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Key Points:

- The Kalman filter is capable of decreasing uncertainties in ring current electron simulations, using Van Allen Probe observations
- The Kalman filter can adjust the model in accordance with observations even in MLT sectors where data are unavailable
- The Kalman filter can improve model predictions for a typical range of ring current electron energies

Supporting Information:

- Supporting Information S1

Correspondence to:

N. A. Aseev,
nikita.aseev@gfz-potsdam.de

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Reanalysis of Ring Current Electron Phase Space Densities Using Van Allen Probe Observations, Convection Model, and Log-Normal Kalman Filter

N. A. Aseev^{1,2} and Y. Y. Shprits^{1,2,3}

¹GFZ German Research Centre for Geosciences, Potsdam, Germany, ²Institute of Physics and Astronomy, University of Potsdam, Potsdam, Germany, ³Department of Earth, Planetary, and Space Sciences, University of California, Los Angeles, CA, USA

Abstract Models of ring current electron dynamics unavoidably contain uncertainties in boundary conditions, electric and magnetic fields, electron scattering rates, and plasmopause location. Model errors can accumulate with time and result in significant deviations of model predictions from observations. Data assimilation offers useful tools which can combine physics-based models and measurements to improve model predictions. In this study, we systematically analyze performance of the Kalman filter applied to a log-transformed convection model of ring current electrons and Van Allen Probe data. We consider long-term dynamics of $\mu = 2.3$ MeV/G and $K = 0.3$ G^{1/2}R_E electrons from 1 February 2013 to 16 June 2013. By using synthetic data, we show that the Kalman filter is capable of correcting errors in model predictions associated with uncertainties in electron lifetimes, boundary conditions, and convection electric fields. We demonstrate that reanalysis retains features which cannot be fully reproduced by the convection model such as storm-time earthward propagation of the electrons down to 2.5 R_E. The Kalman filter can adjust model predictions to satellite measurements even in regions where data are not available. We show that the Kalman filter can adjust model predictions in accordance with observations for $\mu = 0.1, 2.3$, and 9.9 MeV/G and constant $K = 0.3$ G^{1/2}R_E electrons. The results of this study demonstrate that data assimilation can improve performance of ring current models, better quantify model uncertainties, and help deeper understand the physics of the ring current particles.

1. Introduction

The ring current electron population (energies from ~1 to a few 100 keV) is very dynamical and shows strong storm-time variations in magnetic local time (MLT) and radial distance over time scales shorter than 1 hr. Prediction of ring current electron dynamics is of crucial importance for space weather applications, but numerical modeling of this particle population is complicated by uncertainties in electric and magnetic fields, boundary conditions, wave-particle interactions, location of the plasmopause, and magnetopause position. The errors in each component may add up with time and lead to deviation of model results from a true state of the system. An opposite effect may also take place if the errors cancel each other out. In this case, a coincidental agreement between the model and observations for a particular event or for a particular location in space may be misleading.

A lack of detailed MLT-dependent in situ wave and particle observations renders it challenging to quantify model errors in dominant electron loss and transport processes. The ring current models either rely on global statistical electric and magnetic field parameterizations which may differ from real fields or use computationally expensive self-consistent treatment of the fields. Uncertainties in the fields beyond the geostationary orbit (GEO) stem from localized dipolarization fronts (Angelopoulos et al., 1992, 1994) which are believed to effectively inject electrons earthward (e.g., Birn et al., 1998; Gabrielse et al., 2012, 2016, 2017; Turner et al., 2016), but the importance of which in the overall electron transport has not been studied in detail. Within GEO, measured electric fields below $L = 5.5$ appeared to be stronger and more dynamic than the conventional corotation and convection field model suggested (Califf et al., 2014; Rowland & Wygant, 1998). Another important source of model errors originates from uncertainties in the models of different plasma waves such as whistler-mode chorus or plasmaspheric hiss. The errors in wave amplitude and wave-normal angle distributions can lead to incorrect electron scattering rates and inaccurate electron flux predictions.

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Figure 1. Schematic illustration of the Van Allen Probe orbits during the considered period from 1 February 2013 to 16 June 2013.

Data assimilation can offer useful tools which help adjust model predictions in accordance with data and identify uncertainties in the model. Data assimilation methods have been widely used in radiation belt physics (Daae et al., 2011; Koller et al., 2007; Kondrashov et al., 2011; Ni et al., 2009; Schiller et al., 2012; Shprits et al., 2007), but only a few studies on the application of data assimilation methods to the ring current have been performed. Nakano et al. (2008) used a particle filter approach to assimilate energetic neutral atom data into the Comprehensive Ring Current Model (CRCM). They showed that the particle filter can reconstruct the ion distribution in the ring current. Godinez et al. (2016) combined Van Allen Probe Magnetic Electron Ion Spectrometer (MagEIS) ion differential flux data with the kinetic Ring current-Atmosphere interactions Model (RAM) Self-Consistently coupled with a 3-D Euler-potential-based equilibrium magnetic field (SCB) code to estimate the ring current pressure by means of the ensemble Kalman filter. They demonstrated that data assimilation can help reproducing missing enhancement of pressure in the inner magnetospheric region during an isolated substorm event observed on 18 July 2013. These studies give encouraging results of the applicability of data assimilation methods for the ring current model which have to be further validated.

The scarcity of in situ satellite measurements may hinder application of data assimilation methods in space sciences. Models of the Earth's magnetosphere, ionosphere, radiation belts, and ring current are typically multidimensional and involve many degrees of freedom. Spacecraft data are limited in space, and the effects of data assimilation methods can also be localized due to the poor data coverage. The multidimensionality of the models also leads to high computational costs and complicates numerical implementation of data assimilation methods. To tackle such issues, some assumptions about the model have to be made or more sophisticated methods have to be developed. For instance, Bourdarie and Maget (2012) applied the ensemble Kalman filter to perform electron radiation belt reanalysis, using a sparse data set. Shprits et al. (2013) proposed a suboptimal method on the basis of the operator splitting technique to reconstruct three-dimensional dynamics of the radiation belts using Combined Release and Radiation Effects Satellite data. They showed that the results of their method are very close to the optimal results of the full Kalman filter. Merkin et al. (2016) demonstrated that the assimilation of ionospheric magnetic perturbations into the Lyon-Fedder-Mobarry (LFM) global magnetospheric model coupled with the Rice Convection Model (RCM) correctly modifies ionospheric currents and magnetic perturbations and nudges them toward the synthetic observations.

In this study, we present reanalysis of the ring current electron dynamics, using a log-transformed convection model blended with Van Allen Probe measurements by means of the Kalman filter. We limit the computational domain to GEO ($6.6 R_E$) to decrease the computational complexity and increase relative coverage of the domain by the observations. We use the convection portion of the four-dimensional Versatile Electron Radiation Belt (VERB-4D) code (Aseev et al., 2016; Shprits et al., 2015) and Volland-Stern electric field and dipole magnetic field models to drive the transport of ring current electrons. Such an approach allows us to efficiently perform long-term simulations and systematically analyze the applicability of the Kalman filter to the ring current electron modeling.

The manuscript is organized as follows. In section 2, we describe data and methods which are used in the study. In section 3, we present results of numerical experiments with synthetic data which are used to study the performance of the Kalman filter. We illustrate applicability of the Kalman filter in section 4, using real Van Allen Probe observations. We discuss the results in section 5. Section 6 summarizes conclusions of the study.

