus seismischen Oberflächenwellen gewonnen wird, von der Herdtiefe eines Bebens in der Erdkruste.

Mit logarithmisch normalen Verteilungen der Erdbebenhäufigkeit über der Magnitude oder der Energie, der Bestimmung ihrer Koeffizienten und deren Vertrauensintervalle befaßten sich MAAZ und PURCARU sowie NEUNHÖFER und MAAZ. PROCHÁZKOVÁ zeigte die Notwendigkeit einheitlicher Methoden zur Ermittlung der Parameter in der GUTENBERG-RICHTERschen Häufigkeits-Magnituden-Verteilung und gab diesbezügliche Empfehlungen.

Zwei Vorträge von ULLMANN und MAAZ stellten die Verbindung der Seismizitätsfunktion als Energieflußdichte mit der Energieverteilung im Herd vermittels der BÅTH-DUDAschen Formel für das Herdvolumen her und gaben eine Verallgemeinerung der Seismizitätsfunktion, die die Ungenauigkeit der Herdkoordinaten statistisch berücksichtigt.

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Summary

The present number contains the papers read at the conference „Statistical and Tectonophysical Aspects of Seismicity" held from May 16 to 18, 1972, at the Central Earth Physics Institute of the German Academy of Sciences of Berlin, Jena branch, which serve as a basis for the discussions of the working group „Statistical Methods" within the subcommission „Seismicity of Europe" of the European Seismological Commission.

In his opening speech the Director of the institute, Prof. STILLER, pointed out the importance of quantitative representation of seismicity for the complex problems of the project of geodynamics. The chairman of the European Seismological Commission, Prof. SAVARENSKY, Moscow, emphasized the statistical aspects of seismicity and explained the potentialities of physical interpretation of the laws established as well as their application to estimating the earthquake risk under various aspects.

SCHENKOVÁ analysed the chronological succession of the earthquake of the northeastern Mediterranean in the period (1901, 1967) in the light of several models, from which an optimum model was found by the χ^2-test.

PURCARU examined models of the frequency-magnitude dependence, e.g. those of UTSU and of OKADA, from the view-point of information theory.

In two papers read by PANZA the seismicity of Italy including the fundamentals of representation was discussed, where the GUTENBERG-RICHTER magnitude-frequency relation is the underlying principle. Its coefficients were determined for the various seismotectonic units of Italy. Moreover, models of intensity distribution about the epicentres were set up for these regions. On the basis of seismic data going back as far as to the Birth of Christ maps of the seismic risk and of the active faults were then drawn up. Furthermore, PANZA demonstrated the great dependence of the magnitude value as obtained from the seismic surface waves on the focus depth of an earthquake in the earth's crust.

MAAZ and PURCARU as well as NEUNHÖFER and MAAZ were concerned with logarithmically normal distributions of the frequency of earthquakes over the magnitude or energy, the determination of their coefficients and their confidence intervals. PROCHÁZKOVÁ pointed out the necessity of standardized methods for determining the parameters of the GUTENBERG-RICHTER frequency-magnitude distribution and gave recommendations in this respect.

Two papers by ULLMANN and MAAZ established the connection of the seismicity function as energy flow density with the energy distribution in the focus by means of the BÅTH-DUDA formula for focus volumes and rendered a generalization of the seismicity function in which the inaccuracy of the focus coordinates is statistically considered.

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Résumé

Le présent numéro contient les discours qui ont été prononcés pendant la séance à propos des „Aspects statistiques et tectonophysiques du séisme" à l'Institut central „Physique de la Terre" de l'Académie Allemande des Sciences de Berlin, section d'Institut à Iéna, du 16 au 18 mai 1972, discours qui servaient de base pour les délibérations du groupe „Méthodes statistiques" de la sous-commission „Le séisme d'Europe" de la Commission Séismologique Européenne.

Dans son discours d'inauguration, le Directeur de l'Institut, Monsieur le Prof. STILLER, mit en relief l'importance de la présentation quantitative du séisme pour les questions complexes du projet de géodynamique. Le Président de la Commission Séismologique Européenne, Monsieur le Prof. SAVARENSKY, de Moscou, accentua les aspects statistiques du séisme et exposa les possibilités d'interpréter physiquement les conformités déduites aux lois ainsi que l'application de celles-ci à l'appréciation du risque de tremblements de terre à des points de vue différents.

SCHENKOVÁ se mit à analyser la succession temporelle, sur la base de plusieurs modèles, à partir desquels un modèle optimal fut déterminé par le moyen du test χ^2, des tremblements de terre de la mer Méditerranée du nord-est dans la période (1901, 1967).

PURCARU examina des modèles de la fonction fréquence/magnitude, ainsi par ex. ceux d'UTSU et d'OKADA, au point de vue de la théorie d'information.

Dans deux discours prononcés par PANZA, on a discuté le séisme d'Italie, à inclusion des fondements de représentation, sur la base de la relation magnitudes - fréquences d'après GUTENBERG-RICHTER. Ses coefficients ont été déterminés pour les diverses unités séismotectoniques d'Italie. De plus, on a établi, pour ces régions, des modèles démontrant la distribution des intensités autour des épicentres. A l'aide de données séismiques remontant à l'époque de transition, on a établi alors des cartes du risque séismique et des rejets actifs. En outre, PANZA démontra la grande dépendance de l'indication de magnitude telle qu'elle est déduite des ondes séismiques superficielles, de la profondeur du foyer d'un tremblement dans l'écorce terrestre.

La répartition logarithmiquement normale de la fréquence des tremblements de terre sur la magnitude ou l'énergie, la détermination de ses coefficients et intervalles de confiance faisaient l'objet des études de MAAZ et PURCARU ainsi que de NEUNHÖFER et MAAZ. PROCHÁZKOVÁ signala la nécessité de méthodes unifiées pour la détermination des paramètres dans la répartition de fréquence - magnitude d'après GUTENBERG-RICHTER et soumit des propositions correspondantes.

Deux discours prononcés par ULLMANN et MAAZ établirent la relation entre la fonction de séisme comme densité du flux de l'énergie, et la répartition de l'énergie dans le foyer, de conformité à la formule de BÅTH-DUDA concernant le volume du foyer, et proposèrent une généralisation de la fonction de séisme, fonction qui tient compte statistiquement de l'inexactitude des coordonnées de foyer.

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Резюме

В предлагаемой брошюре содержатся доклады, прочитанные на конференции, посвящённой „Статистическим и тектоно-физическим аспектам сейсмичности", организованной Центральным Институтом Физики Земли Германской Академии Наук в Берлине, филиалом института в Иене, с 16 по 18 мая 1972 г., и послужившие основой для совещаний рабочей группы „Статистические методы" в подкомиссии „Сейсмичность Европы" Европейской Сейсмологической Комиссии.

Во вступительной речи директор института тов. проф. ШТИЛЛЕР указал на важность количественной характеристики сейсмичности для общей постановки вопроса геодинамического проекта. Президент Европейской Сейсмологической Комиссии тов. проф. ЗАВАРЕНСКИЙ, Москва, подчеркнул статистические аспекты сейсмичности и указал на возможности физического объяснения полученных закономерностей, а также на использование при оценке опасности землетрясения с различных точек зрения.

ШЕНКОВА подвергла анализу землетрясения в северо-восточной части Средиземного моря в течение времени (1901, 1967) в их временной последовательности с помощью нескольких моделей, из которых через тест χ^2 было получено оптимальное значение.

ПУРКАРУ рассмотрел модели зависимости частоты от интенсивности землетрясения, например, модели УТСУ и ОКАДЫ, с точки зрения информационной теории.

В двух работах ПАНЦЫ, нашедших своё отражение в докладах, речь идёт о сейсмичности Италии, включая основную характеристику, причём в основу положено отношение интенсивности землетрясения и частоты ГУТЕНБЕРГА-РИХТЕРА. Его коэффициенты были получены для разных сейсмотектонических единиц Италии. Кроме того, для этих областей были установлены модели распределения интенсивности вокруг эпицентров. На этой основе с помощью сейсмических данных, которых хватит до начала следующего века, были составлены карты сейсмической угрозы и активных сбросов. Кроме этого ПАНЦА показал сильную зависимость данных интенсивности землетрясения, полученных из сейсмических поверхностных волн, от глубины залегания эпицентра землетрясения в земной коре.

Логарифмически нормальными распределениями частоты землетрясений по интенсивности землетрясения или энергии, определению их коэффициентов и их интервалов надёжности занимались МААЦ и ПУРКАРУ, а также НОЙНХЁФЕР и МААЦ. ПРОХАЦКОВА указала на необдимость разработки единых методов для получения параметров распределения частоты-интенсивности землетрясения по ГУТЕНБЕРГУ-РИХТЕРУ и дала в этом направлении несколько ценных советов.

В своих докладах УЛЬМАН и МААЦ установили связь между функцией сейсмичности как плотности потока энергии и распределением энергии в эпицентре посредством формулы БАТ-ДУДЫ для объёма эпицентра и обобщили функцию сейсмичности, которая учитывает неточность координат эпицентра статистически.

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Opening Remarks:

Seismicity as a Contribution to Geophysical and Geological Complex Interpretation

By

H. STILLER [1)]

Dear Colleagues,

The new founded working group on „Statistical Methods" of the subcommission „Seismicity of Europe" of the European Seismological Commission has organized its first meeting here in the Jena branch of the Central Earth Physics Institute Potsdam of the Academy of Sciences of German Democratic Republic. I am very glad to state that some foreign guests - Dr. SCHENKOVÁ from Czechoslovakia, Prof. SAVARENSKY from USSR, Dr. PANZA from Italy, and Dr. PURCARU from Romania - are taking part at the Jena meeting of the working group.

We know, from the results of the Upper Mantle Project, the great importance of world-wide, regional as well as local seismicity investigations for the development of complex geophysical and geological research work. The new tendencies in investigating the dynamical behaviour of the earth are also closely connected with seismicity research.

In our institute we have gathered some experiences with respect to investigation and mapping of magnetic, gravimetric, and recent crustal movement parameters. Work was also done in the direction of geotectonic map construction. I am very glad to state that in the past years corresponding investigations were also made in the domain of seismicity. Here I have to mention the special work of our colleagues ULLMANN and MAAZ, including aspects of the mathematical theory of seismicity and mapping of seismicity by computers for different regions in Europe.

The results of the Upper Mantle Project have shown the great importance of maps of geophysical and geological parameters as a base of complex interpretation. The handling of geophysical observations with respect to map construction has shown the two aspects of this problem:

a) the statistical and
b) the analytical point of view.

Both of them must be taken into account for seismicity, too. I think that the development of the statistical methods of seismicity has to include the different aspects of the whole complex problem of seismicity, including the various possibilities of application and the connections with other geophysical and geological informations.

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To that effect I feel that seismicity will give good results also for the future tendencies of the Geodynamics Project, especially for the development of the dynamical model of the tectonosphere and for the characterization of regional and global tectonic elements. - I hope that the discussions during the meeting of the working group will be most fruitful.

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General Remarks to the Object of this Meeting

By

E. F. SAVARENSKY [1]

Herr Professor Stiller, meine Damen und Herren!

Zuerst möchte ich Sie im Namen der Europäischen Seismologischen Kommission herzlich begrüßen. Es ist für mich eine große Freude, hier zu sein. Dem Direktor des Zentralinstituts Physik der Erde, Herrn Prof. STILLER, und seinen Kollegen danke ich herzlich dafür.

I am very thankful, as the president of the European Seismological Commission, to the director of this institute and to our colleagues for organizing this possibility to meet here and to discuss - which is most important - some problems concerning the statistical methods in seismicity investigations.

We have the praxis to organize such separate meetings of the ESC working groups. We had two meetings in the Czechoslovakian Socialistic Republic, the first one concerning the nature of the boundaries of the Earth's crust, the second one dealing with the convolution of seismograms; we had also the meeting of the working groups of the North-African as well as of the Iberian regions in Lissabon. Moreover, the conference held in Finland, whose topic was the investigation of the Earth's crust in Northern Europe, has to be mentioned, etc. It is a rather important direction for the activities of our European Seismological Commission. I am personally very glad to attend this meeting here in this institute, which is very well known, as much as our colleagues here at Jena and at Potsdam are. Besides, I have here some other applications connected with my personal interest regarding surface waves. Therefore I am highly pleased to stay here.

For the scientific direction of the investigations concerning the seismicity on the base of statistics I see some problems which are most important. First of all there is the problem to search and to find, by means of the statistics, some laws for the generation of earthquakes in general. And from practical viewpoint it is rather important to find the localities of strong earthquakes, which is very significant for the seismic regionalization too, I suppose. The searching of the well-known linear curve representing the dependence of the logarithm of the frequency on the magnitude or the logarithm of energy, perhaps, was the most popular knowledge concerning the statistics in seismicity. But it is not exact and the problem is much more extensive. The most important question, in my opinion, and a very difficult one is to find out to which place or to which territory the statistical relation belongs because the latter must be connected very closely with the tectonical processes and the physical basement to be investigated.

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It is very difficult to separate aftershocks, foreshocks, and the essential shocks. There is no exact knowledge concerning the definition of „foreshocks" and „aftershocks". Sometimes, we are able to discern these types of shocks after very strong earthquakes. I know the only relative publication submitted by Dr. SUYEHIRO from Japan, who found that the frequency-magnitude relation and especially the coefficient b of the graph changes markedly before and after an earthquake. But when may the process of foreshocks begin, and when will the process of the aftershocks end? We have no knowledge about it till now.

The second and a very important question about the statistics applied to the investigation of seismicity concerns the time. The usual construction of the curve representing the dependence between the frequency and the energy is very difficult, because very strong earthquakes occur rarely and we have very few information. Nevertheless, there are already some results, e. g. from our colleague FEDOTOV, who has published his findings in the Journal Fizika Zemli. He found some peculiarities common to many earthquakes in the Pacific and certain repetitions of energy variations with time as well as of the frequency of shocks in time for many strong earthquakes. Prof. KAWASUMI from Japan announced that the next strong earthquake in the Tokyo region has to be expected, with a probability of 99.7 %, in 1978 at the latest. Having gathered a great deal of material concerning that area, he calculated the probability of the mean periodical law and found certain fundamentals for a prediction. Now the Japanese are preparing special measures for preventing disastrous consequences, as conflagrations and other kinds of serious distructions. - The third important problem is the correlation between the statistical dependences of seismicity and other geophysical parameters, e. g. heat flow, magnetic and gravity anomalies, geology, tectonics, geomorphology etc.

The Seismological Commission is awaiting from the activities of this working group important results and the program of future investigation. The Commission will be pleased to see such results, and I wish to transmit its best compliments.

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Time Distribution of Earthquake Occurrence in the North-Eastern Mediterranean Zone

By

Z. SCHENKOVÁ [1]

Summary

The time distribution of earthquake occurrence in the north-eastern Mediterranean zone is investigated. It is shown that the process with the negative binomial entries as a model describing the occurrence of shallow-focus earthquakes in this zone is better than the POISSON process.

The object of this paper is to investigate one of the characteristic features of the seismicity of the north-eastern Mediterranean zone, i. e. to find laws governing the time distribution of earthquake occurrence using instrumental observations. The original data of shallow-focus earthquakes $(h < 60$ km$)$ are taken from the European catalogue 1901 - 1955 [3] completed by additional information from 1956 - 1967. The period of 67 years, for which the data on earthquakes of $M \geqq 4.5$ are available, might be long enough for the full manifestation of earthquake forces. The one-year interval is applied as a most appropriate unit. Shorter intervals are not feasible because then the number of „empty" intervals increases. The investigated regions are numbered as follows: No. 22 Yugoslavia, the Aegean region as a region of the highest activity in Europe divided into three focal zones - No. 26-1 (Central and South Greece), No. 26-2 (Crete), No. 26-3 (West Turkey), Nos. 27 + 29 + 33 + 34 (the North Anatolian fault zone).

Fig. 1 . European seismic zones

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The occurrence of earthquakes may be considered as a stochastic point process, where the individual events occur separately, or in small groups at a time. The probability structure of a point process is described by the distribution of the numbers of events in a particular interval. The process is stationary if its probability structure is invariant under translations of the time axis. The simplest stochastic point model for the occurrence of earthquakes is the stationary POISSON process. Given a sequence of independent random events, all of which are equally probable, we may assume that the probability is asymptotically proportional to the length of the interval and the probability of more than one event in the small interval is asymptotically negligible.

The probability distribution of a POISSON process [2] is

$$(1)\quad P(\lambda)_i = \frac{\exp(-\lambda)\lambda^i}{i!}, \quad \lambda > 0, \quad i = 0, 1, 2, \ldots,$$

when $P(\lambda)_i$ is the probability that in an arbitrary interval of unit length i events will occur with the intensity λ. To test the actual distribution with POISSON distribution the χ^2 test is used under the assumption that the theoretical frequencies $n\,P(\lambda)_i \geqq 2$, where n is the length of the observation period. If we denote by s_i the number of years with the annual number of earthquakes, i, then the test statistics

$$(2)\quad \chi_o^2 = \sum_{i=0}^{t} \frac{[s_i - n\,P(\lambda)_i]^2}{n\,P(\lambda)_i}$$

has a χ^2 distribution with $t - 2$ degrees of freedom and the parameter of the POISSON distribution

$$(3)\quad \lambda = \frac{\sum_{i=0}^{t} i\,s_i}{\sum_{i=0}^{t} s_i}.$$

The parameters of the POISSON distribution, λ, the test statistics, χ_o^2, and critical values, χ_α^2 (cf.[2]), are given in Table 1.

Table 1. POISSON distribution $P(\lambda)$

Region	22	26-1	26-2	26-3	27+29+ +33+34	26-1 without aftershocks
λ	3.3	7.9	2.6	4.9	6.0	6.0
χ_o^2	116.77	95.97	3.59	90.43	51.28	10.68
$\chi_{1\%}^2(t-2)$	16.8	21.7	13.3	18.5	15.1	15.1
$\chi_{5\%}^2(t-2)$	12.6	16.9	9.49	14.1	11.1	11.1

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The hypothesis about the POISSON distribution is not rejected at the 5 % or 1 % significance levels and the value χ_0^2 is not significant except for shocks in Crete (see Fig. 4, Table 1). It is shown that the agreement between observed and theoretical frequencies is influenced by the presence of aftershocks. As for example aftershocks from the periods 1912, 1914/15, 1953/54 are eliminated in Greece, it is possible to observe a good agreement with the POISSON distribution (see Fig. 7, Table 1).

In order to retrieve more statistical information from the catalogue, it is useful to look for a better distribution or, still more preferably, for a better model. If a two-parameter distribution is found which fits the data then in essence it is possible to describe the data with just two numbers.

