3D TEM Forward Modeling Including Topography

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1 Motivation

Volcanic research using seismic or magnetotelluric investigations is limited to a broad scale length resolving rather big structures at depths of a few kilometres while, e.g., DC resistivity images are limited to shallow subsurface features. Thus, volcanic structures at depths of a few hundred meters are poorly imaged and few evidence of magmatic pathways and hydrothermal systems is found. Understanding details of magma rising in a volcanic conduit is the key to understand vent near eruption dynamics. Here, we want to explore the applicability of the transient electromagnetic method (TEM), a medium-range electromagnetic method, in volcanic environments on the example of Stromboli volcano, Italy. For constraining and evaluating field measurements conducted in June 2019, we use 3D finite element simulations to understand the behaviour of electromagnetic fields in topographically demanding locations. Here, we present the simulation routine and preliminary results.

2 The Numerical Simulation Routine

TEM uses a DC current in a circular or square loop on the Earth's surface which is shut off at a defined time $t_0 = 0\,\mathrm{s}$. According to the law of induction, eddy currents are induced in the underground. While propagating, the current system decays in intensity and expands in space. As a measurand we consider a normalized temporal variation of the magnetic flux density at the Earth's surface in terms of $\partial_t \mathbf{B}$.
Considering the electric field intensity $\mathbf{E}$ we state the time domain formulation of transient electromagnetic induction as an initial-boundary value problem [e.g. Afanasjew et al. (2013); Börner et al. (2015)]

$$\sigma \partial_t \mathbf{E} + \nabla \times (\mu^{-1} \nabla \times \mathbf{E}) = \mathbf{0} \quad \text{on } \Omega \times (0, \infty) \tag{1a}$$
$$\mathbf{E}|_{t=0} = \sigma^{-1} \mathbf{j} \quad \text{on } \Omega \tag{1b}$$
$$\mathbf{n} \times \mathbf{E} = \mathbf{0} \quad \text{on } \partial\Omega \times (0, \infty), \tag{1c}$$

where $\mu = \mu_0$ denotes the magnetic permeability of free space, $\sigma = \sigma(\mathbf{x})$ symbolizes the electrical conductivity varying in space and $\mathbf{j}$ represents the source current density associated with a current shut-off at time t_0.
After spatial discretization using curl-conforming Nédélec elements, the given field equation can be summarized as an ODE initial value problem reading

$$\partial_t \mathbf{u}(t) + A\mathbf{u}(t) = \mathbf{0}, \quad t \in (0, \infty), \quad \mathbf{u}(0) = \mathbf{b}, \tag{2}$$

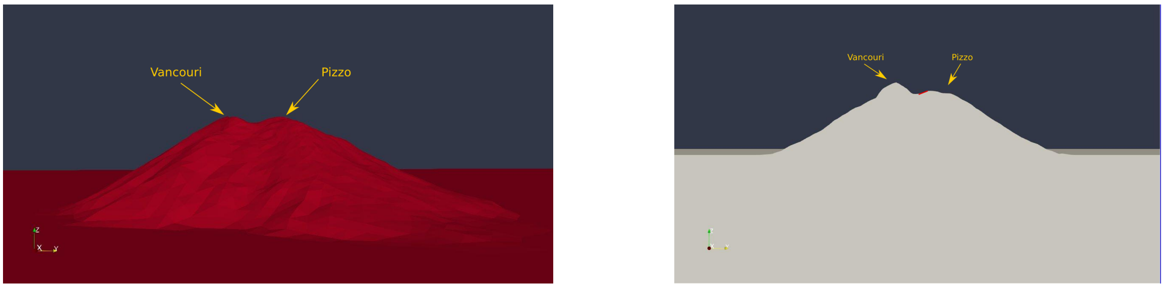

Figure 1: a) DEM of Stromboli volcano including landmarks. b) Vertical section of the model located at the center of the loop. The loop location is indicated by a red line.