2. Data and Methods

2.1. Data

In this study, we use Van Allen Probe data, the “gold standard” of particle and field measurements in the inner magnetosphere. We model evolution of electron phase space density (PSD) for the constant first and second adiabatic invariants, and it is, therefore, necessary to convert local Van Allen Probe observations to

Figure 2. (a) Dependence of energy on L-shell calculated for $\mu = 0.1, 2.3$, and 9.9 MeV/G (red, blue, and green lines, respectively) and constant $K = 0.3$ G^{1/2}R_E. (b) Dependence of pitch angle on L-shell for $K = 0.3$ G^{1/2}R_E. The dipole magnetic field model is used.

the model space. To calculate the invariants and PSD along the satellite orbits, we use Helium, Oxygen, Proton, and Electron Mass Spectrometer (HOPE; Funsten et al., 2013), MageIS (Blake et al., 2013), and Electric and Magnetic Field Instrument Suite and Integrated Science (Kletzing et al., 2013) aboard the satellites.

Figure 1 illustrates Van Allen Probe orbits for the selected period from 1 February 2013 to 16 June 2013. The apogee of the satellites is located at the night side and moves westward from 2.5 hr MLT on 1 February 2013 to 21.5 hr MLT on 1 June 2013. The Van Allen Probes provide observations along one orbit inside a localized MLT sector which may pose difficulties for data assimilation methods to reconstruct global distribution of electron PSD. Note that there is no data on the dayside above ~ 2 R_E for the considered time period, and one of the goals of this paper is to investigate if the Kalman filter can adjust a model to the observations in the regions where data are not available.

We calculate 5-min averages of the measured electron fluxes and then convert the fluxes to PSD according to the formula (Rossi & Olbert, 1970)

$$f = \frac{J}{p^2}, \quad (1)$$

where J is the electron flux, p is the particle momentum, and f is PSD.

We use local magnetic field measurements to compute the first invariant μ . We calculate the second adiabatic invariant K , using T01S magnetic field model incorporated into the IRBEM library (Boscher et al., 2012). We use bilinear interpolation to calculate PSD for given μ and K along the satellite orbits during time interval from 1 February 2013 to 16 June 2013. This time interval allows us to study the response of the ring current electrons to numerous substorms and storms, including the strongest events observed on 17 March 2013 and 1 June 2013, as well as post-storm decay of electron PSD. In this study, we compute PSD for $\mu = 0.1, 2.3$, and 9.9 MeV/G and constant $K = 0.3$ G^{1/2}R_E electrons. The selected invariants cover a typical range of ring current electron energies from ~ 1 to 100 keV at $L = 4$ (see Figure 2).

2.2. Convection Model

We adopt the approach of the VERB-4D code to model the dynamics of the ring current electron population (see a detailed description of the VERB-4D code and underlying equations in Aseev et al., 2016). We use the simplified version of the code, referred to as the VERB-CS code (CS is deciphered as “Convection Simplified”) which solves the convection equation

$$\frac{\partial f}{\partial t} + v_\varphi \frac{\partial f}{\partial \varphi} + v_R \frac{\partial f}{\partial R} + \frac{f}{\tau} = 0, \quad (2)$$

where f is PSD, t denotes time, φ is MLT, R is a radial distance at the geomagnetic equator, v_φ and v_R are drift velocities, and τ are electron lifetimes.

Equation (2) is solved for constant values of the first and second adiabatic invariants. The equation describes electron transport in radial distance and MLT due to $\mathbf{E} \times \mathbf{B}$ and gradient-curvature drifts (second and third

terms on the left-hand side) and electron scattering into the Earth's atmosphere (fourth term). We note that the four-dimensional equation which is solved by the VERB-4D code is reduced to a two-dimensional case by using parameterized lifetimes τ . Such an approach simplifies the implementation of the Kalman filter and allows us to efficiently calculate long-term evolution of electron PSD and test the performance of the Kalman filter.

Equation (2) has to be complemented by initial and boundary conditions. The boundary conditions are periodic in MLT and constant in R . We set up $f = 0$ at $R = 1$, assuming that all electrons are lost in the Earth's atmosphere. We use a statistical model of electron fluxes at GEO developed by Denton et al. (2015) to set up the outer boundary conditions at $R = 6.6 R_E$. The model is based on 82 satellite years of observations at GEO made by magnetospheric plasma analyzer instruments (Bame et al., 1993) aboard Los Alamos National Laboratory Satellites. The model provides Kp-dependent spin-averaged electron fluxes for all MLT sectors and energies from ~ 40 eV to ~ 40 keV. We compute directional flux from the spin-averaged flux, assuming sinusoidal dependence on a pitch angle, and convert the flux to PSD, using equation (1) and assuming the dipole field approximation to calculate particle momentum from the first adiabatic invariant. To specify initial conditions, we calculate PSD, which is computed using Van Allen Probe HOPE and MagEIS measurements, and calculate the MLT-average of PSD over ± 5 hr from the start time of a simulation. We extend initial conditions from Van Allen Probe apogee to GEO, using constant PSD.

Drift velocities are calculated, using the dipole magnetic field approximation and the Volland-Stern Kp-dependent electric field model (Maynard & Chen, 1975; Stern, 1975; Volland, 1973). Inside the plasmasphere, we use a model of electron lifetimes developed by Orlova et al. (2016). Electron lifetimes due to local interaction with chorus waves outside the plasmasphere are adopted from Orlova and Shprits (2014). The plasmopause is calculated using Carpenter and Anderson (1992) model.

2.3. Log-Transformed Convection Model

Equation (2) can be integrated numerically under the presence of strong gradients in f by using high-order discretization schemes and limiters which preserve the monotonicity of the solution and its sign (Aseev et al., 2016; Leonard, 1991). Electron PSD is a nonnegative quantity, but the Kalman filter cannot guarantee its sign conservation (e.g., Cohn, 1997). Cohn (1997) also discussed that an error distribution of positive physical quantities cannot be strictly Gaussian since there exists a nonzero probability of assigning to them negative values, which may pose problems for the optimality of the Kalman filter. An alternative log-transformed formulation of equation (2) can be used to account for these difficulties. We note that the similar approach has been applied to the radiation belt dynamics by Kondrashov et al. (2011).

We transform PSD f to a new variable g

$$g = \log f. \quad (3)$$

Equation (2) can now be rewritten in terms of the new variable g as

$$\frac{\partial g}{\partial t} + v_\varphi \frac{\partial g}{\partial \varphi} + v_R \frac{\partial g}{\partial R} + \frac{1}{\tau} = 0. \quad (4)$$

We note that the transformation (3) guarantees the positivity of electron PSD. Log-transformed convection equation (4) contains the same convection terms as equation (2), and we adapt the VERB-CS code to the log-transformed formulation.

Following the approach of the VERB-4D code, we use the ninth-order explicit numerical scheme and the universal limiter (Leonard, 1991) in the VERB-CS code to solve convection equation (4). A time step of the model is 15 min, and it is adjusted to satisfy the Courant stability condition depending on time-variable drift velocities. We use a spatial grid with 1-hr step in MLT and $0.2 R_E$ step in radial distance.

2.4. Kalman Filter

Log-transformed model (4) includes a number of uncertainties in the parameters such as electric and magnetic fields, electron lifetimes, plasmopause position, and boundary conditions. The model errors can be accumulated with time, and model predictions can be substantially different from the real state of the system. The Kalman filter adjusts model predictions to the available satellite measurements. In this section, we

describe a Kalman filter technique applied to log-transformed convection model (4). We assume that “natural” state and observation variables are the logarithm of PSD defined by equation (3) and the dynamics of the ring current electrons is described by log-transformed convection model (4).