The assumption inherent in the POISSON series, however, is that the probability of an event remains constant, which in practice rarely proves right. Any variation in the expectation of an event - in particular for one event to increase the probability of another - will increase the variance of the distribution relative to the mean, and a negative binomial distribution will invariably better describe the data. Writing the negative binomial distribution in the form $p^k (1 - q)^{-k}$, the probability of i events [4] is given by

$$(4) \quad NB(k, p)_i = \binom{i + k - 1}{k - 1} p^k q^i , \quad i = 0, 1, 2, \ldots,$$

where k, p are parameters, $p + q = 1$. The mean of the negative binomial distribution is

$$(5) \quad M = \frac{k\, q}{p} ,$$

the variance may be written as

$$(6) \quad V = \frac{k\, q}{p^2} .$$

Thus as p is necessarily less than unity, the variance is always greater than the mean, while for the POISSON distribution the variance is equal to the mean λ. Having derived the negative binomial distribution as a generalized POISSON series, it is not surprising to find that the POISSON distribution is obtained as a limiting form of $p^k (1 - q)^{-k}$.

There are several methods of estimating the parameters p and k. One of them is a method of moments, which is used here. If the first two moments of the negative binomial distribution are estimated from the sample moments, then the ratio of the mean to the variance provides an estimate of p, i. e. if the mean of the samples is m and the variance is s^2, then

$$(7) \quad p = \frac{m}{s^2} .$$

When $m = k\, q/p$ and noting that $q = 1 - p$, an estimate of k is given by

$$(8) \quad k = \frac{m\, p}{1 - p} .$$

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Table 2. Negative binomial distribution NB(k, p)

Region	22	26-1	26-2	26-3	27+29+ +33+34	26-1 without aftershocks
k	1.1	0.6	9.0	1.1	1.5	5.0
p	0.25	0.1	0.8	0.18	0.2	0.45
χ^2_o	6.15	41.01	12.89	11.43	14.63	12.26
$\chi^2_{1\%}(t-2)$	18.5	18.5	15.1	21.7	18.5	23.2
$\chi^2_{5\%}(t-2)$	14.1	14.1	11.1	16.9	14.1	18.3
E	0.66	0.53	0.97	0.63	0.68	0.89

The values of these parameters p, k are given in Table 2. The efficiency of estimating p and k by this method (see Table 2) was derived by FISHER [1], and in terms of the parameters used here the reciprocal of the efficiency is given by

$$(9)\quad \frac{1}{E} = 1 + 2\left\{\frac{1}{3} q \frac{2}{(k+2)} + \frac{1}{4} q^2 \frac{2 \times 3}{(k+2)(k+3)} + \frac{1}{5} q^3 \frac{2 \times 3 \times 4}{(k+2)(k+3)(k+4)} + \cdots\right\}.$$

To test the actual distribution with the negative binomial distribution the χ^2 test is used, too, under the same assumption about theoretical frequencies. Test statistics χ^2_o are less than corresponding critical values $\chi^2_{5\%}$ or $\chi^2_{1\%}$ for all the investigated regions except for Greece (Table 2). Therefore, only for Greece with the great aftershock series the hypothesis is rejected at given significance levels. After such sequences are removed, the data show a good agreement with the negative binomial model (Table 2).

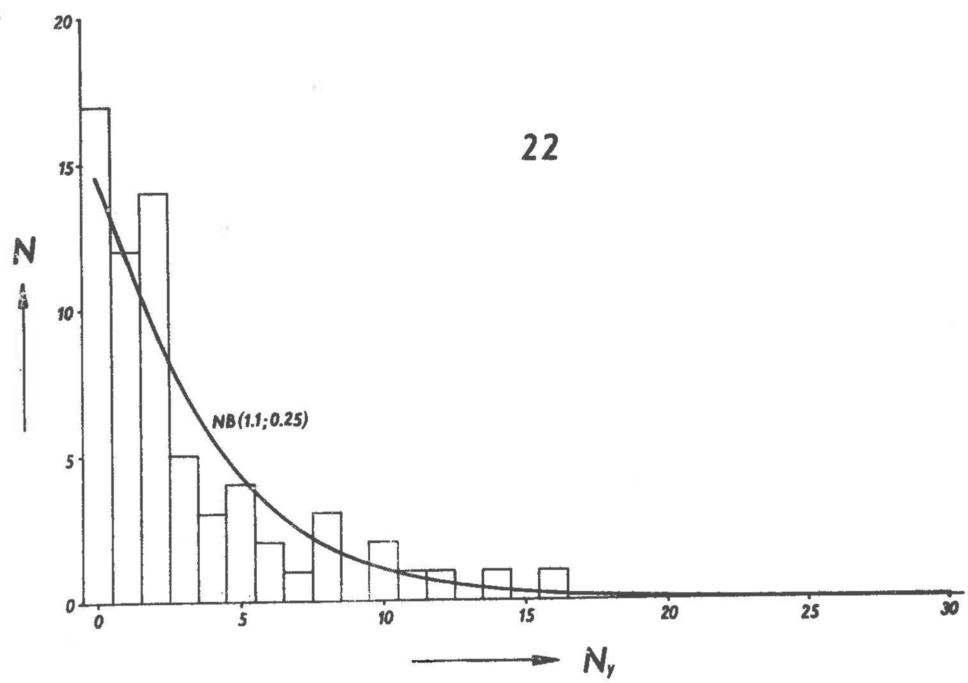

Fig. 2, Histogram of frequency of occurrence of shallow shocks in Yugoslavia

DOI: http://doi.org/10.2312/zipe.1972.018

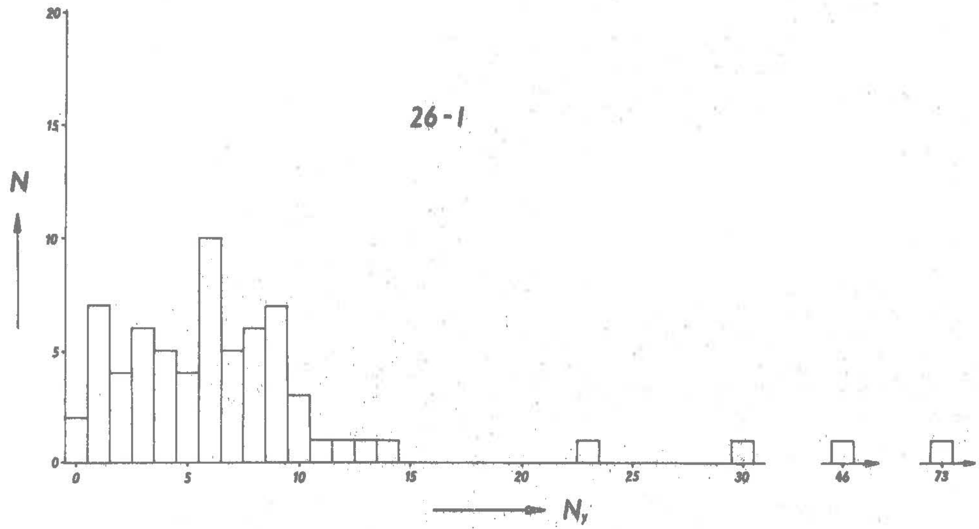

Fig. 3. Histogram of frequency of occurrence of shallow shocks in Greece

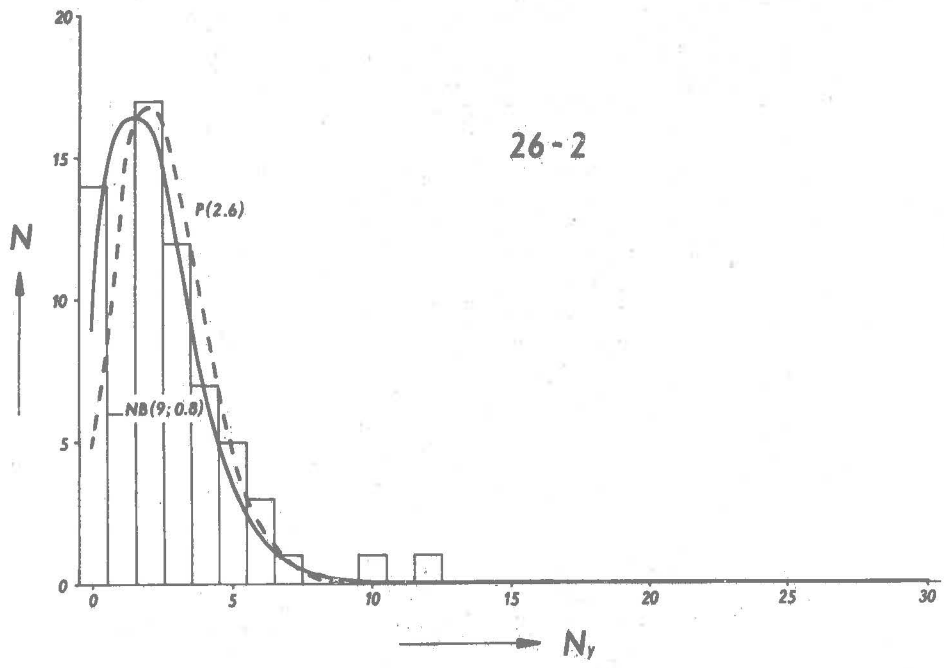

Fig. 4. Histogram of frequency of occurrence of shallow shocks in Crete

DOI: http://doi.org/10.2312/zipe.1972.018

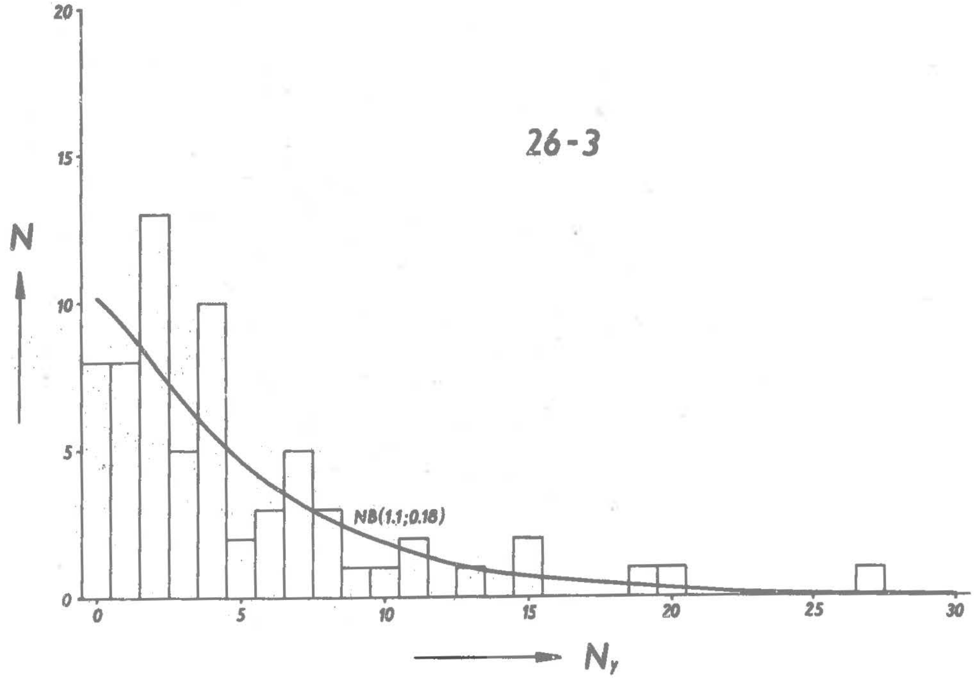

Fig. 5. Histogram of frequency of occurrence of shallow shocks in West Turkey

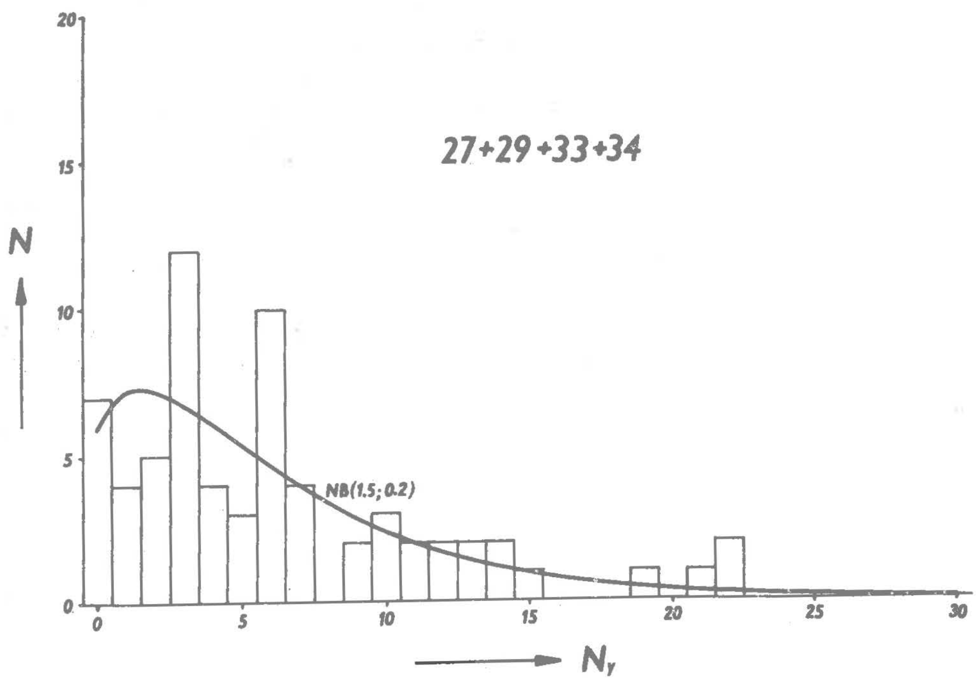

Fig. 6. Histogram of frequency of occurrence of shallow shocks in the North Anatolian fault

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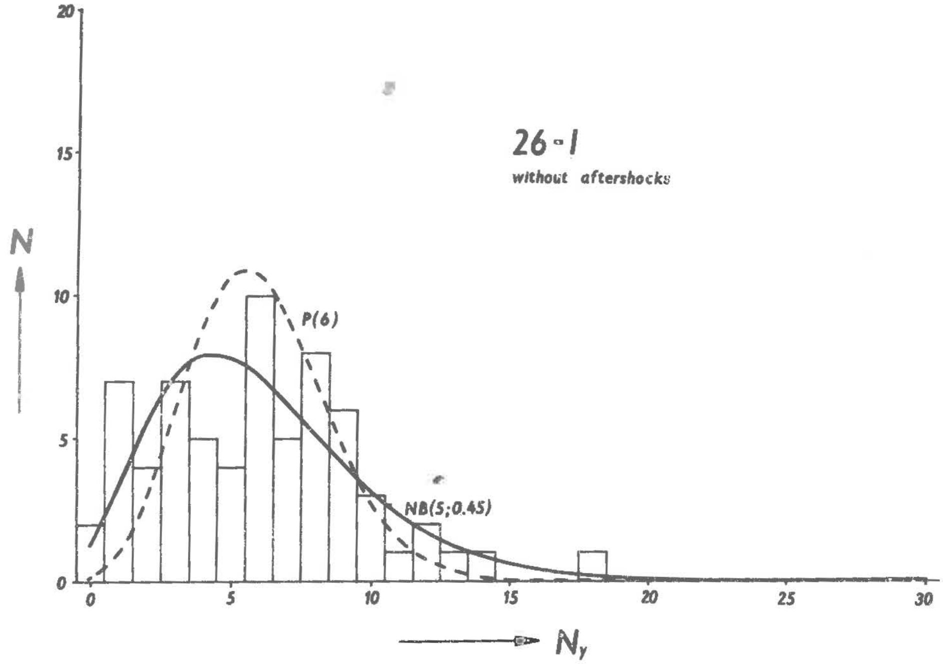

Fig. 7. Histogram of frequency of occurrence of shallow shocks without aftershocks in Greece

In Figs. 2 - 7 the histograms for the frequency of earthquake occurrence in one-year intervals are given for the individual zones (number of years with the given annual number of earthquakes, N, versus number of shocks in one-year intervals, N_y). The curves for the POISSON distribution with the parameter λ are drafted by dashed lines in these figures and the curves for the negative binomial distribution with the parameters k, p are drafted by full lines. From Fig. 7 it is difficult to recognize which distribution describes better the time distribution of earthquake occurrence. For the approximation of the beginning and the end of this histogram the negative binomial distribution and for the middle the POISSON distribution are more convenient.

Conclusion

So far we are limited in the development of ideas based on direct physical evidence or knowledge of physical processes leading to the origin of earthquakes. Therefore, various statistical models are being suggested and their properties are used for estimation of the probability of earthquake occurrence within a given area. According to the results of this paper it may be generally stated that the process with the negative binomial entries as a model describing the occurrence of shallow earthquakes in the north-eastern Mediterranean area is better than the POISSON process. It probably eliminates an influence due to the presence of swarms

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(e. g. West Turkey), which is difficult to define and remove, an influence of spatial inhomogeneity. The observed deviation from the negative binomial distribution must be attributed to the presence of aftershocks.

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Seismicity and Isoseists in Italy

By

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Summary

An analysis of instrumental and historical seismic data has been carried out in order to determine the model of earthquake occurrence and the model of intensity of shakings around the epicentres for the Italian region. The obtained models will be used in the next paper for the estimation of the seismic risk.

1. Introduction

The purpose of this paper is to construct the statistical models of the time series of earthquakes in Italy and of the isoseists of an earthquake of given magnitude. These models will be used in the next paper for the estimation of the seismic risk.

2. The model of earthquake occurrence

Italy was tentatively divided in two different ways:

a) into four regions (Fig. 1) according to the seismotectonic map of Europe (BELOUSSOV et al. 1968 [1]),

b) into three regions (Fig. 2) according to the map of surface faults (MALARODA and RAIMONDI 1957 [7])

in order to have the possibility of cheching the stability of the results.

In each region we assume basically two hypotheses:

1. The events in the magnitude range M_k belong to a POISSON distribution defined by λ_k (the annual average number of events in the range M_k). More precise formulation of this hypotheses is given by MOLCHAN et al. (1970) [8, 9].

