

### Quadrupol ERT

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### Abstract

The quadrupole method in geoelectrics was only applied for long offsets in the past. We scaled down the method to profile-style and grid-style short offset applications. The quadrupole approach yields the apparent resistivity in a tensorial form in contrast to the scalar apparent resistivity obtained from classical geoelectrics. Analysis of the invariants of the apparent resistivity tensor and its representation as ellipse can be used to obtain an estimate of the dimensionality of the subsurface conductivity distribution (1D, 2D, 3D, anisotropic).

### Apparent resistivity tensor

The basic setup to derive an apparent resistivity tensor is made up of a quadrupole source and a quadrupole receiver. Each quadrupole consists of two orthogonal dipoles. The current is injected successively with the two dipoles of the transmitter quadrupole, called AB and CD. For each injection the horizontal electrical field components  $E_x$  and  $E_y$  are measured with the receiver quadrupole. Due to the nature of the dipole field we do not create a uniform polarization direction in the subsurface. Nevertheless, the two current systems of a quadrupole are orthogonal at each point considering an homogeneous halfspace and therefore directional information can be obtained by processing those two injections together. The classical apparent resistivity tensor  $\bar{\rho}_A$  is derived by the product of the matrix  $\bar{E}$  and the inverse of matrix  $\bar{j}$  (I.I).  $\bar{E}$  contains the horizontal electrical field components of two orthogonal current injections (I.II). The corresponding current density vectors at the receiver location form the matrix  $\bar{j}$  (I.III).

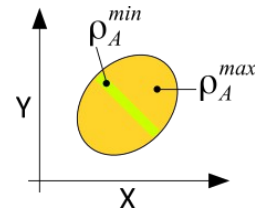
$$\bar{\rho}_A = \frac{\bar{E}}{\bar{j}} \quad (I.I) \quad \bar{E} = \begin{bmatrix} E_x^{AB} & E_x^{CD} \\ E_y^{AB} & E_y^{CD} \end{bmatrix} \quad (I.II) \quad \bar{j} = \begin{bmatrix} j_x^{AB} & j_x^{CD} \\ j_y^{AB} & j_y^{CD} \end{bmatrix} \quad (I.III)$$

In this formulation the distance between current and potential dipoles must be large enough to fulfill the dipole assumption. By reformulation of the apparent resistivity tensor  $\bar{\rho}_A$  with discrete potential differences the distance between source and receiver can be decreased down to profile-style configurations. Therefore, we introduce the discrete apparent resistivity tensor (DART) (II.I). The matrix  $\bar{\Delta U}$  contains a set of two orthogonal potential differences MN and OP for the two different excitation polarizations AB and CD. To obtain the new apparent resistivity tensor  $\rho_A$  the product of  $\bar{\Delta U}$  and the inverse matrix of  $\bar{K}$  is formed (II). The normalization matrix  $\bar{K}$  contains the product of the injected current  $I$  and the geometric factors  $k$  of the electrodes involved in the particular measurements (II.IV).

$$\bar{\rho}_A^{DART} = \frac{\bar{\Delta U}}{\bar{K}} \quad (II.I) \quad \bar{\Delta U} = \begin{bmatrix} \Delta U_{MN}^{AB} & \Delta U_{MN}^{CD} \\ \Delta U_{OP}^{AB} & \Delta U_{OP}^{CD} \end{bmatrix} \quad (II.II) \quad \bar{K} = \begin{bmatrix} I^{AB} \cdot k_{MN}^{AB} & I^{CD} \cdot k_{MN}^{CD} \\ I^{AB} \cdot k_{OP}^{AB} & I^{CD} \cdot k_{OP}^{CD} \end{bmatrix} \quad (II.III)$$

$$k_{MN}^{AB} = \frac{1}{2\pi} \left( \frac{1}{r_{AM}} - \frac{1}{r_{BM}} - \frac{1}{r_{AO}} + \frac{1}{r_{BO}} \right) \quad (II.IV)$$

The DART as tensor of rank two describes an ellipse with minor and major axes. The following depictions of the ellipses are done by coloring the background of the ellipse according to the major axis apparent resistivity value. A small bar inside the ellipse represents the minor apparent resistivity axis and is colored according to its value. All ellipses are normalized to have major axis of equal size.