Let us denote a discretized state variable g at time t_{k-1} as a vector $\mathbf{g}_{k-1}^f$. The state vector $\mathbf{g}_{k-1}^f$ is propagated in time by a nonlinear model operator $\hat{M}_{k-1}$:

$$\mathbf{g}_k^f = \hat{M}_{k-1} \left(\mathbf{g}_{k-1}^f \right). \quad (5)$$

The model operator $\hat{M}_{k-1}$ can be derived from time and space discretization of equation (4).

When applied to a true state $\mathbf{g}_{k-1}^t$ defined on the same model grid as $\mathbf{g}_{k-1}^f$, the model operator advances $\mathbf{g}_{k-1}^t$ in time with some error ϵ_{k-1}^m :

$$\mathbf{g}_k^t = \hat{M}_{k-1} \left(\mathbf{g}_{k-1}^t \right) + \epsilon_{k-1}^m. \quad (6)$$

Error ϵ_{k-1}^m may include uncertainties in model parameters and errors associated with numerical schemes and subgrid processes.

A satellite observation vector $\mathbf{g}_k^{\text{obs}}$ at time t_k also contains random error ϵ_k^{obs} which includes measurement errors (e.g., due to background contamination, noisy detectors, spacecraft charging, or low counting statistics) and error of representativeness due to unresolved scales:

$$\mathbf{g}_k^{\text{obs}} = \hat{H}_k \left(\mathbf{g}_k^t \right) + \epsilon_k^{\text{obs}}, \quad (7)$$

where $\hat{H}_k$ is an observation operator. The operator $\hat{H}_k$ maps model variables into an observation space. The observation space may differ from the model space which is the case of in situ particle measurements made in flux counts. We assume that the conversion from counts to PSD is perfect, and we consider PSD as an actually measured quantity. Such an assumption does not influence results of our study but significantly simplifies implementation of the Kalman filter.

Observations also do not typically coincide with the model grid, and the operator $\hat{H}_k$ may involve interpolation. If it is not possible to make observations in the whole model domain, the size of the vector $\mathbf{g}_k^{\text{obs}}$ can be smaller than the size of the state vector $\mathbf{g}_k^f$.

For purposes of data assimilation, we bin Van Allen Probe observations in space, using model grid nodes as bin centers. The size of each bin corresponds to grid steps in MLT and radial distance. We then calculate mean values of electron PSD inside each bin. The resulting observation vector $\mathbf{g}_k^{\text{obs}}$ is defined on model grid nodes, and the observation operator $\hat{H}_k$ is, therefore, a matrix which contains 0 and 1. We denote this matrix as $\mathbf{H}_k$, and the satellite observation vector can be represented as

$$\mathbf{g}_k^{\text{obs}} = \mathbf{H}_k \mathbf{g}_k^t + \epsilon_k^{\text{obs}}. \quad (8)$$

The Kalman filter gives an optimal solution $\mathbf{g}_k^a$ of the filtering problem ((6) and (8)) which is referred to as analysis:

$$\mathbf{g}_k^a = \left\langle \mathbf{g}_k^t \middle| \left\{ \mathbf{g}_1^{\text{obs}}, \mathbf{g}_2^{\text{obs}}, \dots, \mathbf{g}_k^{\text{obs}} \right\} \right\rangle, \quad (9)$$

where the right-hand side denotes the conditional mean of the true state $\mathbf{g}_k^t$ at time t_k given the observations $\{\mathbf{g}_1^{\text{obs}}, \mathbf{g}_2^{\text{obs}}, \dots, \mathbf{g}_k^{\text{obs}}\}$ made by the time t_k . The model and observation errors ϵ_k^m and ϵ_k^{obs} are assumed to be Gaussian variables with zero mean and white (uncorrelated) in time:

$$\epsilon_k^m \sim N(\mathbf{0}, \mathbf{Q}_k), \quad \langle \epsilon_k^m (\epsilon_l^m)^T \rangle = \mathbf{0}, \quad k \neq l, \quad (10)$$

$$\epsilon_k^{\text{obs}} \sim N(\mathbf{0}, \mathbf{R}_k), \quad \langle \epsilon_k^{\text{obs}} (\epsilon_l^{\text{obs}})^T \rangle = \mathbf{0}, \quad k \neq l, \quad (11)$$

where N denotes multivariate Gaussian distribution, $\mathbf{Q}_k$ and $\mathbf{R}_k$ are model and observation error covariance matrices, respectively, $\mathbf{0}$ is a vector (or matrix, depending on context) with all elements equal to zero, and

$\langle \cdot \rangle$ denotes the expectation operator. Errors ϵ_k^m and ϵ_k^{obs} and the initial true state also have to be mutually uncorrelated (e.g., see Cohn, 1997).

The Kalman filter technique consists of forecast and analysis steps. Let us define the forecast error covariance matrix $\mathbf{P}_k^f$ as

$$\mathbf{P}_k^f = \left\langle \left(\mathbf{g}_k^f - \mathbf{g}_k^f \right) \left(\mathbf{g}_k^f - \mathbf{g}_k^f \right)^T \middle| \{ \mathbf{g}_1^{\text{obs}}, \mathbf{g}_2^{\text{obs}}, \dots, \mathbf{g}_{k-1}^{\text{obs}} \} \right\rangle, \quad (12)$$

and the analysis error covariance matrix $\mathbf{P}_k^a$ as

$$\mathbf{P}_k^a = \left\langle \left(\mathbf{g}_k^a - \mathbf{g}_k^a \right) \left(\mathbf{g}_k^a - \mathbf{g}_k^a \right)^T \middle| \{ \mathbf{g}_1^{\text{obs}}, \mathbf{g}_2^{\text{obs}}, \dots, \mathbf{g}_k^{\text{obs}} \} \right\rangle. \quad (13)$$

At the forecast step, the Kalman filter propagates the analysis $\mathbf{g}_{k-1}^a$ in time and updates the forecast error covariance matrix $\mathbf{P}_k^f$:

$$\mathbf{g}_k^f = \hat{\mathbf{M}}_{k-1} \left(\mathbf{g}_{k-1}^a \right), \quad (14)$$

$$\mathbf{P}_k^f = \mathbf{L}_{k-1} \mathbf{P}_{k-1}^a \mathbf{L}_{k-1}^T + \mathbf{Q}_{k-1}, \quad (15)$$

where matrix $\mathbf{L}_{k-1}$ is referred to as linear tangent model which is obtained by linearizing operator $\hat{\mathbf{M}}_{k-1}$.

At the analysis step, the forecast $\mathbf{g}_k^f$ is corrected according to the observations:

$$\mathbf{g}_k^a = \mathbf{g}_k^f + \mathbf{K}_k \mathbf{d}_k, \quad (16)$$

where $\mathbf{d}_k = \mathbf{g}_k^{\text{obs}} - \mathbf{H}_k \mathbf{g}_k^f$ is an innovation vector, and matrix $\mathbf{K}_k$ is referred to as Kalman gain:

$$\mathbf{K}_k = \mathbf{P}_k^f \mathbf{H}_k^T \left(\mathbf{H}_k \mathbf{P}_k^f \mathbf{H}_k^T + \mathbf{R}_k \right)^{-1}. \quad (17)$$

The analysis error covariance $\mathbf{P}_k^a$ is also updated at this step as follows:

$$\mathbf{P}_k^a = \mathbf{P}_k^f - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_k^f. \quad (18)$$

Equations (14)–(18) constitute a complete set of equations describing the Kalman filter. To compute reanalysis of the ring current electrons, we solve these equations every 30 min which corresponds to every second time step of the log-transformed convection model.