2. The parameters λ_k for different magnitude range M_k in the same region are related to the magnitude M by

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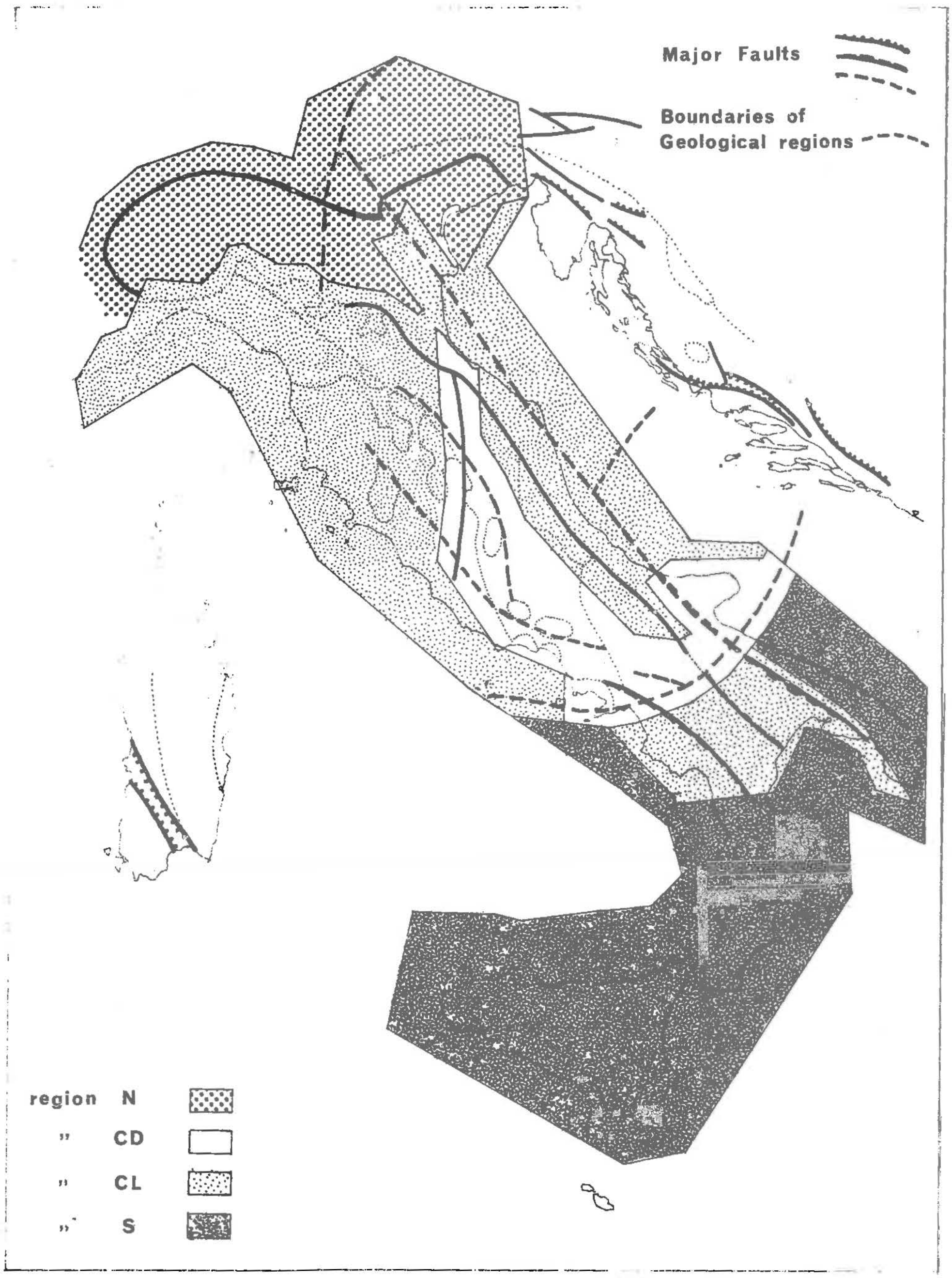

Fig. 1. Seismic regionalization of Italy based on major faults location

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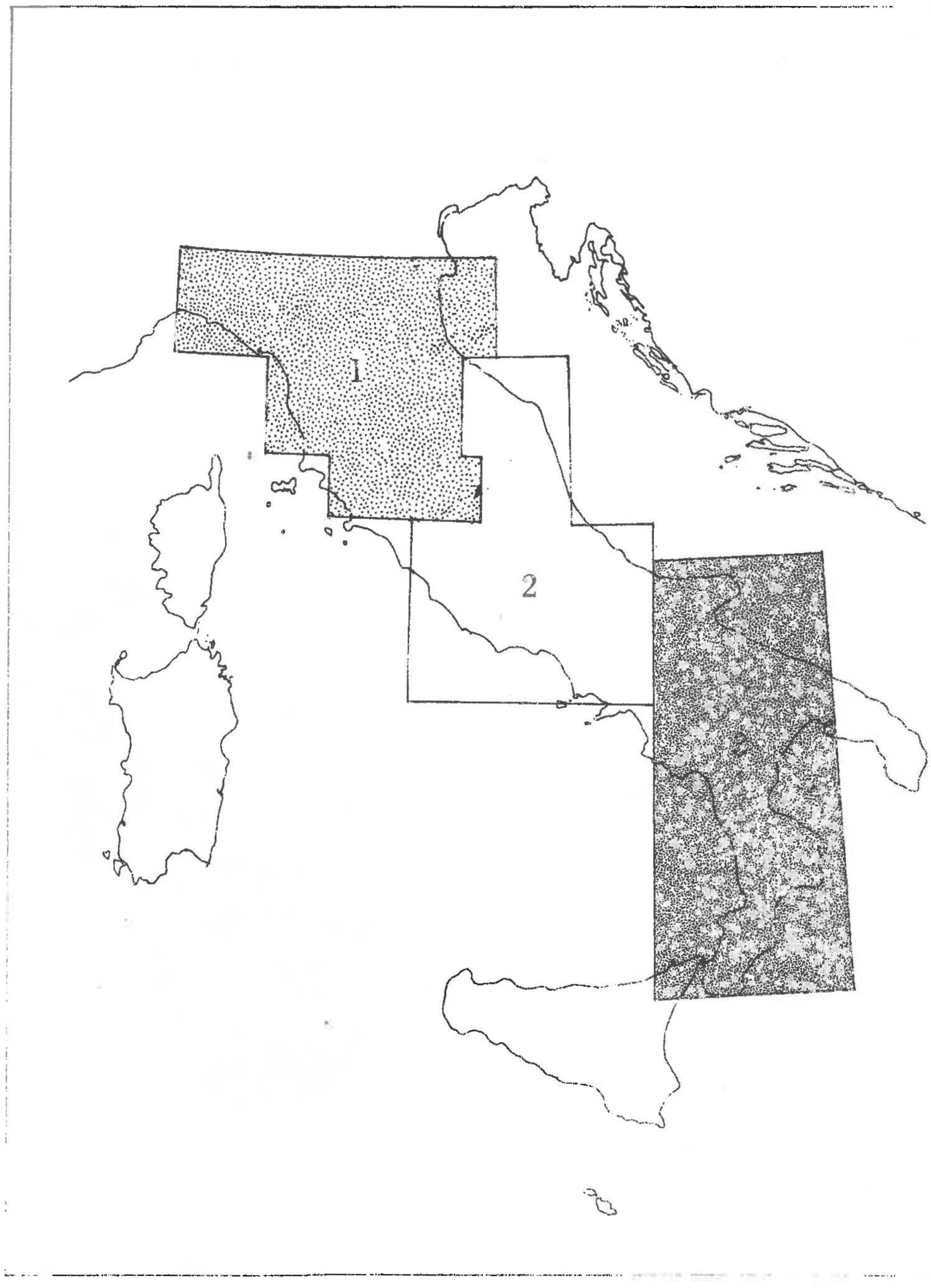

Fig. 2 Seismic regionalization of Italy based on surface faults distribution

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(1) $\lambda_k = \int_{M_k} 10^{\alpha-\gamma(M-M_o)}\, dM$

according to the linear GUTENBERG relation (GUTENBERG and RICHTER 1949 [5]). Details on the statistical consequences of the hypotheses 1. and 2. can be found in CAPUTO et al. (1968) [2].

In order to estimate α and γ we have considered data on 985 seismic events in Italy, instrumentally recorded between 1893 and 1965, which we shall call „instrumental data" (IACCARINO 1968 [6]), and 252 events between 1501 and 1929, recorded in chronicles, which we shall call „historical data" (CAVASINO 1929 [4]). The aftershocks were eliminated using the method of CAPUTO et al. (1972) [3].

For the estimation of α and γ we use the maximum likelihood method. This procedure results in the following main advantages compared with the usual estimation by least squares method:

a) Point estimations are maximum likelihood ones.
b) The confidence areas for α and γ are determined as well.
c) The errors in magnitude are considered.
d) The number of observed earthquakes can be used in the original form, i. e. for any non-intersecting volume of time - magnitude - space.

Table 1 gives the summary of computed variants. As can be seen from an analysis of that table the difference between estimations of α and γ for different M_k is not unsignificant, though less than $\Delta\alpha$, $\Delta\gamma$.

Concerning the magnitude of earthquakes from historical data, for which the macroseismic intensity J was estimated by CAVASINO, we have used the following relations:

$8 < J \leqq 9: \quad 5.6 \pm 0.1 \leqq M \leqq 5.9 \pm 0.1$;
$9 < J \leqq 10: \quad 5.9 \pm 0.1 \leqq M \leqq 7.8$.

Corrections of ± 0.1 were introduced in variants 2 and 3 respectively. Their influence is significant; the indication of magnitude intervals for historical earthquakes is one of the main sources of uncertainty in the estimation of α and γ.

Let us consider now if the joint analysis of the historical and instrumental data is justified. The values of p in Table 2 do not contradict the hypothesis that for each region the intensity of earthquake flow was the same in the historical and instrumental period. Under this hypothesis the ratio of earthquakes numbers ν_1 and ν_2 should approximately correspond to the ratio of periods T_1 and T_2. If ν_1 and ν_2 are independent and distributed by POISSON's law, the conditional distribution of ν_2 for fixed $n = \nu_1 + \nu_2$ is the binomial distribution:

$$F_p(x/n) = \{\nu_1 < x/n = \nu_1 + \nu_2\} = \sum_{k=o}^{[x]} c_n^k\, p^k\, (1-p)^{n-k}$$

with parameter

$p = m\,\nu_2 \,/\, m(\nu_1 + \nu_2)$,

DOI: http://doi.org/10.2312/zipe.1972.018

1. Estimations of α and γ; $\alpha \pm \Delta\alpha$ and $\gamma \pm \Delta\gamma$ are 99 % confidence intervals. Σ is the sum of all regions in corresponding variant

No.	Region	M_k	Data														
			Instrumental							Instrumental and historical							
			γ for different regions are assumed														
			Indipendent				the same			Indipendent				the same			
			γ	$\Delta\gamma$	$-\alpha$	$\Delta\alpha$	γ	$\Delta\gamma$	$-\alpha$	γ	$\Delta\gamma$	$-\alpha$	$\Delta\alpha$	γ	$\Delta\gamma$	$-\alpha$	
	CD		0.79	0.23	4.46	0.14			4.59	0.81	0.13	4.59	0.11			4.62	
	CL		0.75	0.12	4.71	0.10			4.96	0.84	0.10	5.01	0.09			5.02	
	S	0.1	0.92	0.32	5.71	0.16	0.82	0.12	5.40	0.82	0.15	5.34	0.11	0.84	0.07	5.40	
	N		1.09	[illegible]	6.18	0.18			5.29	0.94	0.13	5.62	0.14			5.32	
	Σ		[illegible]	[illegible]	5.48	0.07	-	-	-	0.89	0.07	5.30	0.05	-	-	-	
	CD									0.85	0.13	4.68	0.11			4.78	
	CL	1								0.87	0.11	5.15	0.09	0.86	0.07	5.18	
	S									0.86	0.16	5.51	0.11			5.56	
	N									1.00	0.19	5.82	0.14			5.48	
	Σ									0.94	0.07	5.47	0.05	-	-	-	
	CD									0.77	0.12	4.40	0.11			4.47	
	CL	0.1								0.80	0.10	4.88	0.09	0.80	0.06	4.88	
	S									0.77	0.14	5.19	0.11			5.26	
	N									0.89	0.17	5.44	0.14			5.18	
	Σ									0.85	0.07	5.14	0.05	-	-	-	
	CD		0.67	0.24	4.13	0.16			4.28	0.75	0.14	4.37	0.12			4.53	
	CL	0.4	0.71	0.19	4.56	0.12	0.77	0.13	4.76	0.82	0.11	4.94	0.09	0.81	0.07	4.91	
	S		0.89	0.32	5.57	0.16			5.20	0.81	0.15	5.28	0.11			5.29	
	N		0.98	0.39	5.83	0.17			5.14	0.88	0.17	5.44	0.14			5.25	
	Σ		0.82	0.13	5.08	0.07	-	-	-	0.83	0.07	5.12	0.06	-	-	-	
5	CD+CL	0.4	0.69	0.15	4.42	0.09	0.73	0.14	4.55	0.79	0.60	4.74	0.07	0.79	0.08	4.75	
	S		0.89	0.32	5.57	0.16			5.09	0.81	0.15	5.28	0.11			5.25	
	Σ		0.81	0.15	5.00	0.08	-	-	-	0.84	0.08	5.08	0.06	-	-	-	
6	CD+CL+N	0.4	0.74	0.14	4.72	0.08	0.76	0.13	4.81	0.81	0.08	4.94	0.06	0.81	0.07	4.94	
	S		0.89	0.32	5.57	0.16			5.19	0.81	0.15	5.28	0.11			5.28	
	Σ		0.81	0.13	5.08	0.07	-	-	-	0.83	0.07	5.12	0.06	-	-	-	
7	CD+CL+N	0.4								1.25	0.32	5.81	0.10	1.08	0.24	5.46	
	S									0.75	0.36	5.16	0.15			5.80	
	Σ									1.08	0.24	5.59	0.08	-	-	-	
	CD									0.99	0.48	4.85	0.17			5.02	
	CL	0.4								1.27	0.49	5.85	0.14	1.08	0.24	5.44	
	S									0.75	0.36	5.16	0.15			5.80	
	N									1.68	0.52	7.06	0.21			5.77	
	Σ									1.08	0.24	5.59	0.08	-	-	-	
	I		0.75	0.20	5.17	0.12			5.14								
	II	0.4	0.74	0.15	4.88	0.09	0.74	0.09	4.90								
	III		0.74	0.13	4.95	0.08			4.95								
	Σ		0.74	0.09	4.98	0.05											

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Table 2. Values of $F_p(\nu_2-0)$ and $F_p(\nu_2+0)$. p is the ratio of the number in the second and in the third lines

D a t a	T^{Reg}	N	CL	CD	S	CL + CD	N+CL+CD+S
Historical	400	27	48	38	49	86	162
Instrumental	62	4	11	5	5	16	25
Σ	462	31	59	43	54	102	187
p	0.134	0.129	0.188	0.116	0.094	0.157	0.134
$F_p(\nu_2-0)$		0.39	0.84	0.30	0.13	0.71	0.46
$F_p(\nu_2+0)$		0.60	0.91	0.49	0.25	0.80	0.54

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m being the symbol of average. If the hypothesis is correct,

$$p = T_2 / (T_1 + T_2) .$$

Consequently, for a fixed total number of earthquakes we may consider the observed ν_2 as the quantiles with some confidence level f. Due to the discrete nature of ν_2, f covers the interval $F_p(\nu_2 - 0)$, $F_p(\nu_2 + 0)$. The values of these two functions for $p = 0.134$ are given in Table 2; they confirm our hypothesis. In the worst case, for CL region, any confidence interval for p, symmetrical with respect to $F_p(x)$, includes the hypothetical value $p = 0.134$ with a confidence level more than $0.82 = 1 - 2(1 - 0.91)$.

3. The model of intensity of shakings around the epicentres

We are now going to describe the surface effect of each earthquake in macroseismic terms, i. e. in isoseists, since the majority of the experimental data are represented by isoseists. Because of the lack of information on earthquake mechanism, ground conditions, etc., we will construct a simplified, statistically averaged model of isoseists, which will depend mainly on the magnitude of the earthquake and on the orientation of the major geological fault near its source. - The isoseists are approximated by ellipses.

We considered the 68 events with reliable estimates of the isoseist parameters a, b, A, and Q, which are listed in Table 3 (a = major isoseist semiaxis, b = minor isoseist semiaxis, A = azimuth of the major semiaxis, Q = isoseist area). In Table 3 A_g is the azimuth of the major geological structures near the epicentre, determined from the seismotectonic map of Europe (BELOUSSOV et al. 1968 [1]). The types of these structures are characterized in the same table; m indicates a major fault zone, s and b denote the vicinity of the seashore or of the boundary of different tectonic complexes, respectively. A_l is the azimuth of the local surface fault, indicated by MALARODA and RAIMONDI (1957) [7]. For details about the computation of the relation between the magnitude and the isoseist parameters see CAPUTO et al. (1972) [3]. Here we show only the final results.

We can assume that

(2) $\log Q(M, J) = c_Q(J) + M\, d_Q(J)$,

(3) $\log l(M, J) = c_l(J) + M\, d_l(J)$.

The relation (2) was assumed also in previous studies of isoseists (SHEBALIN, in press [11]), values of $\log Q$ and $\log l$ computed from (2) and (3) are only the mathematical expectance (the average); the deviations of the observed values are considered as random, however, this dispersion may increase for small values of M because of the nature of small faults.

The parameters c_q, d_q were estimated from experimental data of Table 3 with the linear regression method allowing for errors in all observed values Q and M (RADHAKRISHNA 1964 [10]). This procedure has evident advantages, compared with the usual least squares method. The results of computations are given in Table 4.