**Quadrupole ERT Profile**



The Quadrupole ERT method is performed by measuring the horizontal electric field components of a set of two successive and orthogonal current injections.

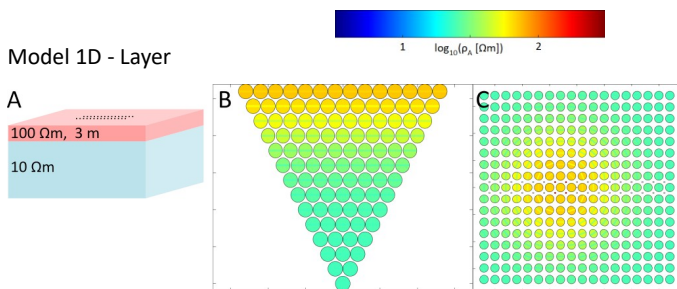
The application in the field can be simplified by setting up two parallel lines of electrodes. The current and potential quadrupoles are then 45° rotated clockwise from the profile direction.

Every adjacent quartet of electrodes can be used to measure the electrical field components and thus be used to calculate a DART. The drawing shows the tensor schematically as an ellipse.

The information of a whole set of quadrupole injections over the profile can conveniently be summarized in a single plot resembling the classical pseudo-sections known from dipole-dipole ERT measurements. In fact, this particular classic pseudo-section can be extracted from the DART pseudo-section by taking the apparent resistivity value in profile direction.

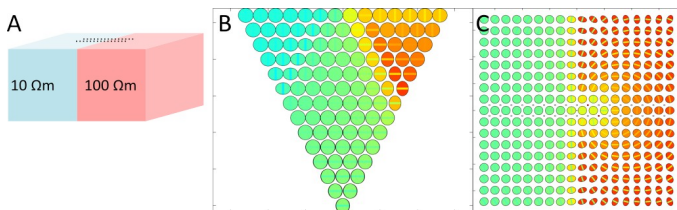
**Numerical Examples**

For the following examples the response (B) of a double profile with 2 x 15 electrodes on different conductivity structures (A) is depicted. The depiction is an overlay of the two reciprocal tensors obtained from interchanging current and potential quadrupole. Additionally, a bird-eye view of one selected injection is presented (C). A tiled arrangement of receivers show the behaviour of the DART over the area. The profiles have 2 m electrode spacing.



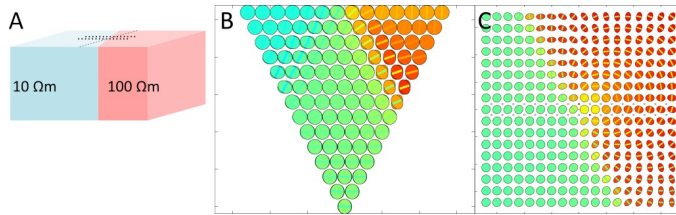
Penetration depths of inline and broadside apparent resistivity are different (Note the split in the second row of the pseudo-section).

**Model 2D – Vertical boundary perpendicular to profile**



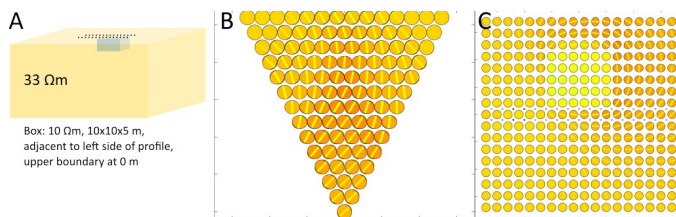
For a profile perpendicularly crossing a vertical boundary the major and minor tensor axes line up with the profile direction and respectively the strike direction.

Model 2D – Vertical boundary oblique to profile



Oblique profile setups yield reciprocal tensors with different skew angles and varying axis orientations. The effect is more pronounced in the vicinity of the boundary (see B, resistive part).