We note that we use the first-order explicit upwind numerical scheme to obtain the linear tangent model $\mathbf{L}_k$ and the ninth-order scheme for operator $\hat{\mathbf{M}}_k$. Such an approach allows us to simplify implementation of the Kalman filter while preserving accuracy of the convection model, but it can potentially affect optimality of the reanalysis.

We adopt the approach of Kondrashov et al. (2011) and set up the model and observation covariance matrices $\mathbf{Q}_k$ and $\mathbf{R}_k$ as diagonal with elements $\log(1 + \alpha_m)$ and $\log(1 + \alpha_{\text{obs}})$, respectively. In this study, we assume equal model and observation errors $\alpha_m = \alpha_{\text{obs}} = 0.5$. The use of equal errors suggests that both model and data contribute equally to the reanalysis, and reanalysis performance is not dominated by either data or model.

3. Experiments With Synthetic Data

In this section, we present results of numerical experiments with “synthetic” data. To generate the synthetic data, we run the log-transformed VERB-CS code with predefined parameters (a control simulation) and assume that the results of the model represent a true state of the system. We then define synthetic data as the results of the control simulation specified at the grid nodes closest to Van Allen Probe A and B trajectories in a given assimilation window. By varying different parameters of the VERB-CS code, we can study deviation of the reanalysis from the known “true” state of the system and performance of the Kalman filter.

For the control simulation, we run the model for $\mu = 2.3$ MeV/G and $K = 0.3$ G^{1/2} R_E electrons. The parameters of the convection model are described in sections 2.2 and 2.3. The model time step is 15 min, and data assimilation is performed every 30 min.

3.1. Metrics

We assess accuracy of the ring current electron PSD reanalysis in fraternal twin experiments by using two metrics which are described in this section. Since a true state of the system is produced by the model in the control simulation, the full true state is available on the same model grid as reanalysis, and assessment of reanalysis accuracy is essentially a comparison between outputs of two simulations.

To understand how well reanalysis reconstructs radial extent of electron PSD and how corresponding errors depend on geomagnetic activity, we use logarithmic difference $\epsilon_{\log 10}$ between MLT-averaged true state and model (or reanalysis) output:

$$\epsilon_{\log 10}(t, R_j) = \log_{10} \left(\frac{1}{N_\varphi} \sum_{i=1}^{N_\varphi} f_{\text{true}}(t, \varphi_i, R_j) \right) - \log_{10} \left(\frac{1}{N_\varphi} \sum_{i=1}^{N_\varphi} f_{\text{model}}(t, \varphi_i, R_j) \right), \quad (19)$$

where f_{true} denotes PSD obtained in the control simulation (true state), f_{model} is the predicted state (model or reanalysis), i and j are grid nodes in MLT and radial distance, respectively, and N_φ is the number of nodes in MLT grid.

Electron PSD varies in space and time by several orders of magnitude, and the use of the logarithmic difference helps us represent scale-dependent errors by illustrating the ratio between the true state and reanalysis. We note that it is impossible to devise a universal metric that would characterize the ability of the model to simultaneously reconstruct all features of the resulting PSD (such as variations in time, MLT, and radial distance), and $\epsilon_{\log 10}$ may hide information about MLT dependence of errors or even mask some errors in the calculation. The limitations which the metric has have to be taken into account during analysis of results of fraternal twin experiments.

To compute a second, quantitative, measure of the general reconstruction of the true state, we follow Morley (2016) and Morley et al. (2018). We require such a measure being a scalar describing accuracy of the reconstruction of the PSD distribution in space. The measure has to be computed for each time step of the model to represent its dependence on geomagnetic conditions. We first calculate “PSD matching ratio” (Morley et al., 2018; Yu et al., 2014) $\xi_{i,j}(t)$ for each grid node

$$\xi_{i,j}(t) = 10^{\eta_{i,j}(t)}, \quad (20)$$

where

$$\eta_{i,j}(t) = \left| \log_{10} \left(\frac{f_{\text{data}}(t, \varphi_i, R_j)}{f_{\text{model}}(t, \varphi_i, R_j)} \right) \right|, \quad (21)$$

and we use the same notation as in equation (19).

The value of $\xi_{i,j}$ represents a factor by which the model (or reanalysis) PSD differ from the true state at a point (φ_i, R_j) , regardless of the fact whether the true state is overestimated or underestimated by the model. Model prediction coincides with the true state at a point (φ_i, R_j) if $\xi_{i,j} = 1$.

We calculate an average of $\xi_{i,j}(t)$ over MLT and radial distance to quantitatively estimate the accuracy of global reconstruction of the true state:

$$\tilde{\zeta}(t) = \frac{1}{N_\varphi} \frac{1}{N_R} \sum_{i=1}^{N_\varphi} \sum_{j=1}^{N_R} \xi_{i,j}(t), \quad (22)$$

where N_R is the number of nodes in the radial grid.

The metric $\tilde{\zeta}$ is symmetric with respect to the swap of true and predicted PSD states. The metric $\tilde{\zeta}$ differs from the metric that Morley (2016) developed, which was named “median symmetric accuracy” (MdSA). Morley (2016) (1) considered a scalar variable changing with time, (2) calculated the median of the matching ratio, (3) subtracted 1 from the median, and (4) multiplied the result by 100 to obtain an equivalent percentage representation.

We do not use percentage representation in this study since the model error may exceed one order of magnitude in some fraternal twin experiments, and the percentage representation is not useful. For this reason, we do not subtract 1 from $\tilde{\zeta}$ and do not multiply it by 100, so $\tilde{\zeta} \geq 1$ and $\tilde{\zeta} = 1$ corresponds to the perfect match between model and data.

We tested the MdSA metric applied to the fraternal twin experiments and found that the median may not reflect well the accuracy of the reanalysis. If the true state coincides (or almost coincides) with the prediction in more than half grid points, which is often true in the case of reanalysis in experiments with errors in boundary conditions and electron lifetimes, the MdSA is zero (or almost zero). Therefore, a perfect match between the true state and reanalysis cannot be distinguished from the case when the model arbitrarily deviates from the true state in less than half grid points. The use of mean in the metric $\tilde{\zeta}$ eliminates such a problem.

The metric $\tilde{\zeta}$ assigns same weights to grid points closer to the Earth, where less changes in PSD are expected, and points near the outer radial boundary, where the system is the most dynamical and PSD is usually higher. By reweighting the mean in equation (22), the metric $\tilde{\zeta}$ can be adjusted to a particular application. In this study, we do not reweight the metric and use equation (22). We note that such a metric is most suitable for assessing the accuracy of the reanalysis in the fraternal twin experiments, and other studies may require a different metric, depending on the problem and desired feature to be reproduced by the model.

We refer to metric $\tilde{\zeta}$ as the mean fractional symmetric error (MFSE). Its name is intentionally made resemble the name median symmetric accuracy introduced by Morley (2016), since MFSE inherits main features from the MdSA. The use of word “error” instead of “accuracy” is more intuitive since the MFSE decreases with the increase in model accuracy.