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N	Day	Epicentre λ	Epicentre φ	M	l_{MAX}	log Q 7	log Q 8	log Q 9	log Q 10	l 7	l 8	l 9	l 10	A° 7	A° 8	A° 9	A° 10	|(Az-Ag)°| 7	|(Az-Ag)°| 8	|(Az-Ag)°| 9	|(Az-Ag)°| 10	A_z-A_g cluster	Structure type	A_z-A_l	$(lgQ-\overline{lgQ})$ 7	$(lgQ-\overline{lgQ})$ 8	$(lgQ-\overline{lgQ})$ 9	$(lgQ-\overline{lgQ})$ 10
44)	23/7/1930	15.4	41.0	6.5	10	3.99	3.52	3.20	2.55	.58	.54	.45	.30	75	-83	-83	-59				20		m	30	-0.10	-0.20	0.15	0.00
45)	5/9/1931	11.4	44.1	4.6	7	1.95				.78				32								80	b		0.03			
46)	3/12/1931	15.8	41.2	3.9	7					.34				89								50	m					
47)	31/7/1936	14.1	41.8	3.7	7					.67				10								60-25	m-b					
48)	26/9/1933	14.2	42.3	5.5	9	3.31	2.98	1.98				.67													0.05	0.15	-0.40	
49)	9/12/1936	13.2	43.1	4.7	8	2.25	1.71			.82	.78														-0.35	-0.45		
50)	17/7/1937	15.4	41.7	4.5	8	2.49	2.11			.84	.83														0.10	0.10		
51)	15/12/1937	15.3	41.7	4.4	7	1.89				.67				6										50	0.10			
52)	15/10/1939	10.2	44.3	5.1	7	2.19				.81															-0.05			
53)	16/10/1940	11.7	42.9	4.4	8	2.99	2.36			.52	.57			72	72										0.75	0.45		
54)	3/11/1941	12.5	41.9	3.9	7					.31				84								85	m					
55)	25/3/1943	13.4	43.2	4.8	7	2.19				.70				30								65	m	55	0.15			
56)	3/10/1943	13.6	43.1	5.5	8	3.28	2.79			.46	.33			-16	-8				25				m		0.05	-0.05		
57)	13/6/1948	12.2	43.5	4.5	8	2.57	1.69			.85	1.0														0.15	-0.30		
58)	18/8/1948	16.1	41.5	5.0	7	3.29								-70				15-55					m	30	0.45			
59)	15/11/1948	11.6	42.9	4.2	7	2.01				1.0															0.35			
60)	29/11/1948	11.6	42.9	4.6	7	1.81				1.0															-0.15			
61)	31/12/1948	12.8	42.5	4.9	7	2.11				.67				71								70	m		0.00			
62)	5/9/1950	13.3	42.5	5.5	8	3.82	3.41				.60				85				75				m		0.55	0.55		
63)	8/8/1951	13.8	42.6	4.9	8	2.52				.47				-76				50					b	75	-0.25			
64)	1/9/1951	13.2	43.1	5.0	7	3.12				.38				37				10					s	75	0.25			
65)	4/7/1952	11.9	43.9	4.4	7	2.31				.61				76								55	m		0.50			
66)	24/6/1956	13.4	42.3	5.0	8	2.73	1.95			.73				-55				70					m		-0.10	-0.45		
67)	31/10/1961	13.0	42.4	4.7	8	2.41	2.11			.43	.43			40	40				60				m	30	-0.20	-0.05		
68)		15.9	40.2	5.5	9	4.01	2.93	2.11				.71				-62				6			m		-0.75	0.10	-0.20	

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Tab. 3

N	Day	Epicentre λ	φ	M	I_{MAX}	log Q 7	8	9	10	l 7	8	9	10	A° 7	8	9	10	$\lvert(A_z-A_g)^{\circ}\rvert$ 7	8	9	10	A_z-A_g cluster	Structure type	A_z-A_l	$(\lg Q-\overline{\lg Q})$ 7	8	9	10
1)	14-16/1/1703	13.1	42.8	6.3	10	3.43	3.19	2.91	2.01				.42				-50				70		m					
2)	2/2/1703	13.5	42.3	6.3	10	3.55	3.25	2.79	2.36			.48	.45	82	80	72	80				25		m					
3)	5/2/1783	15.1	38.3	6.8	10	4.42	3.96	3.41	2.73			.51	.63	58	58	62	55			45			s	0				
4)	7/2/1783	16.2	38.5	6.0	9	3.90	3.27	2.49				.50				62												
5)	27/3/1783	16.5	38.8	6.2	10	4.42	3.75	3.01	2.55			.50	.44			90	90											
6)	23/2/1887	08.2	44.1	7.6	9	4.35	3.96	3.50								40				0			s	42	-0.65	-0.65	-0.40	
7)	22/1/1892	12.7	41.7	4.6	8	2.66	2.19			.95	.56				-80				45				m					
8)	4/3/1898	10.3	44.6	5.1	7	2.25				.67				36				90					b		-0.65			
9)	28/6/1898	12.8	42.4	4.2	8	2.25	1.81			.68	.75			-71	-71				60				m					
10)	3/7/1901	13.7	41.6	5.8	8					.38	.64			25	36							80	m					
11)	5/3/1902	10.5	44.1	4.4	7	1.49																			-0.30			
12)	24/2/1904	13.3	42.1	5.6	8	2.75	2.36			.81	.76														-0.60	-0.60		
13)	12/2/1905	11.5	42.8	4.7	7	2.11				.59				52								90-75	m-b		0.10			
14)	14/3/1905	14.7	41.1	4.9	7	1.71																			-0.45			
15)	25/8/1905	13.9	42.1	4.7	7	2.01				.53				49								90-40	m-b		0.00			
16)	8/9/1905	16.1	38.5	7.0	10	4.27	3.96	3.43	2.86				.29			15	15				0		s		-0.25	-0.15	0.00	-0.05
17)	26/11/1905	15.0	41.1	5.0	7	2.19				.37				74								30	m		0.00			
18)	23/10/1907	16.1	38.0	5.9	10	3.97	3.55	2.86	2.01			.55	.67			57	57			45			s	30	0.30	0.40	0.25	-0.10
19)	10/12/1908	15.1	38.0	5.0	8	2.36				1.0															-0.50			
20)	25/8/1909	11.2	43.1	5.1	8	3.33	2.84			.58	.49			-79	-79				30				m		0.45	0.35		
21)	7/6/1910	15.4	40.9	5.9	10	3.70	3.28	2.89	2.11	.52	.23	.21	.17	-57	-69	-57	-47			20			m		0.10	0.15	0.30	0.00
22)	19/2/1911	12.1	44.2	5.1	7	1.89				.45				-40								10	m		-0.35			
23)	13/9/1911	11.4	43.4	4.8	8	2.81				.42				86				0-90					s		0.15			
24)	28/6/1913	16.2	39.6	5.5	9	3.49	3.18	2.25				.50				-75				20			b		3.25	0.35	-0.05	
25)	4/10/1913	14.7	41.5	5.1	8	2.97	2.22				.25				82				60				m		0.05	-0.30		
26)	27/10/1914	10.4	44.1	5.7	7	2.96				.52				-34				15					b-s	0	-0.45			
27)	13/1/1915	13.6	41.9	6.8	11	4.34	4.04	3.34	2.66				.38				-64				10		m	15	0.00	0.10	0.05	-0.10
28)	22/4/1916	13.4	42.3	5.0	7	2.41				.54				51								55	m	20	0.20			
29)	4/7/1916	13.3	42.8	4.4	8	2.55	2.25			.71	.82								70-35				m-b		0.20	0.35		
30)	16/11/1916	13.2	42.6	5.0	8	2.73	2.18			.73	.71			37				60					m		-0.10	-0.20		
31)	26/4/1917	12.1	43.5	5.5	9	3.23	2.81	2.34		.61	.34	.50		-18	-21	-29				40			m		0.00	-0.05	0.05	
32)	2/12/1917	13.8	41.5	4.8	7	1.89				.56				38								75	m		-0.15			
33)	29/6/1919	11.5	49.3	6.2	9	3.49	3.12	2.71		.53	.41	.48		-45	-50	-50				0			s		-0.35	-0.30	-0.15	
34)	10/9/1919	11.7	42.8	5.3	8	3.21	2.70			.73	1.0														-0.10	0.05		
35)	7/9/1920	10.3	44.2	6.3	9	3.68	3.32	3.01		.56	.56	.43		-53	-41	-41				0			b-s	0	-0.20	-0.20	0.10	
36)	28/8/1921	13.1	43.2	4.7	8	2.61	2.41				1.0														0.05	0.25		
37)	29/12/1922	13.6	41.8	5.5	7	2.86				.34				- 4										35	-0.40			
38)	2/1/1924	13.2	43.7	5.2	8	2.91	2.55			.67															-0.10	-0.05		
39)	11/10/1927	13.4	41.9	5.0	7	2.15				.53				70								75	m		-0.05			
40)	28/10/1927	09.6	44.5	4.8	7	2.19				.70				-42								45-50	m-b	0	0.15			
41)	26/12/1927	12.7	41.7	4.4	8	2.25	1.71			.75	1.0			-10				25					m		-0.19	-0.20		
42)	18/7/1929	11.4	43.9	4.6	7	1.89				.48				-44								0	b	0	-0.05			
43)	27/4/1930	14.6	40.7	4.5	7	1.71				.73				28								90-50	m-b	55	-0.15			

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Table 4. Estimation of parameters of the linear law log $Q(M, J) = c_Q(J) + d_Q(J)\,M$
l.r. = linear regression, l.s. = least square method
Variants 13, 14, 15 refer to the cluster

1	2	3	4	5	6	7	8	9	10	11	12	13
Variant	Intensity	Method	N	δ_m	δ_Q	Quantiles of δ_m 2.5 %	97.5 %	η^2_{n-2}	Quantiles of η^2_{n-2} 2.5 %	97.5 %	c_Q	d_Q
1	7	l.r.		0.2	0.2	0.231	0.417	64.3			-1.134	0.79
2	7	l.r.	37	0.25	0.2	0.198	0.402	52.6	20.6	53.2	-1.31	0.83
3	7	l.s.				0.274	0.440	99.8			-0.60	0.69
4	8	l.r.		0.2	0.2	0.186	0.391	39.7			-1.69	0.82
5	8	l.r.	29	0.25	0.2	0.134	0.373	32.3	14.6	43.2	-1.80	0.84
6	8	l.s.				0.244	0.419	64.0			-1.32	0.75
7	9	l.r.		0.2	0.2	0.045	0.382	8.8			-1.96	0.77
8	9	l.r.	12	0.25	0.2	0.0	0.366	7.2	3.2	20.5	-2.07	0.79
9	9	l.s.				0.163	0.409	13.6			-1.64	0.72
10	10	l.r.		0.2	0.2	0.0	0.166	0.18			-2.16	0.72
11	10	l.r.	5	0.25	0.2	0.0	0.127	0.15	0.2	9.4	-2.17	0.72
12	10	l.s.				0.034	0.219	0.26			-2.13	0.71
13	7*	l.r.		0.2	0.2			16.8			-1.32	0.70
14	7*	l.r.	20	0.25	0.2			13.6	8.2	31.5	-2.71	1.00
15	7*	l.s.						20.7			0.43	0.33

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The variants 13, 14, 15 refer to areas significantly minor to those obtained by extrapolation from larger M. The group of seismic events characterized by such anomalous Q defines the cluster. These values of Q appear when a wide-range variation of the focal depths inside the crust becomes possible.

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Preliminary Epicentre Map of Italy from the Year Zero up to 1970

By

M. CAPUTO and D. POSTPISCHL [1)]

Summary

The seismicity of Italian region is discussed in the sense of the hypocentre distributions. Four maps are constructed referring to the following data division:

a) Map of all known earthquakes from 0 to 1970 (4909 events)
b) map of all instrumental earthquakes
c) map of instrumental earthquakes of class A and B
d) map of instrumental earthquakes of class A

The comparison of the four maps allows to locate the following main seismic areas:

a) Western Alps
b) Eastern Alps
c) Appenines
d) South Italy and Sicily
e) Deep focus earthquakes in the Tyrrhenian Sea.

A summary of all known focal mechanisms for the Italian region is given too.

The map presented here is derived from a catalogue of earthquakes (CAPUTO and POSTPISCHL 1972 [6]) that collects the Italian seismic history from the beginning of the Christian era (year zero) up to 1970. This work is part of a project which was started in 1969 with the aim to draw a map of seismic risk, a map of active faults, and to confirm the stability of seismic activity (BOSCHI et al. 1969 [2], CAPUTO et al. 1969, 1972 [3, 7]).

The first step of the research was the preparation of the above mentioned catalogue, which was intended to be as complete and homogeneous as possible with respect to the parameters involved, representing a basis for further seismological investigations. However, although our inquiries were performed in a careful way, the catalogue may be incomplete.

As far as the time distribution of the earthquakes is concerned the data have been divided into two groups. The first one, called „historical", refers to the data gathered from most different sources, preferably from the works of BARATTA (1901) [1], CAVASINO (1931) [8] and MERCALLI (1897) [11] as well as from the catalogue of GIORGETTI and IACCARINO (1972) [9] and dated from the year zero to 1900. The remaining data result from the existing catalogues, bulletins and the papers published

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n this period. The instrumental or macroseismic informations are divided according to the accuracy of the determination of the epicentres as follows: A: locations $< 0.3^{o}$, B: locations between 0.3^{o} and 0.9^{o} , C: locations $> 0.9^{o}$

As to the epicentral locations the historical events were adapted to the coordinates of the towns or villages that endured the maximum of dammages, according to the historical chronicles. In these cases it is not possible to commit an error regarding the epicentral location and consequently separate the events into the three epicentral accuracy classes. Only in a few instances, when it was possible to draw the isoseists, the epicentre is the central point of the isoseists of maximum degree. Obviously the intensity is to be intended as maximum observed intensity instead of intensity at the epicentre.

In the case of instrumental data the epicentres listed in the catalogue are averaged with respect to the statements of the different sources about the same events. - The epicentre accuracy class is associated to the mean square error. The resulting catalogue covers 4909 earthquakes from the year zero up to 1970.

In order to study the correlation among the epicentre distribution and the major geotectonic pattern of Italy, different epicentre maps, according to the four classes of accuracy of the determination as well as maps for different magnitude classes were drawn. Fig. 1 shows one of these, and precisely the cumulative one, that collects all the events listed in the catalogue.

The mapping of the epicentres provides a very convenient survey of the geographical distribution of seismic activity, even if a differentiation with respect to the intensity or magnitude is lacking. The original scale used in constructing the maps was 1 : 1 000 000, providing a good legibility. The legend of the map explains the different symbols used. These maps represent only a preliminary work, and in fact we are studying a more appropriate way to express the epicentre location, magnitude, depth, accuracy of location etc. in a unique map.

The historical events show a strong correlation between the epicentres and the distribution of the population in the past. It may be mentioned here that we have evidence that many small events have not been recorded instrumentally in the period from 1900 to 1970, due to the lack of seismic stations. In particular an analysis of the maps of the epicentres with different accuracy allows to indicate active seismic units.

Proceeding from north to south we mention the western Alps area, where a marked dispersion of epicentres extending from the Simplon pass to the Ligurian coasts was found. The epicentre locations are generally uncertain, and it is therefore not possible to determine particular alignments. On the contrary, the eastern Alps area is characterized by a very well defined seismogenetic fault, extending from the Lago di Garda towards NE to the Carpathian Mountains, with the maximum activity in the Carnia - Friuli region.

The main activity in Italy, however, takes place along the Apennines, developing to the south into an arcuate structure of the Pacific arc type. We have an alignment

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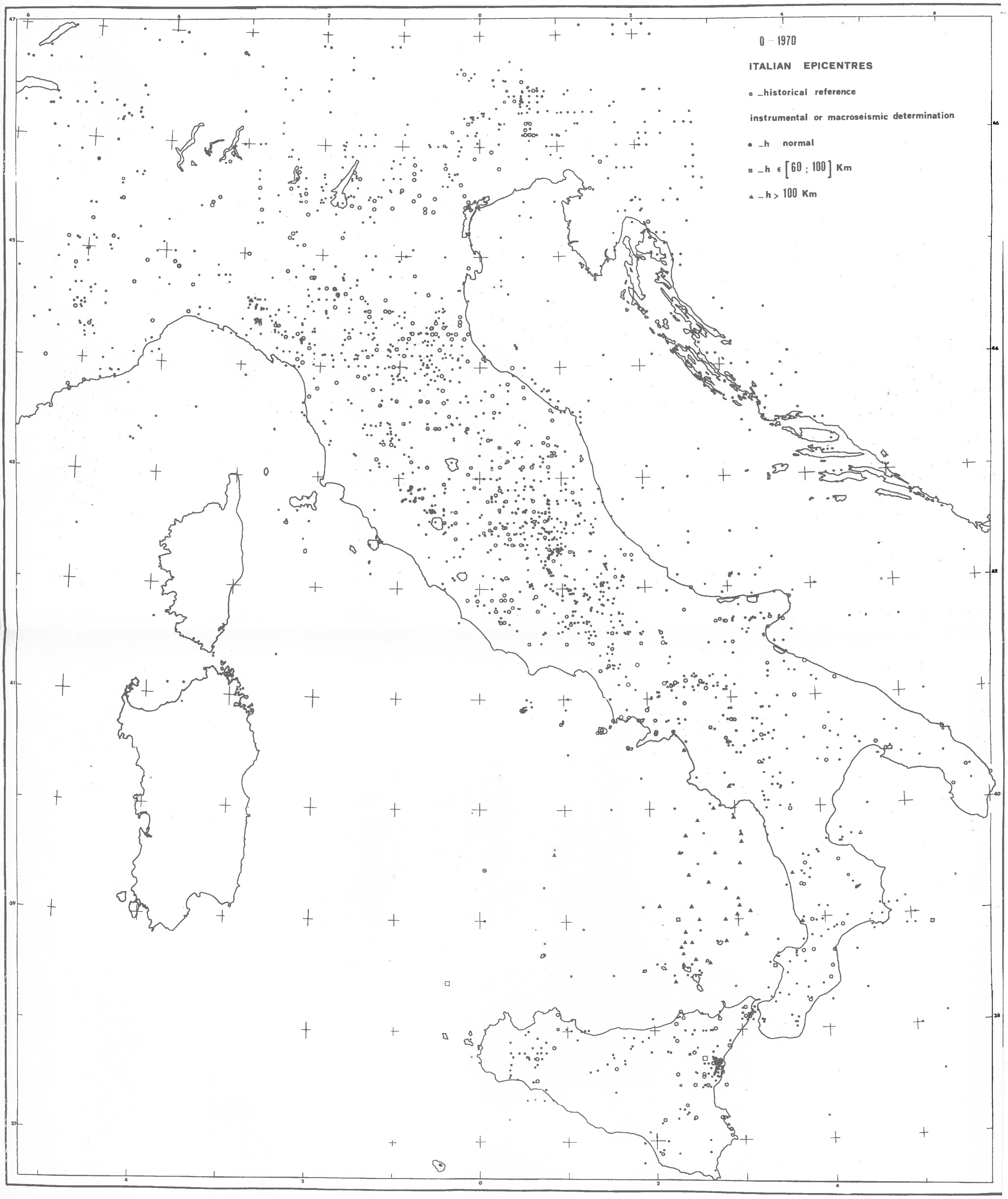

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Focal Effect on M_S Determination from Rayleigh Waves

(Preliminary Report)

By

G.F. PANZA and G. CALCAGNILE [1]

Summary

The theoretical amplitude spectra of RAYLEIGH waves for different focal mechanisms have been computed in the period range 250 - 2 sec, using BEN-MENAHEM and HARKRIDER's formula. The depth dependence of M_S in the range 5 - 75 km is of about three units, while the influence of the orientation of displacement dislocation at a given depth is less than 0.3 units. The depth dependence is decreasing with increasing depth and at about 35 km it becomes comparable with the influence of the other focal parameters.

1. Introduction

In this paper the influence of the focal mechanism on the magnitude determination using RAYLEIGH waves is investigated. We consider earthquakes taking place in the shield structure, considered by PANZA et al. (1972) [3] in the depth range from 5 to 75 km.

2. Purpose

The purpose of this preliminary paper is to point out the problems connected with the magnitude determinations of surface waves from a theoretical point of view and to show the extreme errors in magnitude determination introduced by the lack of knowledge of the focal mechanism. A study of the possible sources of smoothing of such errors remains outside the frame of this note.

