Sparse Joint Inversion of Electromagnetic and Electrical Resistivity Data

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Introduction

- DFG Deutsche Forschungsgemeinschaft German Research Foundation funded project
- Aim: Inversion electromagnetic data, based on a **sparse joint-inversion** scheme using **meshless discretization** techniques

Forward Scheme

- meshless discretization
- singular sources
- primary field

Inversion Algorithm

- sparse joint inversion
- coupling through **Jac.**
- general scheme (DC / CSEM / RMT ...)

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...Work in progress...

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Introduction

- **Xiang Li et al. 2016:** Modified Gauss-Newton full-waveform inversion explained — why sparsity-promoting updates do matter, GEOPHYSICS 81 (3)

'... the constrained model updates remained descent directions, removed subsampling-related artifacts, and improved the overall inversion result...'

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'… the constrained model updates remained descent directions, removed subsampling-related artifacts, and improved the overall inversion result…'

- Improve ,overall inversion quality'
- 'Coupling' in the joint inversion by means of sparse model updates

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'… the constrained model updates remained descent directions, removed subsampling-related artifacts, and improved the overall inversion result…'

- Improve ,overall inversion quality'
- 'Coupling' by means of sparse model updates

→ Compressed Sensing

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Compressed sensing

$$Ax = b$$

Compressed sensing

$$A\Delta m = b$$

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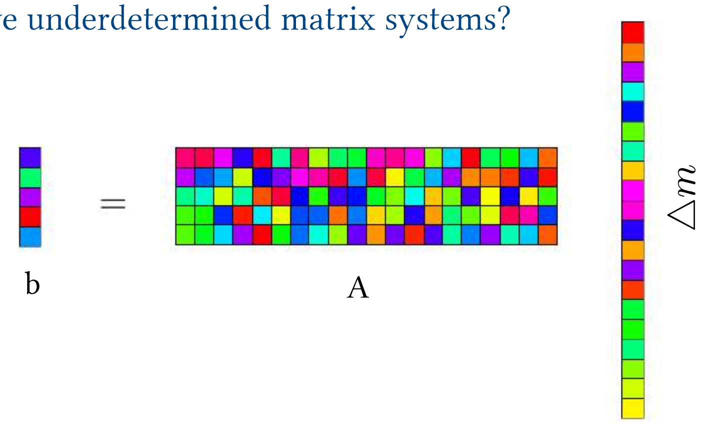
Compressed sensing I

- How to solve underdetermined matrix systems?



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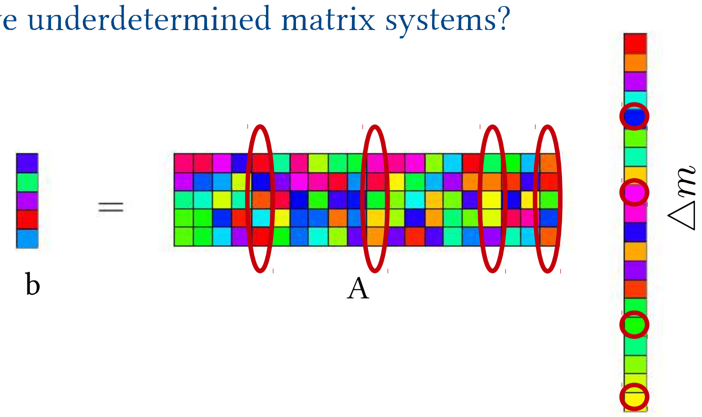
- How to solve underdetermined matrix systems?



- Which parts of A do I need to successfully recover $\triangle m$?

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Compressed sensing I

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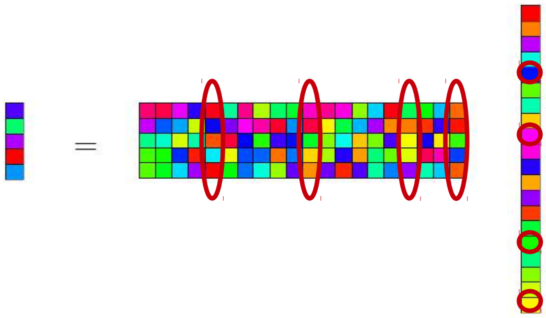

- Successful recovery depends more on its inherent **information content** than on the desired resolution. *(Candès, E.; Romberg, J.; Tao, T. , 2006)*

  $\longrightarrow$ Sub-Nyquist Sampling

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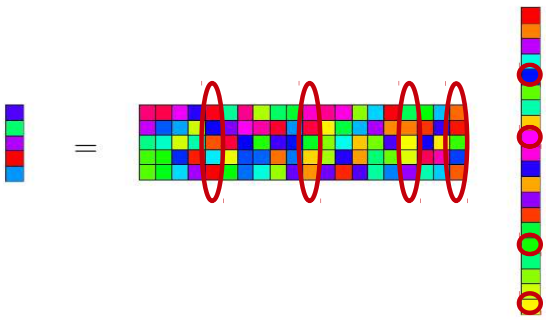

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  $\longrightarrow$ Sub-Nyquist Sampling

- Various algorithms: $\min \ ||\mathbf{x}||_1$ st. $\mathbf{Ax} = \mathbf{b}$ $\longrightarrow$ **sparse solution** (greedy, linear progamming, non smooth optimization etc. ...)