3.2. Fraternal Twin Experiment: Errors in Electron Lifetimes

In a fraternal twin experiment, the model that is used for data assimilation differs from the model that produces a synthetic true state. By changing parameters of the log-transformed convection model, we can artificially introduce a model error and study the performance of the Kalman filter, using the “imperfect” model. In this section, we combine the Kalman filter and the VERB-CS code with tenfold increased lifetimes τ . We keep all other parameters the same for both versions of the model.

Figure 3 illustrates results of the fraternal twin experiment, using the imperfect model with “inaccurate” electron lifetimes. Figure 3a shows MLT-averaged PSD as a synthetic truth, and Figure 3b presents simulation results with the electron lifetimes $\tau \cdot 10$ if the Kalman filter is not applied. The increase in the electron lifetimes by one order leads to drastically different model results. The electrons which are transported down to 3–6 R_E during geomagnetically disturbed times stay longer inside the geostationary orbit, and the resulting PSD can be higher by several orders of magnitude compared to the synthetic data.

Figure 3c shows MLT-averaged reanalysis results obtained by combining an imperfect model, Kalman filter, and synthetic data. The reanalysis is visually indistinguishable from the synthetic true state (Figure 3a). The reanalysis retains the same pronounced features: shorter post-storm PSD decay times and the local minimum between 3 R_E and 5 R_E . Figure 3d shows logarithmic difference $\varepsilon_{\log 10}$ between the MLT-averaged synthetic true state and the reanalysis. The logarithmic difference is generally negative since the model overestimates the “true” state, and the magnitude of the logarithmic difference does not exceed ~ 0.2 and increases with the increase in the Kp index (see Figure 3f).

The model MFSE $\tilde{\zeta}_{\text{model}}$ (see the red line in Figure 3e) grows with time and falls noticeably when the Kp index sharply increases. During such periods, the radial component of electron drift velocity also increases and electrons penetrate closer to the Earth, partially compensating inaccuracies driven by errors in electron lifetimes. The maximum of $\tilde{\zeta}_{\text{model}}$ almost reaches 100, which indicates that the model overestimates the true state by two orders of magnitude on average. In the case of reanalysis, $\tilde{\zeta}_{\text{reanalysis}}$ slightly deviates from 1 and never reaches 2 (see the blue line in Figure 3e). This fact suggests that the Kalman filter can efficiently correct errors in the model predictions.

3.3. Fraternal Twin Experiment: Errors in Boundary Conditions

Modeling the ring current electron dynamics requires the knowledge of MLT-dependent outer boundary conditions. Errors in the boundary conditions due to limited MLT coverage of different spacecraft can be a significant source of uncertainty in the model. Interpolation of satellite data in time and space can only provide general large-scale MLT distribution of electron flux, and smaller-scale particle transport may not be reproduced properly. It is, therefore, important to understand if model errors due to inaccurate boundary conditions can be corrected by the Kalman filter. In this section, we present results of the fraternal twin experiment in which we decrease the boundary conditions 10 times in the VERB-CS model used for data assimilation compared to the model that we use to obtain the synthetic data.

Figure 3. (a) MLT-averaged synthetic “true” state obtained by running the VERB-CS code = Convection Simplified Versatile Electron Radiation Belt code for $\mu = 2.3$ MeV/G and $K = 0.3$ G^{1/2}R_E electrons. (b) MLT-averaged model results when the electron lifetimes are 10 times higher than in the model used to calculate the true state. (c) MLT-averaged reanalysis results. (d) Logarithmic difference $\epsilon_{\log_{10}}$ between the MLT-averaged synthetic true state and reanalysis. (e) Evolution of the MFSE. (f) The Kp index. PSD = phase space density; MFSE = mean fractional symmetric error; MLT = magnetic local time.

Figure 4 presents results of the fraternal twin experiment with the inaccuracies in boundary conditions. Figure 4a shows the MLT-averaged synthetic true state, and Figure 4b shows results of the convection model in which boundary conditions are 10 times lower than the boundary conditions which are used for obtaining the synthetic true state. The decrease in the boundary conditions affects only magnitude of the resulting electron PSD, and the qualitative picture of PSD evolution is similar to the synthetic data.

Figure 4c shows the reanalysis results, using the imperfect model with inaccurate boundary conditions, and Figure 4d shows the logarithmic difference $\epsilon_{\log_{10}}$ between the “truth” and the reanalysis. The reanalysis underestimates the true state by ~ 1 order above 6 R_E. It is an expected result since Van Allen Probe apogee is located near 6 R_E, and there are not enough synthetic measurements for correcting model results beyond 6 R_E. The difference can also reach one order below 6 R_E when the Kp index sharply increases (cf. numerous earthward-looking red spikes in Figure 4d and the Kp index in Figure 4f). During these periods, the convection electric field is enhanced, and electrons are transported from geostationary distances toward the Earth so fast that the Kalman filter needs more time to “pull” the model state toward the synthetic truth.

The model MFSE $\tilde{\zeta}_{\text{model}}$ (see the red line in Figure 4e) increases with time and reaches ~ 4 at the end of the simulation, which corresponds to an average fourfold underestimation of the synthetic true state. During

Figure 4. (a) MLT-averaged synthetic “true” state obtained by running the VERB-CS code = Convection Simplified Versatile Electron Radiation Belt code for $\mu = 2.3$ MeV/G and $K = 0.3$ $G^{1/2}R_E$ electrons. (b) MLT-averaged model results when the boundary conditions are 10 times lower than in the model used to calculate the true state. (c) MLT-averaged reanalysis results. (d) Logarithmic difference $\epsilon_{\log 10}$ between the MLT-averaged synthetic true state and reanalysis. (e) Evolution of the MFSE. (f) The Kp index. PSD = phase space density; MFSE = mean fractional symmetric error; MLT = magnetic local time.

periods of sharp increases in the Kp index, $\tilde{\zeta}_{\text{model}}$ noticeably rises since the errors in boundary conditions propagate inward faster. The reanalysis MFSE $\tilde{\zeta}_{\text{reanalysis}}$ (see the blue line in Figure 4e) is much smaller compared to $\tilde{\zeta}_{\text{model}}$. It does not fall below its background value, which is slightly higher than 1, due to permanent underestimation of the true state above $\sim 6 R_E$. Note that $\tilde{\zeta}_{\text{reanalysis}}$ also increases with the increase in the Kp index and may even reach $\tilde{\zeta}_{\text{model}}$.

This fraternal twin experiment reveals two important aspects of the data assimilation using only Van Allen Probe data. First, errors in boundary conditions cannot be corrected above $6 R_E$ without other data sources, and second, the Kalman filter adjusts the imperfect model with inaccurate boundary conditions to the synthetic state on the time scales comparable to the duration of a storm. We note, however, that the considered fraternal twin experiment is an extreme case of errors in boundary conditions, and errors in real problems can be significantly lower if satellite measurements or statistical models are used to produce the reanalysis.