3. Theoretical magnitude estimation

Recently many authors (e. g. VON SEGGERN 1970 [5]; SYED and NUTTLI 1971 [6]) have discussed the effect of focal mechanism on magnitude determination both for body waves and surface waves. In this paper we analyse the effect of the focal parameters on the magnitude determination from surface waves.

We have considered the amplitude spectra for the fundamental RAYLEIGH mode in the period range 250 - 2 sec, given by

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(1) $U_z^{DC} = \{|R(\omega)|\ e^{i\varphi_0}\}\ k_j^{\frac{1}{2}}\ e^{i3\pi/4}\ \chi(\theta,\ h)\ A_j[\dot{w}(h)/\dot{w}_0]_j$.

Expression (1) was obtained by BEN-MENAHEM and HARKRIDER (1964) [2].

In Fig. 1 examples are given of the amplitude spectra for dip-slip and strike-slip mechanisms as they have been obtained by PANZA et al. (1972) [4]. We can see that the amplitude maximum is strongly depth-dependent.

In Fig. 2 it is shown how (A_2/T_2) changes with depth for different orientations of the displacement dislocation; A_2 is the maximum amplitude and T_2 is the corresponding period. The shaded area refers to the case $\theta = 30^o$, $45^o \leqq \delta \leqq 90^o$, $\lambda = 180^o$, 270^o, where θ is the azimuth, δ is the dip angle, λ is the slip angle, as defined by BEN-MENAHEM and HARKRIDER (1964) [2]. As it can be seen in Fig. 2, the relation $\log(A_2/T_2)$ versus h is not linear and it seems that for deep earthquakes depth variations of the focus are not so critical, hence they become comparable with variations in the dislocation orientation. Over the depth range 5 - 75 km we can have 3 units of error in magnitude determination.

VON SEGGERN, computing the theoretical surface wave magnitudes for different sources at the same depth, has found an error of more than one unit considering the amplitude at about 20 sec, as it is practically done in routine work (BÅTH 1966 [1]). This is an error which is larger than that one shown in Fig. 2 and it is critically connected with the choice of T around 20 sec.

In fact, for mechanisms of the strike-slip type (e. g. $\delta = 90^o$, $\lambda = 180^o$) it can be shown that at a focal depth of about 18 km the periods around 20 sec can be characterized by the well pronounced and narrow minimum which in Fig. 1A occurs at about 29 sec and 10.8 sec, respectively, for h = 35 km and h = 7 km. Obviously, this fact can affect significantly the magnitude determination if we consider $T \approx 20$ sec, while to consider $T = T_2$, as we have seen, makes the dislocation orientation less critical at least for $h \leqq 35$ km.

Finally, an analysis of the azimuthal dependence of the magnitude has shown that such a parameter causes a modulating effect. This effect is not relevant for our considerations, at least neglecting determinations from a "station" within 5^o azimuth off nulls, even if it can have a smoothing effect on the theoretical errors previously mentioned.

4. Discussion

It seems important to emphasize that a correction of focal mechanisms must be applied only for certain purposes. In fact the magnitude correction for the focal mechanism is appropriate if one is concerned with an estimate of the seismic energy or in order to set up an $M_s - m_b$ discriminant between earthquakes and explosions which is as free as possible of regional effects, while it is not appropriate for seismic risk evaluation. In the latter case an accurate determination of the focal depth can give useful information about which instrument (long-, intermediate-, short-period) must be used in order to have the most accurate possible measurements of the ground displacement.

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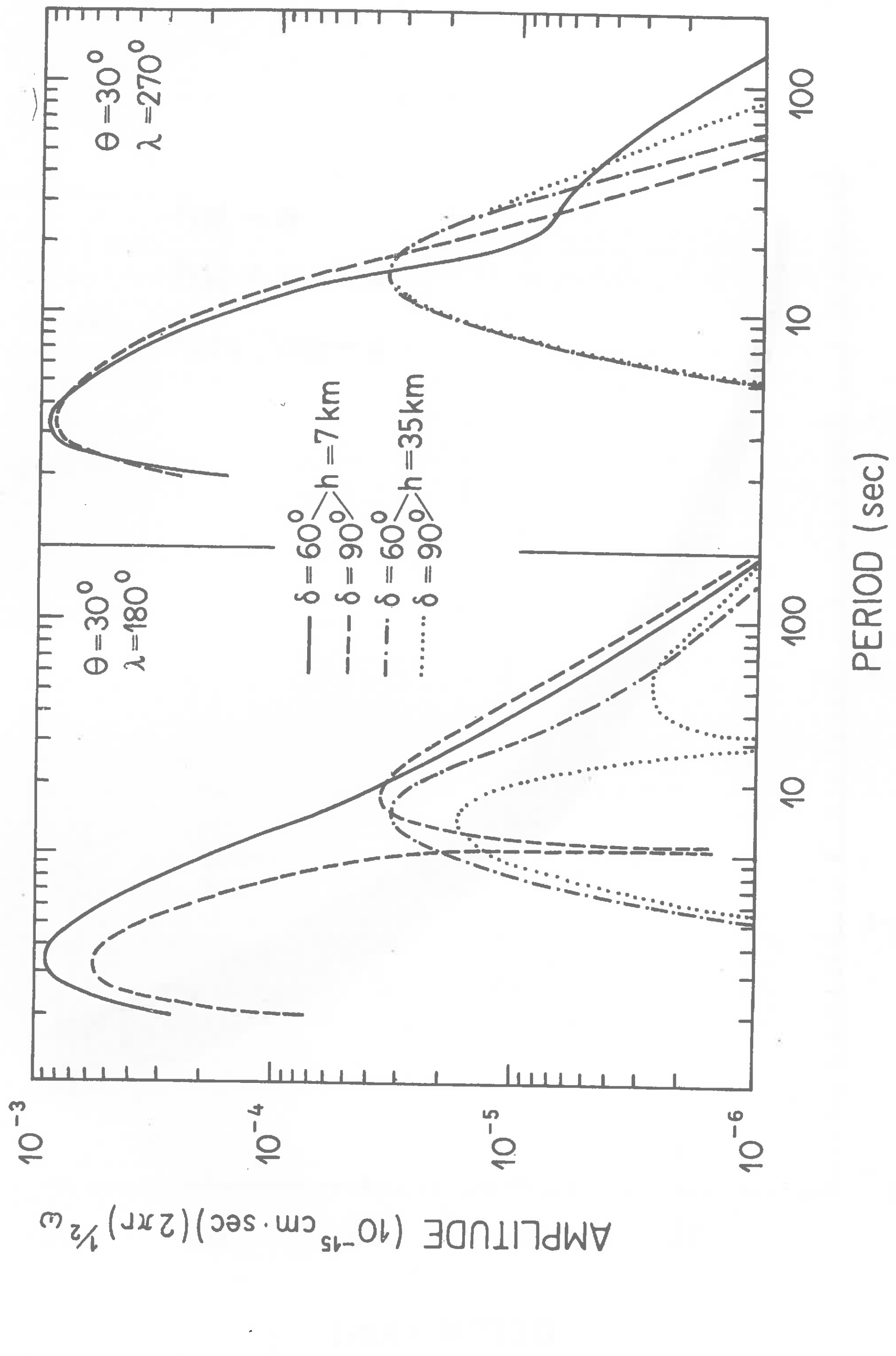

Fig. 1. Fundamental mode amplitude spectra for some examples of dip-slip and strike-slip mechanisms with the geometrical spreading factor removed

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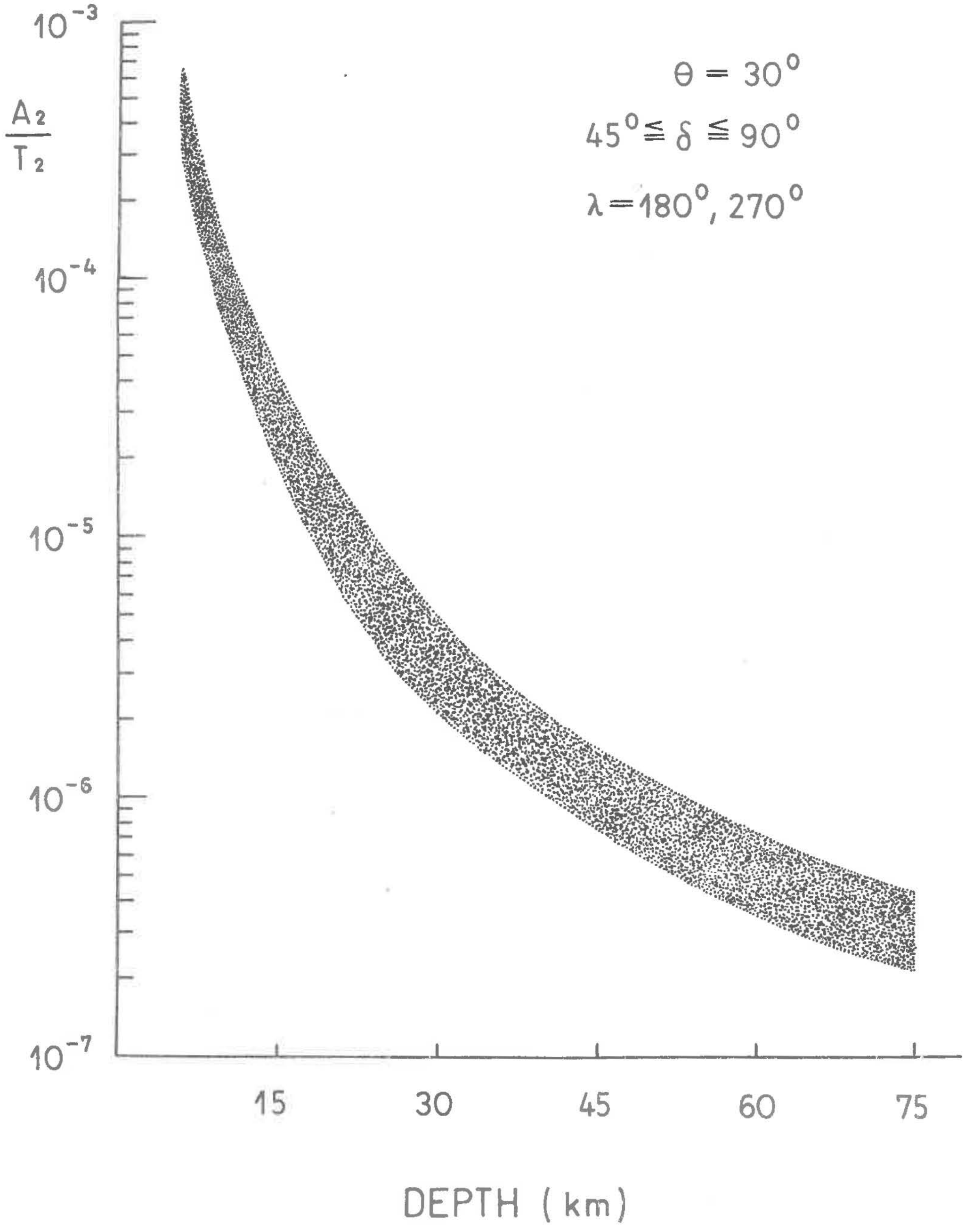

Fig. 2 Depth dependence of A_2/T_2

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5. Conclusion

We have considered the possible theoretical sources of errors in magnitude determination from RAYLEIGH waves assuming a double-couple source representation of constand vector magnitude. Our results can be summarized as follows:

a) The focal depth is the critical focal parameter for $0 \leqq h \leqq 35$ km; for deeper sources the influence of the other focal parameters becomes relevant as well.

b) For the considered focal mechanisms the obtained dispersion in magnitude determinations is of about 3 units.

In order to have a more complete picture of the problem it will be necessary to deal with a structure significantly different from the one considered here and the possible contribution of the higher modes. The introduction of a source of finite dimension may be important, too. This study will be the subject of a future paper.

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On the Magnitude Distribution and Prediction of Earthquakes

By

G. PURCARU [1] and R. MAAZ [2]

Summary

Some problems of the frequency-magnitude distributions are analysed and the possibilities of extrapolation of these relations in the range of small and great magnitudes are considered from the point of view of earthquake prediction. One of these distributions, the lognormal magnitude distribution , is studied in detail. Moreover, the confidence intervals of the coefficients of the corresponding frequency-magnitude relation and some discussions are given.

Introduction

One of the principal ways leading to the prediction of earthquakes is the study of the statistical laws of their occurrence in space and time, which generally has a stochastic character; therefore, the knowledge of its structure is possible by consideration of the earthquakes as systems of random events.

The stochastic character of the occurrence of earthquake events implies that if

1. the statistical laws reflect more exactly the distribution of different parameters of the earthquakes and
2. stochastical models describe more precisely the behaviour of the system in space and time,

the problem of earthquake prediction can be solved more exactly. In literature many papers exist which deal with both of these aspects, and some important theoretical and practical results have been obtained.

In earthquake forecasting three parameters must be predicted, the location, time of occurrence, and magnitude or energy, the second of which is most difficult to be determined. Concerning the magnitude or energy the fundamental problem consists in the determination of the real magnitude distribution for a given earthquake event system and the possibility of extrapolation of observed data on magnitude (energy) in the range of its small and great values. In the present paper we shall present some results regarding this problem and make different considerations especially on the lognormal distribution of magnitude.

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2. On the frequency-magnitude relation

At present in almost all papers the GUTENBERG-RICHTER (G.-R.) relation is used to express the dependence between the log frequency of earthquakes and magnitude or energy. This fact implies that the magnitude follows a negative exponential distribution while the energy obeys the PARETO distribution.

But it has been observed in some seismic regions that the G.-R. law can be inadequate, and the function $\log N(M) = \alpha - \beta M$ does not satisfy in all cases the observed distribution of earthquake frequencies with respect to M or $\log E$. Some time ago a generalized frequency law for earthquakes in the detailed study of the seismic regime has been introduced [20]. The deviations from the G.-R. law have been observed both at left-hand and right-hand sides of the linear graph of the frequency-magnitude relation. In other words, the extrapolation in the range of minimum or maximum magnitudes is not correct in these cases.

Therefore, other frequency-magnitude (energy) relations have been introduced to satisfy the real conditions, the forms of the graphs $\log N(M)$ being nonlinear. For instance, in two areas of Tadzhikestan it has been observed [11] that the mean long-time graph $\log N(K) = \delta - \gamma K$ ($K = \log E$, joule) is nonlinear in the range of strong earthquakes, and a better relation was introduced.

For the earthquakes of Japan and its vicinity it was established [13] that, although the G.-R. relation is available, the magnitude distribution has a form different from the negative exponential one. The given type of frequency-magnitude relation agrees very well with the observed values of magnitude of the largest earthquake occurred in or near Japan.

In the papers [7, 8] the frequency of earthquakes with $K = 7 - 15$ has been investigated for some zones of USSR, and the following results were obtained: The frequency graphs $\log N(K)$ are formed by two straight lines with a common point situated in the range $K = 11 - 12$, i. e. in the domain of relatively strong earthquakes. However, the frequency function proposed in [8] results in discrepancies from the observational data at both ends of the graph, and the authors suppose a quadratic form to be valid for the frequency-magnitude relation. A similar phenomenon has been observed for the form of the earthquake frequency graph with respect to magnitude in some aftershock sequences [3, 14]. But in comparison with the deviations from the linear graph of $\log N(M)$ or $\log N(K)$ in the range of relatively large or large magnitudes the deviations in the domain of small magnitudes or energies represent a more difficult problem. Here the difficulty consists in the problem if these deviations are real or only appear to be a consequence of an insufficiently correct estimation of the representativeness of earthquake frequencies (of a given M or class K) with small values of magnitude or energy. In any case, according to [4] the use of nonrepresentative classes (due to low magnification of seismometers) in plotting the frequency-magnitude graph has a strong influence, and considerably falsified results may be obtained by extrapolating the left-hand side of the G.-R. graph. - The same problem has been taken into consideration for the statistical interpretation of BÅTH's law [16, 24].

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But it was established [2, 19] that in some seismic areas the observed frequencies of earthquakes in the range of small M and K are smaller than those obtained by left-hand extrapolation of the linear frequency graph. It must be mentioned the result obtained in [2] that the general linear form of the graph of frequency function of a vast region is a consequence of the superposition (summation) of more curviliniar graphs of the individual parts of the region, taking into consideration all depth intervals (for small zones and different depth classes as well as for large zones the $\log N(K) = f(K)$ has a curviliniar form [2,3]). In the same paper [2] a detailed analysis of the above conclusion is given, especially on the basis of breakage processes for which the lognormal distribution is used [6].

In [6] it has been demonstrated that in breakage processes the size of particles in crushed rocks obeys to a lognormal distribution. Basing on this result and on stringent physical relations between rockbursts and earthquakes, in [9] the first suggestion is given as to that the frequency-magnitude law has to be interpreted as a lognormal distribution rather than a negative exponential distribution. This is mentioned also in [24]. Later the same question was treated in [5, 12]. It was considered that this distribution better approximates the nonlinear form of the graph of frequency-magnitude and especially the left-hand side in the domain of small values of magnitude or energy.

The problem of the lognormal distribution of earthquakes is treated again in [10], and in [12] a detailed and critical analysis of the problem of linear adjustment of the graph of frequency-energy relation has been performed. In the same paper the generality of the lognormal distribution is shown, and it has been verified that for great energy values the straight line is a sufficient approximation.

In the papers [17, 18, 21, 22] the lognormal distribution of magnitude is applied to the study of frequencies of Vrancea intermediate earthquakes. It resulted that the lognormal distribution of M is better than the negative exponential one, and the calculated frequencies with lognormal law are in very good agreement with the observed data. (For the Vrancea case it is a real fact [9] that the frequencies of small classes of M are smaller than those obtained by left-hand extrapolation of the linear graph of the G.-R. relation.) A further step has been made in [21], where an estimation of the coefficients of the frequency-magnitude relation for the lognormal distribution without using the least square method was given.

In this paper we offer a new method for estimating these coefficients, more simple than in [21] (in the quoted paper the method of moments has been used), and moreover, we shall determine the confidence intervals of these parameters at a given confidence level.

3. Estimation of coefficients of the magnitude-frequency relation

Let a pattern of n earthquakes have been occurred in a given space and time. The random variable is the magnitude M. M_i $(i = 1, 2, \ldots, n)$ are the values of this variable in the instance. In addition, let $N(M_i)$ be the observed frequencies of earthquakes with magnitude M_i and $p(M_i) = N(M_i)/n$ be the statistical probability of M_i. According to the above analysis the variable magnitude can follow

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different distribution laws with corresponding magnitude-frequency relations (a synthesis is given in [15]). We assume that the error in magnitude determination is negligible, the earthquakes being independent of each other with respect to their magnitudes and the magnitude following a lognormal distribution.