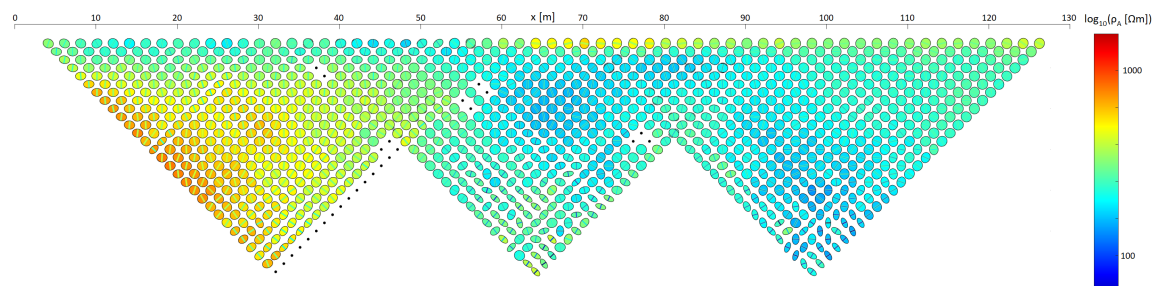
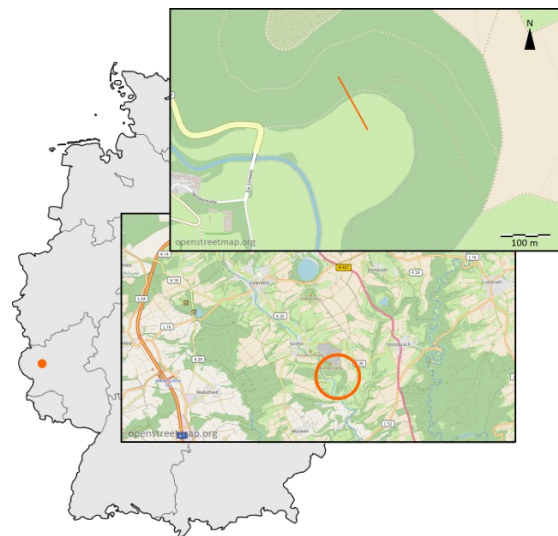
Model 3D – Buried box beside profile



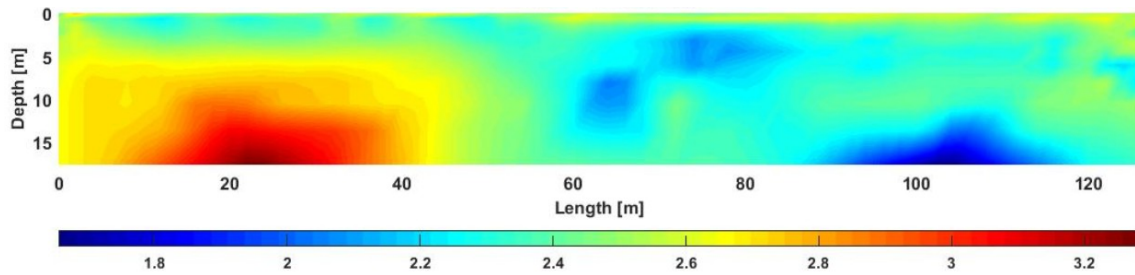
The conductor beside the profile would create a resistive anomaly in a classical pseudo-section. The ellipses reveal the conductor aside the profile.

Field Example

The following data set was collected at the Sprink Maar (Eifel, Germany) in June 2019 using the quadrupole ERT method with three overlapping 62 m double-profiles with 64 electrodes each, yielding 126 m in total. The profile was oriented perpendicular to the geologic boundary consisting of the volcanic rim and the Maar interior sediments. The data was acquired with our 64-channel CR-Device. The CR-Device is an insitute-built geoelectric field instrument capable of sampling 64 input channels simultaneously with a sampling rate up to 2 kHz. All voltages are measured towards a common reference (CR) electrode. This approach enables the calculation of potential differences between any given pair of electrodes from the original dataset. Further, any channel can be used for current injection.



The dataset reflects a structure which in general appears 2D as the major and minor axes line up with the profile direction . Minor deviations are the result of small scale anomalies. We used a 2D inversion code and fitted the major and minor axes of the DARTs. The inversion result is depicted below.



The high resistive domain to the left of the model corresponds to the pyroclastic material of the crater rim. The transition to the conducting Maar interior is sharp and well defined. The interior consists of sands and clays. The latter are responsible for the low resistivities .

#### **Conclusion and Outlook**

The method of Quadrupole ERT bears still a lot unexplored details which need deeper investigations. One major point to elaborate is the principle of reciprocity. Our studies found out that by interchanging current and potential electrodes the apparent resistivity tensor elements may change sign. Further, analysis of the behaviour of the DART skew angle may reveal more information about subsurface conductivity structures.