3.4. Fraternal Twin Experiment: Errors in Convection Electric Field

Determination of the global convection electric field is one of the most challenging tasks in modeling the ring current electron dynamics. Due to unavailability of global in situ electric field measurements, ring current

Figure 5. (a) MLT-averaged synthetic “true” state obtained by running the VERB-CS code = Convection Simplified Versatile Electron Radiation Belt code for $\mu = 2.3$ MeV/G and $K = 0.3$ G^{1/2} R_E electrons. (b) MLT-averaged model results when the convection electric field is 4 times higher than in the model used to calculate the true state. (c) MLT-averaged reanalysis results. (d) Logarithmic difference $\epsilon_{\log_{10}}$ between the MLT-averaged synthetic true state and reanalysis. (e) Evolution of the MFSE. (f) The Kp index. PSD = phase space density; MFSE = mean fractional symmetric error; MLT = magnetic local time.

codes have to rely on empirical models or self-consistent calculations of the electric field. Uncertainties in the electric field model directly result in uncertainties in drift velocities, affect electron transport time scales, and may lead to significant errors in the distribution of electron PSD. In this fraternal twin experiment, we study whether the Kalman filter can improve predictions of ring current electron PSD in the case of an inaccurate electric field model. We multiply the convection electric field by 4 and assimilate the synthetic true state provided by the “perfect” model. We note that the corotation electric field remains unchanged.

Figure 5 shows results of the fraternal twin experiment. The synthetic true state (Figure 5a) is obtained with the same parameters as has been used in the previous experiments. Figure 5b presents results of the VERB-CS code with the fourfold increase in the convection electric field. The results of the imperfect model show fast earthward propagation of the ring current electrons since the convection electric field is responsible for radial transport of the particles in the model. The difference between the true state and the model is most prominent between 2.5 and 6 R_E . The model overestimates the true state by several orders of magnitude, and the difference grows with enhanced geomagnetic activity. One of the most noticeable features in the model results (Figure 5b) is an enhancement of electron PSD at $\sim 3 R_E$ that occurs during the 17

Figure 6. (a) Evolution of electron PSD for $\mu = 2.3$ MeV/G and $K = 0.3$ G^{1/2}R_E obtained from Van Allen Probe data and binned in time and radial distance. (b) Magnetic local time-averaged model results when the Kalman filter is not applied. (c) Magnetic local time-averaged reanalysis results. (d) Innovation vector binned in time and radial distance. (e) The Kp index. PSD = phase space density.

March 2013 geomagnetic storm and lasts until the end of the simulations, being intermittently refilled during periods of increases in the Kp index.

Figure 5c presents reanalysis results which combine the imperfect model and synthetic data obtained along the orbits of Van Allen Probes being flown through the synthetic true state. The reanalysis contains features observed both in synthetic data (Figure 5a) and model predictions (Figure 5b). The reanalysis eliminates long-term features produced by the model, such as the “unrealistic” enhancement at 3 R_E. However, the Kalman filter is not able to correct the shorter-term behavior of the model on the time scales of approximately less than a day. The logarithmic difference $\epsilon_{\log_{10}}$ between the data and the model (see Figure 5d) is located near the inner edge of the enhanced PSD and exceeds one order of magnitude. The difference is shifted earthward with the increase in the Kp index, when the model electric field sharply becomes larger. This fact suggests that more measurements at different radial distances and MLT sectors are required to reconstruct global convective transport and loss of the electrons for typical storm time scales.

The reanalysis MFSE $\tilde{\zeta}_{\text{reanalysis}}$ (see Figure 5e) is noticeably decreased in comparison to the model $\tilde{\zeta}_{\text{model}}$. During periods of the increase in the Kp index, $\tilde{\zeta}_{\text{reanalysis}}$ also grows sharply and can be comparable to $\tilde{\zeta}_{\text{model}}$. We note that $\tilde{\zeta}_{\text{reanalysis}}$ in this experiment is generally larger than in the previous experiments, being sometimes higher than 100. The correction of errors associated with overestimated drift velocities seems to be more challenging for the Kalman filter than the correction of errors in boundary conditions and electron lifetimes.

We also performed an experiment with the convection electric field being four times smaller in the model that was used for data assimilation than in the model that provided synthetic data. Supporting information Figure S1 shows results of the experiment. We note that we simply swapped used in the previous experiment

Figure 7. (a) Innovation vector for $\mu = 2.3$ MeV/G and $K = 0.3$ $G^{1/2}R_E$ binned in the Kp index and radial distance. (b) Number of samples which are used to calculate the mean value in each bin. PSD = phase space density.

models for performing data assimilation (Figure 5b) and obtaining synthetic true state (Figure 5a) to avoid very low PSD values in the synthetic data due to slow radial transport. The results indicate that the overall difference between the synthetic data and reanalysis is much smaller in the case when the model electric field is underestimated than in the case of overestimation.

4. Reanalysis of Ring Current Electrons Using Van Allen Probe Data

The experiments with synthetic data give a promising result that the Kalman filter can improve predictions of the ring current electron dynamics by decreasing model errors associated with the uncertainties in electron lifetimes, outer boundary conditions, and drift velocities and provide reasonable reanalysis results, using only Van Allen Probe synthetic “measurements.” In this section, we apply the Kalman filter to combine the VERB-CS code with real Van Allen Probe measurements by means of data assimilation.

4.1. Reanalysis of Ring Current Electron PSD from 1 February 2013 to 16 June 2013

We adopt the model parameters described in sections 2.2 and 2.3 which have been used to produce the synthetic true state for the fraternal twin experiments. The model relies on several simplifications which likely introduce noticeable errors. Errors in the electron drift velocities are associated with the magnetic dipole field approximation, which is not valid during storm times at high radial distances, and the Kp parameterization of the convection electric field, which is naturally driven by the dayside reconnection and varies on the time scales much shorter than the cadence of the Kp index. Uncertainties in boundary conditions stem from the used statistical model, conversion from the spin-averaged to directional flux, and calculation of the PSD under the dipole field approximation. Parameterized electron lifetimes and simplified plasmopause model may also contribute to the overall model error.

Figure 6a presents evolution of $\mu = 2.3$ MeV/G and $K = 0.3$ $G^{1/2}R_E$ electron PSD derived from Van Allen Probe HOPE and MageIS measurements between 1 February 2013 and 16 June 2013. The PSD values are binned in time and radial distance with 10 hr and $0.2 R_E$ steps, respectively. The PSD varies by more than three orders of magnitude within GEO, and electrons can be propagated down to $2.5 R_E$ during geomagnetic storms (see the Kp index in Figure 6e).

Figure 6b shows MLT-averaged VERB-SC code results for the considered period of time. Note that these results contain real model errors, yet they have been treated as the synthetic truth in the fraternal twin experiments. The model results noticeably differ from the observed PSD. The model underestimates the data between $3 R_E$ and ~ 4.5 – $5 R_E$ by several orders of magnitude (see the dark-blue color in Figure 6b). The observed storm-time enhancements of electron PSD are reproduced by the model above $\sim 4.5 R_E$. However, the model overestimates the post-storm PSD for some events such as those on 17 March 2013 and 29 March 2013. The model-data comparison suggests that violated balance between transport and loss processes may lead to significant deviations of the predicted PSD on time scales from several weeks to months.

Figure 8. The Kp (green) and Dst (red) indices during the geomagnetic storm observed on 1 June 2013.

Figure 6c presents MLT-averaged reanalysis results combining the VERB-CS code and Van Allen Probe observations. The Kalman filter corrects for inaccuracies in the model and improves the results to better match the observations. The reanalysis captures observed penetration of the ring current electrons from GEO to $2.5 R_E$, which cannot be reproduced by the model, and the overall radial extent of the reanalysis shows the same variability as seen in the data. The reanalysis results resemble the observations and significantly differ from the model below $\sim 5 R_E$. The features of both model and data can be seen in the reanalysis above $5 R_E$, where the electron PSD reaches the highest values.