With these assumptions it results that the log magnitude is normally distributed and, if $P(\ln M)$ is the probability density, we have [1]:

(1) $$P(\ln M)\, d(\ln M) = \frac{d(\ln M)}{\sigma_{\ln} \sqrt{2\pi}} \exp\left[-(\ln M - \ln \gamma)^2/(2\sigma_{\ln}^2)\right],$$

$$-\infty < \ln M < \infty .$$

From (1) it results that the probability density of magnitude, $P(M)$, and corresponding distribution function, $F(M)$, are of the lognormal distribution, and we can write

(2) $$F(M) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t_s} \exp\left(-\frac{t^2}{2}\right) dt, \quad t_s = (\ln M - \ln \gamma)/\sigma_{\ln}$$

and

(3) $$P(M)\, dM = \frac{dM}{M\, \sigma_{\ln} \sqrt{2\pi}} \exp\left[-(\ln M - \ln \gamma)^2/(2\sigma_{\ln}^2)\right] \quad M > 0 .$$

Because of (1) and (3) it follows that

(3a) $$\text{Prob}\,(\zeta \leqq M \leqq \eta) = \text{Prob}\,(\ln \zeta \leqq \ln M \leqq \ln \eta),$$

and if $n(M)$ is the frequency of earthquakes with given M this is equal to the frequency of given $\ln M$, whence

(4) $$N(\ln M)\, d(\ln M) = N(\ln M) \frac{d(\ln M)}{dM} dM = N(M)\, dM .$$

The density (1) means that

(5) $$E(\ln M) = \ln \gamma, \quad D^2(\ln M) = \sigma_{\ln}^2$$

and the estimates $\widetilde{\ln} \gamma$ and $\tilde{\sigma}_{\ln}^2$ for the mean and variance of log magnitude are given by

(6) $$\widetilde{\ln} \gamma = \frac{1}{n} \sum_{i=1}^{n} \ln M_i ,$$

(7) $$\tilde{\sigma}_{\ln}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (\ln M_i - \widetilde{\ln} \gamma)^2 .$$

According to (3) we have

(6a) $$E(M) = \exp\left(\ln \gamma + \frac{1}{2} \sigma_{\ln}^2\right),$$

(7a) $$D^2(M) = \exp(2 \ln \gamma + \sigma_{\ln}^2)\left[\exp(\sigma_{\ln}^2) - 1\right].$$

Further, it can be observed that

(8) $$P(\ln M) \frac{d(\ln M)}{dM} dM = P(M)\, dM .$$

Taking the logarithm of (1) it results that we can write

DOI: http://doi.org/10.2312/zipe.1972.018

(9) $\ln P(\ln M)\, d(\ln M) = [A_1 + B_1 \ln M - C_1 \ln^2 M]\, d(\ln M)$, $C_1 > 0$,

where

(10) $A_1 = -\ln(\sigma_{\ln} \sqrt{2\pi}) - \ln^2 \gamma/(2\sigma^2_{\ln})$,

(11) $B_1 = \ln \gamma/\sigma^2_{\ln} = \log \gamma \log e/\sigma^2_{\log}$,

(12) $C_1 = 1/(2\sigma^2_{\ln})$.

Because $P(\ln M)$ is a probability density, $N(\ln M)$ a frequency function, and $n P(\ln M)\, d(\ln M) = N(\ln M)\, d(\ln M)$ it results that the magnitude-frequency relation has the form:

(13) $\ln N(\ln M)\, d(\ln M) = A + B \ln M - C \ln^2 M\, d(\ln M)$, $C > 0$,

where

(13a) $A = A_1 + \ln n$, $B = B_1$, $C = C_1$.

This method was partially used in [17]. It must be observed that (9) and (13) are not identical because (9) represents the logarithm of the density function and (13) the logarithm of the absolute frequency N, depending on the total number of earthquakes, n, in the given instance.

Practically, for working up of the earthquake observational data it is more suitable to use the logarithm on base 10. In this case instead of (13) we have

(14) $\log N(\log M)\, d(\log M) = (A_o + B_o \log M - C_o \log^2 M)\, d(\log M)$, $C_o > 0$.

Taking into account (13a) we obtain

(15) $A_o = A \log e = A_1 \log e + \log n$,

(16) $B_o = B = B_1$,

(17) $C_o = C/\log e = C_1/\log e$.

Similar relations can be given for probability density (3) as follows:

(18) $\ln P(M) = a' + b' \ln M - c' \ln^2 M$, $c' > 0$.

Using (10), (11), and (12) we obtain

(19) $a' = A_1$, $b' = B_1 - 1$, $c' = C_1$.

As $\ln N(M) = \ln n + \ln P(M)$, it results

(20) $\ln N(M) = A_1 + \ln n + (B_1 - 1) \ln M - C_1 \ln^2 M$

or

(21) $\log N(M) = A_1 \log e + \log n + (B_1 - 1) \log M - \frac{C_1}{\log e} \log^2 M$

and

(22) $N(M)\, dM = 10^{a_o + b_o \log M - c_o \log^2 M}\, dM$,

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where

(23) $a_o = A_1 \log e + \log n$, $b_o = B_1 - 1$, $c_o = C_1/\log e$.

Finally, we obtain the following relations between the coefficients used above:

(24) $a_o = A_o = A \log e = A_1 \log e + \log n = a' \log e + \log n$,

(25) $b_o = B_o - 1 = B - 1 = B_1 - 1 = b'$,

(26) $c_o = C_o = C/\log e = C_1/\log e = c'/\log e$.

Now the coefficients from (24) - (26) can be given as a function of A_1, B_1, C_1 from (10) - (11), respectively. The results are identical with those given in [17], where A_o, B_o, and C_o are exactly the coefficients a, b, and c from [17] and $A_1 \log e$, b_o, c_o are respectively the a^*, b^*, c^* from the quoted paper.

It may be mentioned here that because the frequency N is likewise a function of M or log M and considering also (4) and (23), it results that the relation $\log N(M) = a + b \log M - c \log^2 M$ given in [17] must be considered with respect to d(log M). This is true because in [17] the estimation of the coefficients of the above relation - by moment method - have been determined with respect to ln M (and log M). At the same time the equation (22) must be considered with respect to dM. The relations (24) - (26) can likewise be obtained by only using $P(\log M) =$ $= [\overline{K}/(\sigma \log \sqrt{2\pi})]$:

(26') $\exp[-(\log M - \log \gamma)^2/(2\gamma^2_{\log})]$

where $\overline{K} = \log e$.

4. Confidence interval formulas for the coefficients of magnitude-frequency relation

In this section we shall give the estimation of the confidence intervals of the coefficients of the magnitude-frequency relation for the distribution used in this paper.

For the lognormal distribution of magnitude the probability density is given by (1) for ln of the magnitude. The magnitude-frequency relation has the form (13) or (14) or (22), which can be obtained by changing the variable,

(27) $\frac{1}{M} P(\ln M)\, dM = P(M)\, dM$.

Because $E(\ln M) = \ln\gamma$, $D^2(\ln M) = \sigma^2_{\ln}$, the estimators $\widetilde{\ln\gamma}$ and $\tilde{\sigma}^2_{\ln}$, according to the above formulas, are given in the expressions

(28) $$\widetilde{\ln\gamma} = \frac{\hat{B}_1}{2\hat{C}_1} = \frac{\hat{B}_o}{2\hat{C}_o \log e} ,$$

(29) $$\tilde{\sigma}^2_{\ln} = \frac{1}{2\hat{C}_1} = \frac{1}{2\hat{C}_o \log e} ,$$

where $\hat{B}_1$, $\hat{C}_1$, $\hat{B}_o$, $\hat{C}_o$ are the estimators of B_1, C_1, B_o, C_o, respectively, which can be determined easily by the help of (6) and (7).

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If $I_\beta(\ln \gamma)$ is the confidence interval at the confidence level (probability) β, we have [25]:

$$(30)\quad \text{Prob}\,(\tilde{\ln}\,\gamma - t_\beta(n-1)\frac{\tilde{\sigma}_{\ln}}{\sqrt{n}} \leqq \ln\gamma \leqq \tilde{\ln}\,\gamma + t_\beta(n-1)\frac{\tilde{\sigma}_{\ln}}{\sqrt{n}}) = \beta\ ,$$

where the parameter $t_\beta(n-1)$ has the STUDENT distribution (given in special tables) as a function of β with $n-1$ degrees of freedom. Thus, it can be obtained easily that the confidence limits for E_o satisfy

$$(31)\quad \hat{B}_o - t_\beta(n-1)\sqrt{\frac{2\,\hat{C}_o \log e}{n}} \leqq B_o \leqq \hat{B}_o + t_\beta(n-1)\sqrt{\frac{2\,\hat{C}_o \log e}{n}}\ ,$$

which gives the interval $I_\beta(B_o)$. In (31) the parameter $\hat{C}_o$ is fixed and given by (29).

To calculate the confidence interval of the coefficient C_o at the confidence level β we use the fact that the random variable $(n-1)\ \tilde{\sigma}^2_{\ln}$ has a χ^2 distribution with $n-1$ degrees of freedom. Therefore,

$$(32)\quad \text{Prob}(\chi^2_{1-\alpha/2}\,(n-1) < \frac{(n-1)\ \tilde{\sigma}^2_{\ln}}{\bar{\sigma}^2_{\ln}} < \chi^2_{\alpha/2}\,(n-1)) = \beta\ ,$$

where $\chi_{\alpha/2}\,(n-1)$ means the χ^2 distribution with $n-1$ degrees of freedom and $\alpha/2 = 1 - (\beta/2)$.

From (32) and using (29), the confidence limits for C_o satisfy the condition

$$(33)\quad \frac{\hat{C}_o\ \chi^2_{1-\alpha/2}\,(n-1)}{n-1} \leqq C_o \leqq \frac{\hat{C}_o\ \chi^2_{\alpha/2}\,(n-1)}{n-1}\ ,$$

and therefore we have immediately the confidence interval $I_\beta(C_o)$ for a given probability $\beta = 1-\alpha$ (for given β and n the values of $\chi^2_{\alpha/2,1-\alpha/2}\,(n-1)$ exist in special tables).

5. Conclusions

The above analysis results in the following conclusions:

1. In different seismic regions variant magnitude (or energy) distributions besides the negative exponential distribution are existing, hence the G.-R. relation cannot be accepted in all cases.
2. To give an extrapolation in the range of great values and expecially small values of M, for earthquake prediction firstly the magnitude distribution type must be known.
3. The determination of magnitude distribution type is very strongly conditioned by the representativeness of the magnitude (or energy) classes.
4. By a method distinguished from that given in a previous paper estimations of the coefficients of the magnitude-frequency relation for the lognormal distribution of magnitudes are presented.
5. The confidence intervals of the coefficients are also given.

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Refined Determination of the Parameters of the Lognormal Energy-Frequency Law

By

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Summary

Till now, some methods are known for calculating the parameters of a lognormal energy-frequency law of earthquakes. But they are not even suitable because they effect an unwanted weighting of the observation. Therefore, another method of adjustment is described. Starting from a first approximation, the parameters of the lognormal distribution are calculated as exactly as wanted by an iterative process, using the rules of trial and error and the method of least squares.

1. Introduction

Let us consider a manifold $\mathfrak{E}_0$ of n earthquakes observed inside the region of investigation, Γ, during the period ϑ. Γ may be a simply connected part of the earth. The seismic energy released by the i-th earthquake of $\mathfrak{E}_0$ is denoted by $E_i > 0$. We suppose that the earthquakes considered are not interdependent. For that reason the energy E_i of any possible earthquake is treated like a random variable. It belongs to the energy interval $(E - 0.5\,dE,\ E + 0.5\,dE)$ with the probability $w(E)\,dE$,

$$(1)\quad w(E) \geqq 0\ , \quad \int_0^\infty w(E)\,dE = 1\ .$$

In recent years the probability density $w(E)$ arose interest with respect to the physical processes in seismically active regions and the seismic risk as well. In this connection the exact determination of a suitable function $w(E)$ is very important. There exist a lot of different approaches for $w(E)$. One of them is

$$(2)\quad w(E) = \frac{1}{\sqrt{2\pi}\,\sigma\,E}\exp\left[-\frac{(\ln E - \ln E^*)^2}{2\,\sigma^2}\right]\ ,$$

introduced first by NEUNHÖFER [1] with regard to theoretical considerations. Expression (2) represents a logarithmic normal distribution over E. The adequate one over $\ln E$ is

$$(3)\quad \widetilde{w}(\ln E) = \frac{1}{\sqrt{2\,\pi}\,\sigma}\exp\left[-\frac{(\ln E - \ln E^*)^2}{2\,\sigma^2}\right]\ .$$

It represents a normal distribution over $\ln E$ with the standard deviation σ and the mean $\ln E^*$. (2) and (3) are connected by $w(E)\,dE = \widetilde{w}(\ln E)\,d(\ln E)$.

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σ and E^* or $\ln E^*$, which characterize $w(E)$ and $\tilde{w}(\ln E)$, can be estimated from the set $\mathfrak{E}_o$ by means of

(4) $\ln E^* = \frac{1}{n} \sum_{i=1}^{n} \ln E_i$, $\sigma^2 = \frac{1}{n-1} \sum_{i=1}^{n} (\ln E_i - \ln E^*)^2$,

if $\mathfrak{E}$ is representative for the manifold of all earthquakes in Γ. But in practice there are many weak earthquakes which cannot be observed entirely. Therefrom follows that the calculated $\ln E^*$ and σ become too great or too small, respectively. Therefore, it seems to be reasonable to compute the wanted parameters by an adjustment, for instance by the least squares method, using only such energy classes the events of which are observed entirely. We suggest an iterative procedure which is suitable for the use of digital computers.

2. Determination of the parameters

First of all, a reasonable first approximation is needed. It can be yielded by the formulas (4). But it seems to be more favourable to use the following considerations:

Let $\bar{\mathfrak{E}}$ be the set of all earthquakes which occurred in Γ during ϑ, $\mathfrak{E}_o \leqq \bar{\mathfrak{E}}$, and be $\bar{n} \geqq n$ the number of events of $\bar{\mathfrak{E}}$. The set may be representative for the manifold $\mathfrak{E}$. Consequently, the mathematical statement for the frequency distribution, $\bar{N}(\ln E)$, is proportional by the factor $\bar{n}$ to the probability density, $\tilde{w}(\ln E)$:

(5) $\bar{N}(\ln E) = \bar{n}\, \tilde{w}(\ln E)$.

Of course, analogous relations are valid for $w(E)$, too.

From (3) there follows

(6) $\ln \tilde{w}(\ln E) = \alpha_1 + \beta_1 \ln E - \gamma_1(\ln E)^2$,

where

(7) $$\begin{cases} \alpha_1 = -\ln \sigma \sqrt{2\pi} - (\ln E^*)^2/2\sigma^2 , \\ \beta_1 = (\ln E^*)/\sigma^2 , \\ \gamma_1 = 1/2\sigma^2 . \end{cases}$$

Because of (5) we obtain

(8) $\ln \bar{N}(\ln E) = A_1 + B_1 \ln E - C_1 (\ln E)^2$,

where

(9) $A_1 = \alpha_1 + \ln \bar{n}$, $B_1 = \beta_1$, $C_1 = \gamma_1 > 0$.

The coefficients A_1, B_1, and C_1 of equation (8) can be determined by the least squares method. It is necessary to classify the events of $\mathfrak{E}_o$ and than the logarithm of the frequencies of the different classes must be computed. But only such

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classes can be taken into account the events of which are expected to be observed entirely. Otherwise the subset inside a class is not representative. Therefore, the set $\underline{\mathfrak{E}}$ of earthquakes used to determine the A_1, B_1, and C_1 is a subset of $\mathfrak{E}_o$, i. e. $\underline{\mathfrak{E}} \subset \mathfrak{E}_o \subset \overline{\mathfrak{E}}$.

From (7) and (9) it follows

$$(10) \quad \sigma = (2\, C_1)^{-1/2} , \quad \ln E^* = B_1\, (2\, C_1)^{-1} ,$$

$$(11) \quad \ln \overline{n} = A_1 = \frac{1}{2} B_1^2 + \frac{1}{2} \ln(\pi/C_1) .$$

Such a fitting of $\overline{N}(\ln E)$ to the empirical data $\underline{\mathfrak{E}}$ is not a correct one because there are used the logarithms of the frequencies N_j of a class j and not the frequencies themselves. That means a not wanted weighting of the N_j . We consider the $\overline{n}$, σ, and $\ln E^*$ so computed as the wanted approximations $x^{(o)}$, $y^{(o)}$, and $z^{(o)}$ of the unknown parameters x, y, z.

The question is how to construct in a simple manner a sequence $x^{(m)}$, $y^{(m)}$, $z^{(m)}$ $(m = 0, 1, 2, \ldots)$ converging to a triplet x, y, z which minimizes

$$(12) \quad \sum_j [\overline{N}(\ln E_j , \zeta, \eta, \xi) - N_j]^2 \equiv S(\xi, \eta, \zeta)$$

so that

$$(13) \quad \lim_{m\to\infty} S(x^{(m)}, y^{(m)}, z^{(m)}) = S(x, y, z) = \text{Min.}$$

is valid.

Constructing the point sequence $x^{(m)}$, $y^{(m)}$, $z^{(m)}$ the assumption is made that all points and also the final one x, y, z are situated in a surrounding of the first approximation $x^{(o)}$, $y^{(o)}$, $z^{(o)}$. Suitably the surrounding of the starting point is bounded by planes which are perpendicular to the axis of the Cartesian coordinate system. The length of the edges of the rectangular parallelepiped must be adapted to the values $\overline{n}$, σ, and E^* as well as to their inaccuracies. For constructing the sequence mentioned above, it is recommendable to compute (12) for the net points defined by the three families of coordinate planes

$$(14) \quad \begin{cases} \xi = x_{h_x}^{(m)} & \text{with} \quad x_{h_x}^{(m)} = q^{\pm h_x/H_x^m}\, x^{(m)} \quad \text{and} \quad h_x = 0, 1, \cdots, H_x , \\ \eta = y_{h_y}^{(m)} & \text{with} \quad y_{h_y}^{(m)} = q^{\pm h_y/H_y^m}\, y^{(m)} \quad \text{and} \quad h_y = 0, 1, \cdots, H_y ; \\ \zeta = z_{h_z}^{(m)} & \text{with} \quad z_{h_z}^{(m)} = q^{\pm h_z/H_z^m}\, z^{(m)} \quad \text{and} \quad h_z = 0, 1, \cdots, H_z . \end{cases}$$

For each m the outer planes bound a rectangular parallelepiped around $x^{(m)}$, $y^{(m)}$, $z^{(m)}$. Any family of planes refines the former one. There must be given the values

$$(15) \quad \xi = q_x^{\pm H_x} \equiv A_x^{\pm 1} , \qquad \eta = q_y^{\pm H_y} \equiv A_y^{\pm 1} , \qquad \zeta = q_z^{\pm H_z} \equiv A_z^{\pm 1}$$

obtained from (14) for $m = 0$ and $h_x = \pm H_x$ by which the boundaries of the first

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surrounding are characterized. Then the equations (15) have to be comprehended as conditions for the choice of q_x, H_x, etc. It is necessary to use a sufficiently fine grid for being sure that the procedure converges to the absolute minimum and not to another possibly existing relative one.