where $\mathbf{u}$ denotes the coefficient vector of the finite element approximation of $\mathbf{E}$ with respect to the Nédélec basis at times $t > 0$, $\mathbf{b}$ the vector of initial values and the matrix A includes the spatial approximation of the curl-curl operator and the spatially varying conductivity $\sigma(\mathbf{x})$, respectively.
Now, we find a solution of (2)

$$\mathbf{u}(t) = e^{-tA}\mathbf{b} \tag{3}$$

which represents a matrix exponential function and can be evaluated using the Rational Arnoldi approximation at any given time $t > 0$ [Afanasjew et al. (2008), Börner et al. (2015)].
For simulation purposes, $\partial_t \mathbf{B}$ can be easily extracted from Faraday's law of induction

$$\partial_t \mathbf{B} = -\left(\nabla \times \mathbf{E}\right). \tag{4}$$

3 Application Stromboli

Incorporating an open-access digital elevation model (DEM) from NASA into a cuboid simulation domain, we may run forward modeling of both the electric and magnetic field simulating TEM measurements at topographically demanding locations.
Due to its accessibility and frequent, low explosive activity we chose Stromboli volcano, Italy, as an appropriate target for a field experiment. Consequently, our numerical experiments focus on this specific mountain.
The resulting topographic model of Stromboli volcano and a slice through the middle of the transmitter loop (its position is indicated in red) are qualitatively shown in Fig. 1 a) and b), respectively.
For evaluating the use of TEM within this rough terrain, a detailed understanding of the propagation process of the electromagnetic fields in the underground is of great importance. Therefore, animated time sequences on slices of the computational domain can provide a meaningful tool of visualization. Due to the nature of a print medium, we restrict the visualization to three stages of field interaction in the topographic environment as shown in Fig. 2. The computational domain $(x, y, z) \in \Omega = [-2000, 2000]^2 \times [-3000, 1000]\,\mathrm{m}^3$, containing an air half-space and conductive underground, is decomposed into 637 008 tetrahedra resulting in 747 855 degrees of freedom. The conductivities of the Earth and air layer are set to $\rho_{Earth} = 10^3\,\Omega\mathrm{m}$ and $\rho_{air} = 10^6\,\Omega\mathrm{m}$, respectively.
Three stages of the diffusing magnetic field are depicted in Fig. 2 in terms of $\partial_t B_z$. In Fig. 2 a), we can see a center minimum in the $\partial_t B_z$ field which is bounded by the wired loop,

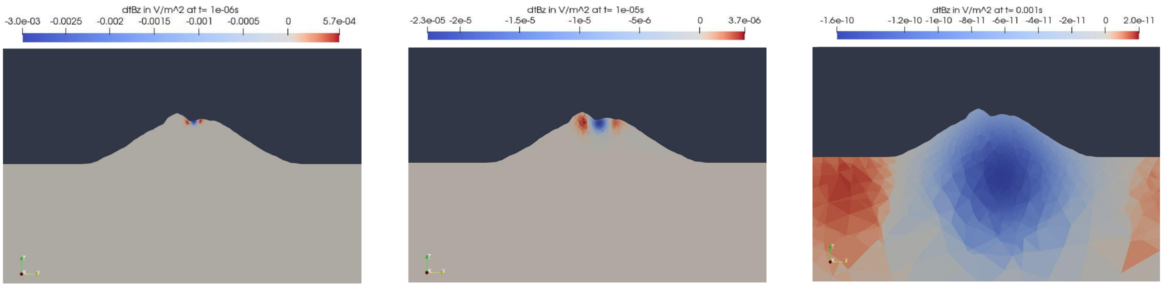

Figure 2: a) Minimum in the center of the loop at small time $t = 10^{-6}$ s. b) Propagation of the field along the edge of the relatively conductive underground, $t = 10^{-5}$ s. c) Field propagates downward similar to the homogeneous half-space at relatively late times $t = 10^{-3}$ s.

positive values occur only outside the loop. The $\partial_t B_z$ field propagates into the underground and along the Earth-air interface as seen in Fig. 2 b). Hence, the field travels along the topographic boundary. At relatively late times $t \geq 10^{-3}$ s seen in Fig. 2 c), the $\partial_t B_z$ field has migrated to depths at which it is less affected by topography. Further propagation is similar to the propagation within the homogeneous half-space.

4 Conclusion and Outlook

3D simulations of electromagnetic fields are highly useful to image electromagnetic fields in the underground. This leads to a more sophisticated understanding of physical processes and the nature of electromagnetic fields. Furthermore, applied to the DEM of a measurement location, optimized measurement configurations can be computed before the field survey is carried out. Last but not least, 3D modeling including topography can prevent misinterpretation of data sets recorded in steep terrain and can be used as the forward solver to inversion routines.

Acknowledgements

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