The correction that the Kalman filter adds to the model is most clearly seen in an innovation vector. The innovation is a difference between observations and forecast transformed into the observation space (vector $\mathbf{d}_k$ in equation (16)). We note that the innovation in our formulation is the natural logarithm of the ratio between observed PSD and the model since we use the natural logarithm of PSD as a model and observation variables (see equation (3)). For simplicity, we transform the ratio from natural to common logarithm. Negative values of the innovation indicate that the model overestimates the data, and positive values mean that the observed PSD is higher than those predicted by the model.

Figure 6d shows the innovation transformed back into the model MLT-R space. The abundance of both positive and negative values of the innovation indicates that the model may underestimate as well as overestimate the data. For the events when electrons are transported down to $2.5\text{--}3 R_E$, the innovation shows high positive values below $4 R_E$, implying that the radial transport in the model is less efficient than in the reality. The ring current electron PSD experiences intermittent enhancements and decreases between $4 R_E$ and $6 R_E$, and the innovation shows the highest magnitude and more frequent variations in that region.

Figure 7a presents the innovation binned in Kp index and radial distance, and Figure 7b shows the number of samples used to calculate the innovation. During relatively quiet geomagnetic times ($Kp < 4$), the innovation shows the smallest deviations from zero. The innovation is almost zero between $2 R_E$ and $4 R_E$, and the deviations are higher above $4 R_E$. The innovation is significantly bigger during disturbed geomagnetic conditions ($Kp \geq 4$) than during quiet times. The innovation shows high positive values between $3 R_E$ and $4 R_E$ when $Kp \geq 4$ which indicates that the model cannot reproduce fast earthward propagation of ring current electrons. This fact together with almost zero innovation in this region during quiet times suggests that the underestimation of electron PSD below $4\text{--}5 R_E$ produced by the model (see Figure 6b) is associated with the errors in the storm-time electron transport. At higher radial distances, the uncertainties in boundary

Figure 9. Global evolution of PSD for $\mu = 2.3 \text{ MeV/G}$ and $K = 0.3 \text{ G}^{1/2} R_E$ electrons during the geomagnetic storm observed on 1 June 2013: (a–e) model results when the Kalman filter is not applied, (f–j) reanalysis, and (k–o) the difference between common logarithms of reanalysis and model results. PSD = phase space density.

Figure 10. (a) Evolution of electron PSD for $\mu = 0.1$ MeV/G and $K = 0.3$ G^{1/2}R_E obtained from Van Allen Probe data and binned in time and radial distance. (b) Magnetic local time-averaged model results when the Kalman filter is not applied. (c) Magnetic local time-averaged reanalysis results. (d) Innovation vector binned in time and radial distance. (e) The Kp index. PSD = phase space density.

conditions start playing more important role during disturbed times, and the model both underestimates and overestimates the data.

4.2. Reanalysis of Ring Current Electron PSD During 1 June 2013 Storm

In the previous sections, we have analyzed the performance of the Kalman filter on time scales of months. We have shown that the Kalman filter is able to improve model predictions but may not perform well during storm times. In this section, we apply the Kalman filter during the storm observed on 1 June 2013 to study the ability of the Kalman filter to reconstruct MLT-dependent distribution of the ring current electron PSD on time scales of several days when the magnetospheric convection is most intense. During the chosen storm, the Kp index reaches 7, and the Dst index goes down to -124 nT at $\sim 08:00$ UT on 1 June 2013 (see Figure 8).

We initialize the VERB-CS code at 00:00 UT 30 May 2013 by MLT-averaging Van Allen Probe HOPE and MagEIS data over ± 5 hr, which approximately corresponds to one satellite orbit. We model evolution of $\mu = 2.3$ MeV/G and $K = 0.3$ G^{1/2}R_E electron PSD and use the same parameters of the convection model which are described in sections 2.2 and 2.3 and have been used in previous numerical simulations.

Figure 9 illustrates the global PSD evolution during the storm. The first row (Figures 9a–9e) shows the model results if the Kalman filter is not applied, the second row (Figures 9f–9j) presents the reanalysis, and the last row (Figures 9k–9o) shows the logarithm of the ratio between the reanalysis and the model. Both reanalysis and model are initialized with the same PSD (see Figures 9a, 9f, and 9k). The model results do not noticeably differ from the reanalysis 30 hr after the beginning of the simulation (see Figures 9b, 9g, and 9l), most likely due to the fact that geomagnetic conditions were very quiet.

During the main phase of the storm, the model results and the reanalysis are almost identical (see Figures 9c, 9h, and 9m) which may indicate that the Kalman filter is not able to correct the model errors during the

Figure 11. (a) Evolution of electron PSD for $\mu = 9.9$ MeV/G and $K = 0.3$ G^{1/2}R_E obtained from Van Allen Probe data and binned in time and radial distance. (b) Magnetic local time-averaged model results when the Kalman filter is not applied. (c) Magnetic local time-averaged reanalysis results. (d) Innovation vector binned in time and radial distance. (e) The Kp index. PSD = phase space density.

storm main phase when PSD is significantly enhanced and electrons are rapidly transported earthward. One day later at 6:00 UT on 2 June 2013, model predicts lower PSD values below 3 R_E than the reanalysis shows (see Figure 9n). The reanalysis is generally lower than the model results between 3 R_E and 5 R_E (cf. Figures 9d and 9i), and a spiral-like structure of enhanced PSD which is observed in the model results at ~4 R_E (Figure 9d) is almost absent in the reanalysis (Figure 9i). The difference between the reanalysis and the model becomes slightly smaller 2 days after the main phase of the storm (see Figure 9o).

The presented results indicate that assimilation of Van Allen Probe point measurements is able to correct the errors in the predictions of the log-transformed convection model across different MLT sectors in accordance with the observations. The Kalman filter adjusts the model to local Van Allen Probe observations within the assimilation window, and the model spreads the locally modified PSD across different MLT sectors and radial distances. This mechanism is responsible for pronounced differences in the regions which are not covered by the satellite observations (e.g., $R > 2$ on the dayside for the considered period of time). A lack of data near GEO and in different MLT sectors leads to the limitations described in the fraternal twin experiments in sections 3.3 and 3.4: The reanalysis is likely dominated by the model contribution rather than the data during storm times and near GEO during both quiet and disturbed geomagnetic conditions.

4.3. Reanalysis of Ring Current Electron PSD for Different Values of μ

Adiabatic invariants are natural variables which are used to formulate equations describing radiation belt and ring current electron dynamics. However, it is more instructive to predict evolution of electron flux as a function of energy and pitch angle for real applications such as calculating fluences which help estimate potential effects of surface and deep dielectric charging of satellites. Calculation of electron flux for given energy and pitch angle requires the knowledge of PSD for a range of the first and second adiabatic invariants.

In this section, we apply the developed data assimilation framework for reconstruction of electron PSD for different values of μ which cover a typical energy range of the ring current electrons.

We use the same approach as described in section 2 to assimilate PSD obtained from Van Allen Probe HOPE and MagEIS data for $\mu = 0.1$ and 9.9 MeV/G and constant $K = 0.3$ G^{1/2}R_E, using the convection model and the Kalman filter. The selected adiabatic invariants roughly correspond to 1 and 100 keV at ~ 4 R_E, respectively (see Figure 2).