3. Accuracy

In practice the procedure must be finite, hence $m \leqq M$. After $m + 1$ steps the point x', y', z' of the grid around $x^{(m)}$, $y^{(m)}$, $z^{(m)}$ may show the minimum $S(x', y', z')$ of all computed values $S(\xi, \eta, \zeta)$. This point x', y', z' represents the point $x^{(m+1)}$, $y^{(m+1)}$, $z^{(m+1)}$ of the wanted sequence if and only if it is situated inside the parallelepiped. But if x', y', z' is situated on the boundary, the family of planes must be supplemented by additional outer planes. The computations must be done so often until a possibly new triplet x', y', z' is an inner point.

Finally a triplet $x^{(M+1)}$, $y^{(M+1)}$, $z^{(M+1)}$ characterized by special values h'_x, h'_y, and h'_z approximates x, y, z in such a way that all further triplets of the sequence, including its limes, are placed in the surrounding of $x^{(M+1)}$, $y^{(M+1)}$, $z^{(M+1)}$ defined above. Therefore, it holds concerning x

$$(16)\quad |x - x^{(M+1)}| < x^{(M)}\, q_x^{(h'_x+1)/H_x^M} - x^{(M)}\, q_x^{h'_x/H_x^M} ,$$

hence

$$(17)\quad \frac{|x - x^{(M+1)}|}{x} \leqq \frac{x^{(M)}}{x}\, q_x^{h'_x/H_x^M} \left(q_x^{1/H_x^M} - 1\right) = \frac{x^{(M+1)}}{x} \left(q_x^{1/H_x^M} - 1\right)$$

or approximately

$$(18)\quad \frac{|x - x^{(M+1)}|}{x} \leqq Q_x - 1$$

with

$$(19)\quad Q_x = q_x^{1/H_x^M} .$$

Consequently, $Q_x - 1$ represents the bound of a relative accuracy which must be fixed, e. g. to 10^{-2}, that means $Q_x = 1.01$. Because of (15) equation (18) yields the condition

$$(20)\quad H_x^{M+1} = \frac{\lg A_x}{\lg Q_x} .$$

Similar relations hold for y and z. A special example is:

$$A_x = 10 , \qquad Q_x = 1.01 , \qquad H_x = 4 \longrightarrow M = 3 , \quad q_x = \sqrt[4]{10} .$$

With each of the $M + 1$ steps at least $(2\,H_x + 1)(2\,H_y + 1)(2\,H_z + 1)$ values $S(\xi, \eta, \zeta)$ are computed. So it is possible to estimate the computational expense. In the realistic case

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$$A_x = A_y = A_z = 10 \;, \quad Q_x = Q_y = Q_z = 1.01 \;, \quad H_x = H_y = H_z = 4$$

at least $4 \times 9^3 = 2916$ computations of (12) are necessary. The postulation $Q_x = Q_y = Q_z = 1.001$ requires two further steps, that implies 50 p. c. additional expense.

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Projective Seismicity and Focal Volume

By

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Summary

The projective seismicity of a single earthquake results from the local seismicity of the concerned quake. For the focal region a spherical and a cylindrical model are discussed. The volume of the focal region is identified with the focal volume. For calculation of the radius of both the sphere and the cylinder a common formula based on the BÅTH and DUDA relation between the focal volume and the magnitude of an earthquake is used.

Studying the distribution of earthquakes with respect to space and time requires, besides the systematic collection of the seismic data, a theoretical conception for working up these data. Such a concept is up to now a matter of scientific research and has to be adequate to the special aim of the study of distribution of the earthquakes in space and time. With respect to tectonical processes leading to the release of earthquakes it seems to be essential to have regard on the deformation energy of single quakes for the quantitative description of their space-time distribution. These energy values can be estimated only from that energy amount which is radiated by seismic waves.

The tectonical aspect of earthquake activity as well as the physical aspect led the authors to start from, and to carry on, the old idea to comprehend seismicity of a region and a time interval ϑ as a sum of all values E_i $(i = 1, 2, \ldots)$ of concerned earthquakes related to the volume or area of the investigated region and to the duration $|\vartheta|$ of the considered time interval [4]. The suitable generalization of this definition yields seismicity $S[Z,t]$ as a continuous function of space point Z and time point t by superposition

$$(1) \quad S[Z, t] = \sum_i s_i[Z, t]$$

of the contributions $s_i[Z,t]$ belonging to the single earthquakes marked by their number, i. The function $s_i[Z,t]$ has to be comprehended as the distribution of energy E_i in space and time. Therefore, we put suitably

$$(2) \quad s_i[Z, t] = E_i \, p_i(t) \, q_i(Z) , \quad \int_{\sigma_i} p_i(t) \, dt = \int_{\omega_i} q_i(Z) \, dv = 1 ,$$

where σ_i and ω_i mean the time interval and the focal region, respectively,

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DOI: http://doi.org/10.2312/zipe.1972.018

yielding an effective contribution of that quake to the seismicity value in Z and t. It is clear that dv and dt denote the time element and volume element in Z,t.

Furtheron, seismicity is regarded not as an explicit function of time t but of a time interval ϑ which may be large enough, say some decades. The seismicity funtion $S[Z, \vartheta]$ at point Z for the time interval ϑ is expressed by

$$(3) \quad S[Z, \vartheta] = \frac{1}{|\vartheta|} \int_{\vartheta} S[Z, t]\, dt = \frac{1}{|\vartheta|} \sum_i [E_i \int_{\vartheta} p_i(t)\, dt]\, q_i(Z) .$$

The integral

$$(4) \quad \int_{\vartheta} p_i(t)\, dt = \int_{\vartheta \cap \sigma_i} p_i(t)\, dt = (P_{\vartheta})_i \leqq 1$$

gets the value 1 if the corresponding time range σ_i of the i-th quake is totally part of the studied time interval ϑ. To put $p_i(t)$ as a DIRAC function with its maximum at the i-th focal time t_i is mathematically very simple but physically rather irrelevant. Nevertheless, if we do so, all earthquakes inside of ϑ contribute with $(P_{\vartheta})_i = 1$ and all quakes outside of ϑ with $(P_{\vartheta})_i = 0$; so they are of no interest. That is the mathematical expression for the present situation of studying seismicity of a time interval.

It is not convenient to represent seismicity of a space. Therefore, it is usual to give a two-dimensional representation of space seismicity maps drawing lines of equal level. Then it is necessary to collect the concerned information along the vertical straight line coinciding with the arbitrary point Z' at the surface of the Earth. That means, we have to give up information about the variability of the seismicity depending on depth z. This is obtained certainly in the best manner by defining seismicity $S'[Z', \vartheta]$ of surface point Z' as the integral mean

$$(5) \quad S'[Z', \vartheta] = \frac{1}{|\varepsilon|} \int_{\varepsilon} S[Z, \vartheta]\, dv$$

with respect to the infinitesimal projection cone ε. This field function is called projective seismicity. It is again a superposition of certain contributions $s_i'[Z, \vartheta]$

$$(6) \quad S'[Z', \vartheta] = \sum_i s_i'[Z, \vartheta] ,$$

where

$$(7) \quad s'[Z', \vartheta] = \frac{3\, E\, P_{\vartheta}}{R\, |\vartheta|} \int_{\zeta} q(Z)\, dz , \quad Z \in \zeta , \quad Z' \in \zeta .$$

(Furtheron the index i is omitted.) Here R means the radius of the spherical Earth. The function $s'[Z', \vartheta]$ represents like former heuristic functions a two-dimensional distribution of the individual energy E without any fixation of the seismicity distribution with depth. Especially this expression does not contain the focal depth which is a problematical parameter in former statements.

The procedure to define seismicity demonstrated here has the advantage to reduce the „projective seismicity" to the individual physically motivated function q(Z). But now there is the problem to find a suitable function q(Z). - The most obvious statement for q(Z) is

$$(8) \quad q(Z) = f(x) , \quad x = \overline{QZ} , \quad f(x) = 0 \quad \text{for} \quad x \geqq r ,$$

DOI: http://doi.org/10.2312/zipe.1972.018

where q(Z) depends only on the distance, x, of point Z from the hypocentre. The focal region ω is limited in this case to the interior of the sphere $x < r$. The most simple analytical statement for f(x) is a polynomial of at least third degree. For example, the expression

$$(9) \quad f(x) = \frac{15}{4 \pi r^2} [1 - 3 (\frac{x}{r})^2 + 2 (\frac{x}{r})^3] \quad \text{for} \quad 0 \leqq x \leqq r$$

realizes the obvious requirements

$$(10) \quad f(0) \geqq f(x) > f(r) = 0 , \quad f'(0) = f'(r) = 0$$

and (2), that means the condition of normalization

$$(11) \quad \int_\omega q(Z)\, dv = 1 .$$

Now the expression (9) for q(z) helps by integration along the vertical straight line ζ to the formula

$$(12) \quad s_I'[Z', \vartheta] = \frac{45 E P_\vartheta}{8 \pi R r^2 |\vartheta|} [-3 v' + 5 v'^3 + \frac{3}{2} (1 - v'^2)^2 \ln \frac{1 + v'}{1 - v'}] ,$$

where

$$(12a) \quad v' = |\sqrt{1 - (\frac{x'}{r})^2}| \quad \text{with} \quad x' = \overline{Q'Z'} .$$

The function fulfils again essential requirements listed above, including the condition that the integral taken over the projection ω' of ω yields the amount E. Because formula (12) seems to be somewhat complicated to compute seismicity (5), in practice a second statement should be tried.

Representing approximately the real focal region ω by a vertical circular cylinder with centre Q, the hypocentre of the individual earthquake, radius a and vertical dimension l, it is obvious to put $q(Z) = f_1(x')\, f_2(Z)$. This statement separates a priori the horizontal and the vertical variations of q(Z), and the integration in formula (7) along the vertical straight line ζ yields a value independing of distance x' being the epicentral distance. In analogy to former requirements we have the conditions

$$(13) \quad f_1(0) \geqq f_1(x') > f(a) = 0 , \quad f_1'(0) = f_1'(a) = 0 ,$$

$$(14) \quad f_2(0) \geqq f_2(|z - h|) > f_2(\frac{l}{2}) = 0 , \quad f_2'(0) = f_2'(\frac{l}{2}) = 0 ,$$

and that of normalization. They can be realized by the polynomials

$$(15) \quad f_1(x') = \frac{10}{3 \pi a^2} [1 - 3 (\frac{x'}{a})^2 + 2 (\frac{x'}{a})^3] ,$$

$$(16) \quad f_2(|z - h|) = \frac{2}{l} [1 - 3 (\frac{2|z - h|}{l})^2 + 2 (\frac{2|z - h|}{l})^3] ,$$

so that we get the contribution

$$(17) \quad s_{II}'[Z', \vartheta] = \frac{3 E P_\vartheta}{R |\vartheta|} f_1(x') .$$

Besides, we have been led to this cylindrical model of ω by discussions with Prof. GORSHKOV and Dr. SHENKAREVA at the Lomonossov University.

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To compare the results one should put $r = a$ so that the horizontal dimensions of the two focal models are equal. The values $s'_I[Q', \vartheta]$ and $s'_{II}[Q', \vartheta]$ at the epicentre Q' are related by

(18) $s'_I[Q', \vartheta] : s'_{II}[Q', \vartheta] = 9 : 8$.

Thence $s'_I[Z', \vartheta]$ is more concentrated near the epicentre than $s'_{II}[Z', \vartheta]$ and diminishes more rapidly. From the seismologically obvious conditions $s_I[Q, \vartheta] = s_{II}[Q, \vartheta]$ and $r = a$ results

(19) $l = \frac{8}{9} 2 r$

as the length of the cylinder.

The open question on the dimension of the focal region ω reduces now after all these considerations to the determination of r, that means to the seismological estimation of ω where the radiated seismic energy was concentrated before the quake.

With the study of aftershock series of strong earthquakes of magnitude M BÅTH and DUDA stated statistically that the hypocentres of a complete series determine a volume V according to

(20) $\lg V = -5.42 \pm 0.51 + (1.47 \pm 0.14) M , \quad 5.3 \leqq M \leqq 8.7$

(cf. [1]). It has to be supposed that a great part of the energy of the main shock is concentrated in the so defined focal volume. But the amount of energy of the main shock outside that volume may be comparable with the first energy part because the main shock lies usually at the border of this region. Therefore, in the sense of a heuristical procedure it is allowed to identify the focal volume V of the earthquake series after BÅTH and DUDA with the volume $|\omega|$ of the focal region discussed above, so we have to put in the case of the spherical model

(21) $V = |\omega_I| = \frac{4}{3} \pi r^3$,

whence results

(22) $\lg r = -2.01 \pm 0.27 + (0.49 \pm 0.05) M$,

or to put

(23) $V = |\omega_{II}| = \frac{16}{9} \pi r^3$

in the case of the cylindrical model, leading to the equation

(24) $\lg r = -2.06 \pm 0.27 + (0.49 \pm 0.05) M$.

Despite of the discrepancy that the ratio of the focal volumina equals $4 : 3$ the two relations for r do not differ seriously having in mind the value of the variance. Therefore, a mean equation to compute r should be used in general.

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The Projective Seismicity of an Earthquake with Regard to the Probability Distribution of its Epicentre

By

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Summary

For the analytical representation of the projective seismicity of an earthquake the inaccuracy of the epicentral coordinates is taken into account by means of a heuristical probability density distribution of the epicentre. The product of that function with the projective seismicity of the concerned quake is the integrand of a surface integral which defines the probable projective seismicity at any point on the Earth's surface for the considered period. The intersection of the projection of the focal region to the Earth's surface with a region belonging to that point, determined by the definition area of the probability density of the epicentre, represents the integration region. Practically, the integration proves to be very complicated. But in the case of a probability function being independent of the direction of the epicentre dislocation and defined at the whole Earth's surface an explicit analytical formula can be obtained if the model of the focal region and the probability function are assumed to be mathematically simple.

For determination of seismicity the localization of earthquakes is necessary. The co-ordinates of a punctual seismic source, however, can never be computet exactly. Therefore, the principally inevitable inaccuracies of the focal data should be expressed by means of suitable probability distributions of the hypocentres also in the analytical representation of seismicity. For the ascertainment of the focal coordinates using instrumental data recently practicable procedures have been offered [1, 2], which take into account systematic errors due to irregularities of the underground structure and insufficient seismogram analysis. Such near-field methods will prove very appropriate for investigation of seismicity of a seismically active region.

All seismicity functions hitherto existing do not contain any information on the probability distributions of hypocentres. This holds also for the projective seismicity of a single earthquake, ε, at a point Z' of the Earth's surface, $\overline{\Gamma}^*$, and in time interval, ϑ, which is of interest in the following investigation. It is represented by

$$(1)\quad s'[Z', \vartheta] = \frac{3\,E\,P_\vartheta}{R\,|\vartheta|} \int_\zeta q(Z)\,dy$$

[5]. In this formula E means the seismic energy of ε, $|\vartheta|$ the length of ϑ, P_ϑ a temporal valuation of ε with property $0 < P_\vartheta \leq 1$ (cf.[3]), R the radius

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of the spherical earth, ζ the intersection of the radius through $Z' \in \overline{\Gamma}^*$ and the focal region ω of the earthquake ε, Z an arbitrary point of ζ, dy the linear element of ζ at $Z \in \zeta$, and $q(Z)$ the spatial distribution of E in the Earth's interior. Per definitionem of ω the point function $q(Z)$ equals zero then, and only then, if Z lies outside of ω. The expression for $s'[Z', \vartheta]$ does not contain explicitly the focal depth h of ε, hence here the problem of the localization of the earthquake ε is reduced to the ascertainment of the epicentre, Q'.

A simple, strongly idealizing condition for ω and $q(Z)$ consists in the fact that the surface of ω is an upright circular cylinder with radius a and altitude l, whose axis passes through Q' and the hypocentre Q, and that the distribution density of E can be represented by the product

$$(2)\quad q(Z) = f_1(x')\, f_2(y) ,$$

where

$$(3)\quad x' = \overline{Q' \cdot Z'} , \quad 0 \leqq x' \leqq a$$

and

$$(4)\quad y^2 = \overline{Q\, Z}^2 - x'^2 , \quad -\frac{l}{2} \leqq y \leqq \frac{l}{2} .$$

It is obvious that Q is simultaneously the centre of the cylinder. Then because of

$$(5)\quad \int_\zeta f_2(y)\, dy = 1$$

for the seismicity $s'[Z', \vartheta]$ the expression

$$(6)\quad s'[Z', \vartheta] = \frac{3\, E\, P_\vartheta}{R\, |\vartheta|} f_1(x')$$

is obtained.

The mathematically most simple representation of $f_1(x')$ which satisfies all physically plausible conditions is

$$(7)\quad f_1(x') \doteq \frac{10}{3\, \pi\, a^2} \left[1 - 3\left(\frac{x'}{a}\right)^2 + 2\left(\frac{x'}{a}\right)^3\right]$$

(cf. [3]). Instead of this relation the function

$$(8)\quad f_1(x') = \frac{3}{\pi\, a^2} \left[1 - \left(\frac{x'}{a}\right)^2\right]^2 \quad (0 \leqq x' \leqq a)$$

with the properties

$$(9)\quad f_1(0) \geqq f_1(x') \geqq f_1(a) = 0 ,$$

$$(10)\quad \frac{df_1(0)}{dx'} = \frac{df_1(a)}{dx'} = 0 ,$$

and

$$(11)\quad \int_0^a x'\, f_1(x')\, dx' = \frac{1}{2\, \pi} ,$$

which also characterize the former expression in (7), will be used. The reason for

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this modification of $f_1(x')$ will not be clear before the end of further considerations.

Because of the principally unavoidable uncertainties of the localization of the earthquake ε it is not sure that the fixed point $Q' \in \overline{\Gamma}^*$ represents the epicentre of ε. As the locus of the real epicentre, however, certainly a point Q'' of the region $\mathfrak{G}(Q') \subseteq \overline{\Gamma}^*$ comes into question. The probability that the real epicentre is situated in the surface element df of $\mathfrak{G}(Q')$ at point Q'' is expressed by $W(\overrightarrow{Q'\ Q''})\, df$, where the vector function $W(\overrightarrow{Q'Q''})$ defined in $\mathfrak{G}(Q')$ describes the probability density of the position of the epicentre of ε, hence

$$(12) \quad \int_{\mathfrak{G}(Q')} W(\overrightarrow{Q'\ Q''})\, df = 1$$

holds and with regard to the special meaning of Q'

$$(13) \quad W(\vec{0}) > W(\overrightarrow{Q'\ Q''}) > 0 \quad \text{for} \quad Q'' \neq Q' .$$

At the border of $\mathfrak{G}(Q')$ the vector function $W(\overrightarrow{Q'\ Q''})$ vanishes.

The projection of the focal region on the Earth's surface is denoted by ω'. The probable dislocation of the epicentre from Q' to Q'' effects the virtual displacement of ω' into the congruent region $\omega'' \subset \overline{\Gamma}^*$. It may be assumed that ω'' is produced from ω' always by the translation indicated by the vector $\overrightarrow{Q'\ Q''}$. In that way the arbitrary point $Z' \in \omega'$ is transformed into $Z'' \in \omega''$, and we obtain

$$(14) \quad \overrightarrow{Z'\ Z''} = \overrightarrow{Q'\ Q''} .$$

For each point $Z' \in \omega'$ the set of possible translations $\overrightarrow{Z'\ Z''}$ yields a region $\mathfrak{G}(Z')$ parallel and congruent to $\mathfrak{G}(Q')$. The union of these regions defines the point set

$$(15) \quad \overline{\omega} = \bigcup_{Z' \in \omega'} \mathfrak{G}(Z')$$

of all the points $Z'' \in \overline{\Gamma}^*$ coming into question.

The probability that the picture of $Z' \in \omega'$ gets into the surface element $df'' \subset \overline{\Gamma}^*$ at point $Z'' \in \mathfrak{G}(Z') \subset \overline{\omega}$ is expressed by $W(\overrightarrow{Z'\ Z''})\, df''$. Because of (14) we have

$$(16) \quad W(\overrightarrow{Z'\ Z''}) = W(\overrightarrow{Q'\ Q''}) .$$

Consequently, for the contribution $ds''[Z'', \vartheta]$ to the probable projective seismicity at the point $Z'' \in \overline{\omega}$ and in the time interval ϑ the displacement $\overrightarrow{Z'\ Z''}$ helps to get the relation

$$(17) \quad ds''[Z'', \vartheta] = s'[Z', \vartheta]\, df'\, W(\overrightarrow{Z'\ Z''}) .$$

Herefrom by integration (at first on the whole region ω')

$$(18) \quad s''[Z'', \vartheta] = \int_{\omega'} s'[Z', \vartheta]\, W(\overrightarrow{Z'\ Z''})\, df'$$

results. To each point $Z'' \in \overline{\omega}$ belongs a region $\mathfrak{G}(Z'') \subseteq \overline{\Gamma}^*$, characterized by

DOI: http://doi.org/10.2312/zipe.1972.018

(19) $Z' \in \mathfrak{G}(Z'') \quad \leftrightarrow \quad W(\overrightarrow{Z'\ Z''}) > 0$.

With it the formula

(20) $s''[Z'', \vartheta] \;=\; \int_{\omega' \cap \mathfrak{G}(Z'')} s'[\overrightarrow{Z'\ Z''})\ df'$

takes the place of (18). This relation is identical with (18) exactly if $\mathfrak{G}(Z'') = \bar{\Gamma}^*$ and therefore $\mathfrak{G}(Q') = \bar{\Gamma}^*$ comes true.

The vector equation

(21) $\overrightarrow{Q'\ Z'} \;=\; \overrightarrow{Q'\ Z''} - \overrightarrow{Q'\ Q''}$

obtained by means of (14) helps to understand the geometry of $\mathfrak{G}(Z'') \subset \bar{\Gamma}^*$. Because Q' and also Z'' in (19) are fixed points the $\overrightarrow{Q'\ Z''}$ in (21) means a constant vector. On the one hand, let all possible vectors $-\overrightarrow{Q'\ Q''}$ with $Q'' \in \mathfrak{G}(Q')$ be carried to Z''. Then (21) describes the set of these points Z' which define the region $\mathfrak{G}(Z'')$. On the other hand, the head of the vector $-\overrightarrow{Q'\ Q''}$ with Q' as its origin and $Q'' \in \mathfrak{G}(Q') \in \bar{\Gamma}^*$ lies in a region $\mathfrak{G}(Q'') \subset \bar{\Gamma}^*$ which obviously results from $\mathfrak{G}(Q')$ by a half rotation in the plane Earth's surface $\bar{\Gamma}^*$ about the fulcrum Q'. Consequently, the region $\mathfrak{G}(Z'')$ follows from a virtual parallel displacement of $\mathfrak{G}(Q'')$ in the plane $\bar{\Gamma}^*$ which transfers Q' to the fixed point Z''. These considerations, notabene, are superfluous in the case $\mathfrak{G}(Q') = \bar{\Gamma}^*$.

The evaluation of the integral in (20) is nearly always very complicated, but in general it can be carried out technically. The analytical problems above all consist in the limitation of the region $\mathfrak{G}(Z'')$ and the structure of the probability density $W(\overrightarrow{Z'\ Z''})$. In order to get a closed expression for the probable projective seismicity $s''[Z'', \vartheta]$ it must be assumed $\mathfrak{G}(Q') = \bar{\Gamma}^*$ and

(22) $W(\overrightarrow{Z'\ Z''}) \;=\; w(\varrho) \quad$ with $\quad \overrightarrow{Z'\ Z''} \;\equiv\; \varrho$.

Besides let be

(23) $\overline{Q'\ Z''} \;\equiv\; x''$, $\quad \sphericalangle\, Z'\ Q'\ Z'' \;\equiv\; \alpha$,

and represent $s'[Z', \vartheta]$ like in (6) by means of the function (8), then because of

(24) $\varrho^2 \;=\; x'^2 + x''^2 - 2\, x'\, x'' \cos\alpha$

and

(25) $df' \;=\; x'\ dx'\ d\alpha$

(20) and (18) respectively become

(26) $s''[Z'', \vartheta] \;=\; \dfrac{9\ E\ P_\vartheta}{\pi\ R\ a^2\ |\vartheta|} \int_0^a x'\ [1 - (\frac{x'}{a})^2]^2 \int_0^{2\pi} w(\sqrt{x'^2 + x''^2 - 2x'x''\cos\alpha})\ d\alpha\ dx'$.

Obviously, the aspired aim can be attained only by use of an extremely simple analytical expression for $w(\varrho)$. Therefore,

(27) $w(\varrho) \;=\; \dfrac{1}{\pi} \dfrac{b^2}{(b^2 + \varrho^2)^2} \quad$ with $\quad b \;=\; const \quad (b > 0)$

DOI: http://doi.org/10.2312/zipe.1972.018

is adopted. This function satisfies the normalization condition (12). The surface over plane $\overline{\Gamma}^*$ geometrically representing the function (27) has the shape of a bell with its absolute maximum $w(0) = \frac{1}{b^2 \pi}$ at the point Z" and the infinitely far edge on $\overline{\Gamma}^*$. The points in which the surface is curved parabolically form a circle parallel to $\overline{\Gamma}^*$ with the radius $\frac{b}{\sqrt{5}}$, because it holds $\frac{d^2 w(\frac{b}{\sqrt{5}})}{d\varrho^2} = 0$.

For seismological interpretation of the parameter b the notion of the standard deviation, σ, of a one-dimensional (direction invariant) distribution of the probable epicentre deviation $\overline{Q'\,Q''} = \varrho$ is helpful. After (27) such a distribution is defined by

$$(28)\quad \overline{w} = C\, w(\varrho)\ ,$$

where the positive constant C has the dimension of a length and because of the normalization condition

$$(29)\quad 2\,C \int_0^\infty \frac{b^2\, d\varrho}{(b^2 + \varrho^2)^2} = 1$$

submits to

$$(30)\quad C = \frac{b}{2}\ .$$

The variance σ^2 follows per definitionem from the equation

$$(31)\quad \sigma^2 = \int_{-\infty}^{+\infty} \varrho^2\, C\, w(\varrho)\, d\varrho = \frac{b^2}{\pi} \int_0^\infty \frac{u^2\, du}{(1 + u^2)^2} = \frac{b^2}{2\pi}\, B(\tfrac{3}{2}, \tfrac{1}{2})$$

(cf. [4]). Regarding the relation

$$(32)\quad B(\xi, \eta) = \frac{\Gamma(\xi)\, \Gamma(\eta)}{\Gamma(\xi + \eta)}$$

between the beta and the gamma function $B(\xi, \eta)$ and $\Gamma(\xi)$ respectively as well as the functional equation $\Gamma(\xi + 1) = \xi\, \Gamma(\xi)$ and, finally, the special value $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$$(33)\quad b^2 = 8\, \sigma^2$$

follows from (31). It is remarkable that $\sigma = b/\sqrt{8}$ does not considerably differ from the radius $b/\sqrt{5}$ of the circle which is the geometrical locus of all points of parabolic curvature of the bell-shaped surface representing the probability density.

With the substitution

$$(34)\quad c \equiv \frac{b^2 + x'^2 + x''^2}{2\, x'\, x''} = \frac{b^2 + (x' - x'')^2}{2\, x'\, x''} + 1 > 1$$

and because of (24) the expression (27) gets the form

$$(35)\quad w(\varrho) = \frac{b^2}{4\, \pi\, x'^2\, x''^2}\, \frac{1}{(c - \cos\alpha)^2}$$

and one obtains

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$$(36)\quad \int_0^{2\pi} w(\sqrt{x'^2 + x''^2 - 2x'\,x''\cos\alpha})\,d\alpha = \frac{b^2}{4\,\pi\,x'^2\,x''^2}\int_0^{2\pi}\frac{d\alpha}{(c-\cos\alpha)^2} =$$

$$= \frac{b^2 c}{2x'^2 x''^2\,(c^2-1)^{3/2}}\,.$$

From (34)

$$(37)\quad c \pm 1 = \frac{b^2 + (x' \pm x'')^2}{2\,x'\,x''}$$

follows so that

$$(38)\quad c^2 - 1 = \frac{1}{4\,x'^2\,x''^2}\,[b^2 + (x' - x'')^2]\,[b^2 + (x' + x'')^2] =$$

$$= \frac{1}{4\,x'^2\,x''^2}\,[x'^4 + 2\,(b^2 - x''^2)\,x'^2 + (b^2 + x''^2)^2]\,.$$

Using (36) and (38) the representation (26) can be transformed into

$$(39)\quad s''[Z'', \vartheta] = \frac{9\,E\,b^2\,P_\vartheta}{\pi\,R\,a^6\,|\vartheta|}\,I\,,$$

where

$$(40)\quad I = \int_0^a \frac{2\,x'\,(x'^2 + b^2 + x''^2)(x'^2 - a^2)^2\,dx'}{[x'^4 + 2\,(b^2 - x''^2)\,x'^2 + (b^2 + x''^2)^2]^{3/2}}\,.$$

For evaluation of this integral the substitutions

$$(41)\quad x'^2 + b^2 - x''^2 = u\,, \qquad a^2 + b^2 - x''^2 = A$$

prove suitable. Hereby (40) is transformed into

$$(42)\quad I = \int_{A-a^2}^{A}\frac{(u + 2x''^2)(u - A)^2\,du}{(u^2 + 4\,b^2\,x''^2)^{3/2}} = \sum_{\varkappa=0}^{3} I_\varkappa$$

with

$$(43)\quad I_0 = 2\,A^2\,x''^2\int_{A-a^2}^{A}\frac{du}{\varphi(u)}\,,$$

$$(44)\quad I_1 = A\,(A - 4\,x''^2)\int_{A-a^2}^{A}\frac{u\,du}{\varphi(u)}\,,$$

$$(45)\quad I_2 = 2\,(x''^2 - A)\int_{A-a^2}^{A}\frac{u^2\,du}{\varphi(u)}\,,$$

$$(46)\quad I_3 = \int_{A-a^2}^{A}\frac{u^3\,du}{\varphi(u)}\,,$$

where

$$(47)\quad \varphi(u) = (u^2 + 4\,b^2\,x''^2)^{3/2}\,.$$

Instead of (39) the explicit formula

$$(48)\quad s''[Z'', \vartheta] = \frac{9\,E\,b^2\,P_\vartheta}{\pi\,R\,a^6\,|\vartheta|} = \sum_{\varkappa=0}^{3} I_\varkappa$$

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is used with the terms

$$(49) \quad I_0 = \frac{1}{2} \left(\frac{A}{b}\right)^2 \left(\frac{A}{B} - \frac{b^2 - x''^2}{b^2 + x''^2}\right) ,$$

$$(50) \quad I_1 = A \, (A - 4 \, x''^2) \left(\frac{1}{b^2 + x''^2} - \frac{1}{B}\right) ,$$

$$(51) \quad I_2 = 2 \, (A - x''^2) \left(\frac{A}{B} - \frac{b^2 - x''^2}{b^2 + x''^2} + \ln \frac{2 \, b^2}{A + B}\right) ,$$

$$(52) \quad I_3 = \frac{A^2 + 8 \, b^2 \, x''^2}{B} - b^2 - x''^2 - \frac{4 \, b^2 \, x''^2}{b^2 + x''^2}$$

and the substitution quantity

$$(53) \quad B = \sqrt{(a + x'')^2 + b^2} \quad \sqrt{(a - x'')^2 + b^2} \, .$$

These extensive calculations in consequence of the underlying simple functions (8) and (27) for analytical representation of the projective seismicity and the probability density $W(\overrightarrow{Q' \, Q''})$ show, on the one hand, the difficulties arising in connection with the determination of $s''[Z'', \vartheta]$ according to (20) and justify, on the other hand, the choice of the function concerned.

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Comparison of the Results of Different Procedures of the Calculating of the Magnitude - Frequency Relation

By

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Summary

It is demonstrated that the values of the parameters in the magnitude-frequency relation of GUTENBERG and RICHTER depend strongly on the method of their determination. Corresponding recommendations are given.

The relation between the number of earthquakes and their magnitude is of primary importance in seismology. This relation is usually approximated by a straight line, exceptionally by two or three straight lines. It is usually written in the form

$$\log N = a - b M ,$$

where N is the number of earthquakes with magnitude $M \pm dM$ and a, b are constants. The values of these constants vary from region to region. They depend, however, also on the length of the period, on the preliminary treatment of material before calculation, on the method of calculation and on application of simple or cumulative frequency. The influence of these quantities may be seen on Table 1, which is calculated for the seismic region in western Greece [2].

In Table 1 the values in the first and second lines correspond to the period 1901 - 1955 and to the extended period 1901 - 1967, respectively. The first and the second parts of the table correspond to the class 0.5 of magnitude unit. The values in the first row are calculated using centres of classes, the values in the second row correspond to the weighted means within each class, and the third row corresponds to the class 0.1 of magnitude unit. The first column contains values a, b calculated by the least squares method. In the second column values a, b are calculated by the generalized least squares method, and in the third column they are determined by the maximum likelihood method. The fourth and fifth columns give values of b calculated by the formulas of PAGE and of UTSU, respectively.

The values in the upper part of the table correspond to a simple frequency of earthquakes, whereas the values in the lower part correspond to a cumulative frequency of earthquakes. By comparing all values listed in the table we see that a and b depend on the procedure of the preparation of data and on the method of calculation. The individual values usually do not lie within the limits of errors of the other values.

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Table 1. Region 10 + 13

	Least squares		Generalized least squares		Maximum likelihood		PAGE	UTSU
	a	b	a	b	a	b	b	b
Centres of	6.03 ± 0.30	0.78 ± 0.05	5.24 ± 0.36	0.62 ± 0.08	5.31	0.64	0.59	0.62
classes	5.71 ± 0.30	0.70 ± 0.05	0.70 ± 0.05	0.56 ± 0.07	5.06	0.57	0.53	0.58
Weighted	6.19 ± 0.29	0.81 ± 0.05	5.49 ± 0.39	0.67 ± 0.08	5.54	0.68	0.66	0.69
mean	5.84 ± 0.26	0.73 ± 0.05	5.25 ± 0.35	0.61 ± 0.07	5.29	0.62	0.61	0.65
Step 0.1	4.78 ± 0.29	0.69 ± 0.05	4.23 ± 0.37	0.56 ± 0.08	4.05	0.53	0.53	0.59
	4.54 ± 0.26	0.62 ± 0.05	4.10 ± 0.33	0.52 ± 0.07	3.90	0.49	0.49	0.55
Cumulative frequency								
Centres of	6.75 ± 0.32	0.88 ± 0.06	5.89 ± 0.28	0.71 ± 0.06	5.95	0.72	0.66	0.68
classes	6.52 ± 0.32	0.81 ± 0.06	5.34 ± 0.23	0.65 ± 0.05	5.89	0.67	0.61	0.64
Step 0.1	6.79 ± 0.14	0.92 ± 0.03	5.92 ± 0.14	0.74 ± 0.03	5.96	0.75	0.75	0.77
	6.69 ± 0.17	0.87 ± 0.03	5.79 ± 0.11	0.69 ± 0.03	5.83	0.70	0.70	0.72

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From Table 1 it follows that we must respect the following rules before we can compare the parameters a and b valid in different regions:

1. All values of a, b must be calculated using either the simple or the cumulative frequency of earthquakes.
2. The original material must correspond to the same period and the same range of M or E and the same magnitude scale, M or m.
3. The parameters have to be calculated by the same method.
4. Identical magnitude classes must be used.

Only if these principles are applied the parameters a and b can be compared for different regions. If they are ignored, differences occur, which have no relation to the process of earthquake generation itself, having a purely formal character. The detailed analysis of this problem is given in the papers [1, 3].

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The Informational Energy and Entropy in Statistics and Prediction of Earthquakes

By

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Summary

The paper introduces new characteristics of earthquakes, the informational energy, ε, and entropy, H. They are given by corresponding formulas for different types of magnitude-frequency relations and models of magnitude distributions of earthquakes.

The estimations of the coefficient b are also obtained, giving its new interpretation in terms of H and ε. The coefficient b in the GUTENBERG-RICHTER relation reflects the informational state of the given earthquake sequence with respect to magnitude and appears as a measure of the degree of disorder in earthquake event systems. The following formulas were derived: $b = K_o \, 10^{-H}$ and $b = K_1 \, \varepsilon$, where K_o and K_1 are constants.

Some interpretations of H and ε are made for earthquake prediction in space, time and size, and it is concluded that the degree of prediction can be estimated only in a probabilistic way. For earthquake analysis the efficiency of H and ε is given by introducing the utility (priority) measure of an event with respect to a given object. Finally, the term of informational earthquake activity is introduced for the construction of corresponding maps as a function of different purposes.

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