Figure 10 presents reanalysis of $\mu = 0.1$ MeV/G and $K = 0.3$ G^{1/2}R_E PSD in the same format as Figure 6. The PSD obtained with the model (Figure 10b) is generally higher than the observations (Figure 10a) below ~ 4.5 R_E. The model shows fast earthward transport during periods of the Kp index $\gtrsim 4$, and the inner extent of the PSD produced by the model is located closer to the Earth by 1–1.5 R_E. The reanalysis (see Figure 10c) is able to adjust the model to the measurements, with the biggest corrections made between 3 and 4.5 R_E during storm times (see Figure 10d). As in the case of the fraternal twin experiment with artificially increased velocities presented in section 3.4, the Kalman filter is more effective on the time scales longer than ~ 1 day. During geomagnetic storms, the reanalysis mostly resembles model rather than the data, which is most probably caused by lack of global in situ measurements. We also note that MLT averaging may also affect the comparison of the results presented in Figure 10.

Figure 11 shows reanalysis for $\mu = 9.9$ MeV/G and $K = 0.3$ G^{1/2}R_E electrons. Such values of the adiabatic invariants correspond to relatively high energies, and the electrons are effectively transported radially by the diffusion process (Lyons & Schulz, 1989). Convection equation (2) does not take into account radial diffusion, and the model results (Figure 11b) significantly underestimate observations (Figure 11a). The reanalysis (Figure 11c) shows that even in the case of a missing physical process in the model equation, the Kalman filter can correct model predictions, using Van Allen Probe data. As expected, the innovation is the highest and positive for periods of high Kp index which indicates that the Kalman filter recognizes the underestimation of data by the model and adjusts the predictions accordingly.

5. Discussion

In this work, we used the log-transformed VERB-CS model, which was specially developed for this study on the basis of the approach of the VERB-4D code, Van Allen Probe in situ measurements, and the Kalman filter to reconstruct the global dynamics of ring current electron PSD from 1 February 2013 to 16 June 2013. We used the model with Kp-dependent boundary conditions, drift velocities, and electron lifetimes to efficiently perform the long-term simulations and study performance of the Kalman filter. While the simplified model can deviate from the data on time scales of weeks or months, the reanalysis agrees better with the observations. Numerical experiments with synthetic data showed that the Kalman filter can be effectively applied to correct errors in model predictions associated with electron lifetimes, outer boundary conditions, and drift velocities. We showed that the Kalman filter adjusts predictions of the convection model to the observations across different MLT sectors, using only local Van Allen Probe measurements.

We demonstrated that the Kalman filter is a powerful tool that can be used in real applications and long-term studies of the ring current electron dynamics. Existing models include many physical processes such as charged particle transport and wave-particle interactions and numerous parameters and parameterizations which may contain significant uncertainties (e.g., plasmopause and magnetopause location, drift velocities, and spatial outer boundary conditions). Our results suggest that the Kalman filter can provide global reanalysis which is more accurate than either interpolated data or a global model.

Reanalysis of the ring current electron dynamics can be used to identify the missing physical processes in the model. By inspecting the innovation, it is possible to determine in which regions and under which conditions the model overestimates or underestimates observations and study balance between transport and loss processes. As an example, we show that the earthward transport of $\mu = 2.3$ MeV/G and $K = 0.3$ G^{1/2}R_E electrons below ~ 4 R_E during storm times cannot be reproduced by the simplified Kp-dependent convection model.

The Kalman filter can correct model predictions for typical ring current electron energies from 1 to 100 keV. Convection model (2) can be further applied to a range of different values of the first and second adiabatic invariants and blended with Van Allen Probe data by means of the Kalman filter to reconstruct electron

fluxes for particular energy and pitch angle inside GEO. Such an extension is beyond the scopes of this manuscript and will be performed in our future studies.

Since in situ particle measurements are localized in a narrow region of space at a particular time, assimilation of Van Allen Probe observations has inherited several limitations. During the main phase of geomagnetic storms, electron PSD is significantly enhanced, and ring current electrons undergo fast earthward transport. The Kalman filter may require a time period comparable to that of a typical storm main phase to reconstruct global distribution of electron PSD. As a result, the reanalysis of global storm-time electron distribution can be primarily driven by the model rather than observations during a storm main phase. The lack of data near GEO also renders the reanalysis to rely on the model. The performance of the Kalman filter can be significantly improved by simultaneously assimilating multiple satellite observations such as made by ERG, THEMIS, LANL, or GOES missions. More detailed global evolution of electrons fluxes or PSD can be obtained by assimilating POES and MetOp satellite data which simultaneously provide measurements at different MLT sectors and across different L-shells with high temporal resolution. We note that data assimilation of multiple spacecraft data requires careful intercalibration and specification of observation errors for each satellite.

The Kalman filter relies on the assumption of Gaussian distribution of model and observation errors. In this work, we use the logarithm of PSD as model and observation variables. The choice of the log-transformed variables is made to eliminate the computational issues which are associated with the possibility of the Kalman filter to produce negative PSD values. By using the log-transformed variables, we implicitly assume log-normal distribution of model and observation errors. Yet non-negativity of PSD indicates that the distribution of errors cannot be strictly Gaussian, log-normal distribution of the errors is also not guaranteed. The quantification of model and observation errors is a challenging problem which we will address in future studies.

The 2-D log-transformed convection model that we use in this study can be potentially incorporated in the more sophisticated space weather codes and frameworks. For instance, the VERB-4D code includes both ring current and radiation belt electron transport, loss and acceleration processes driven by convection, radial diffusion, and local wave-particle interactions. Methods of data assimilation have been already developed for the radiation belt portion of the code (Kellerman et al., 2014; Shprits et al., 2013), and the approach presented in this study supplements the developed methods to globally reconstruct the full 4-D state of the ring current and radiation belt electrons by means of the Kalman filter and the VERB-4D code.

6. Summary and Conclusions

We presented the long-term reanalysis of the ring current electron dynamics, using Van Allen Probe observations, the Kalman filter, and the 2-D log-transformed VERB-CS code. We showed that the Kalman filter can be applied to reconstruct global evolution of ring current electron PSD using Van Allen Probe point measurements. The main conclusions are summarized below.

1. Even when only Van Allen Probe data are used, the Kalman filter is capable of decreasing errors in model predictions which are associated with electron lifetimes, boundary conditions, and drift velocities.
2. Reanalysis retains features which are observed in data and cannot be fully reproduced by the model, such as the earthward propagation of $\mu = 2.3$ MeV/G and $K = 0.3$ G^{1/2}R_E electrons down to ~ 2.5 R_E and shorter loss time scales.
3. The innovation allows us to study performance of the model and identify model uncertainties. With regard to the Kp-dependent convection model applied to $\mu = 2.3$ MeV/G and $K = 0.3$ G^{1/2}R_E electrons, the magnitude of the innovation increases (and hence, the model accuracy decreases) with radial distance and the Kp index.
4. Assimilation of Van Allen Probe measurements globally adjusts model predictions in accordance with the observations even in the sectors in which the data are not available.
5. The Kalman filter can adjust model predictions for $\mu = 0.1, 2.3$, and 9.9 MeV/G and constant $K = 0.3$ G^{1/2}R_E using Van Allen Probe data, which corresponds to a typical energy range of ring current electrons.
6. Assimilation of multiple spacecraft data is required to reconstruct the ring current electron dynamics during storm main phases and near GEO.

The results obtained in this study indicate that the Kalman filter can efficiently combine a physics-based model of the ring current electrons and sparse satellite observations to improve model predictions. The further studies will include more sophisticated models, multispacecraft data, and combined reanalysis of the ring current and radiation belt electron dynamics.

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