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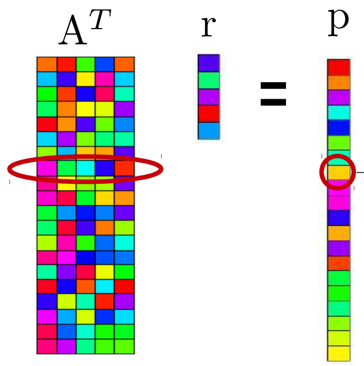
CS II – (Orthogonal) Matching Pursuit

Correlate residual
with sensing matrix
$\longrightarrow$ signal proxy



$r = b - A\widehat{\Delta m}$

$\langle a_k, r\rangle = p_k$

Select largest
correlation

Enlarge support set:

$\Omega_k = supp(p_k)\bigcup t$

Invert over support:

$d_k = A^{\dagger}_{\Omega_k} b$

Truncate and compute
Estimate:

$t_k = supp(p_k)$

$\widehat{\Delta m} = d_k$

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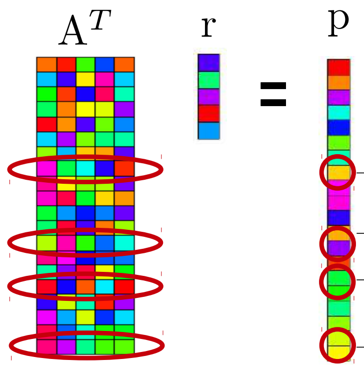

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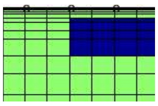
2d Inversion – normal - Gauss-Newton

$(\rho_a(f), \varphi(f))$

(ρ_a)



$$\phi^\beta(\mathbf{m},\mathbf{d}) = \phi(\mathbf{m},\mathbf{d}) + \beta\phi_{\mathbf{W}}\ (\mathbf{m})$$
$$= \frac{1}{2}||\mathbf{F}[\mathbf{m}] - \mathbf{d}||_2^2 + \beta||\mathbf{W}(\mathbf{m} - \mathbf{m}_{ref})||_2^2 \quad \rightarrow \min_{\mathbf{m}}\ \phi^\beta(\mathbf{m},\mathbf{d})$$

$$\begin{bmatrix} \mathbf{J}\left(\mathbf{m_k}\right) \\ \sqrt{\beta}\mathbf{W} \end{bmatrix} \triangle\mathbf{m} = \begin{bmatrix} \mathbf{d} - \mathbf{F}\left[\mathbf{m_k}\right] \\ -\sqrt{\beta}\mathbf{W}\left(\mathbf{m}_k - \mathbf{m}_{ref}\right) \end{bmatrix}$$

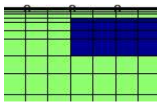
$$\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha\triangle\mathbf{m}$$

Adjoint approach:

$$\mathbf{J}\left(\mathbf{m}\right) = -\mathbf{P}\frac{\partial E_x}{\partial\mathbf{m}}$$

$$\left.\frac{\partial E_x}{\partial\sigma_k}\right|_{x_j} = \int_D \mathbf{E}^\dagger \cdot E_x\phi_k\ dv$$

$$\phi_{\nabla^2_m} = ||\nabla^2(\mathbf{m} - \mathbf{m}_{ref})||_2^2$$



Implementation based on:

Candansayar, M.E., & Tezkan, B. 2008. Geophysical Prospecting, 56

Candansayar, M.E., 2008. Journal of Geophysics and Engineering, 5

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2d Inversion – sparse

$$\phi^\beta(\mathbf{m},\mathbf{d}) = \phi(\mathbf{m},\mathbf{d}) + \beta\phi_{\mathbf{W}}\ (\mathbf{m})$$
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- sparse model update

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$$\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha\triangle\mathbf{m}$$

$$\underset{\triangle\mathbf{m}}{\text{minimize}} \quad ||\triangle\mathbf{m}||_1$$

$$\text{subject to} \quad \min_{\mathbf{m}} \phi^{\beta}(\mathbf{m},\mathbf{d})$$

(Gulliksson & Oleynik, 2017)

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$$\begin{bmatrix} \mathbf{L}_k \\ \sqrt{\beta}\mathbf{W}_{\mathbf{\Omega_k}} \end{bmatrix} \triangle\mathbf{m} = \begin{bmatrix} \mathbf{d} - \mathbf{F}\left[\mathbf{m_k}\right] \\ -\sqrt{\beta}\mathbf{W}_{\mathbf{\Omega_k}}\left(\mathbf{m}_k - \mathbf{m}_{ref}\right) \end{bmatrix}$$

$$\mathbf{L}_k = \mathbf{J}\left(:, \Omega_k\right)$$

subset of model parameters

$$loop: \ [\Omega_{i+1} = \Omega_i \cup t_{max}]$$

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2d Inversion – sparse

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$$t_{max} = \underset{t\in\bar{\Omega}_i}{\text{argmax}} \left| \mathbf{r}_i^T \frac{\mathbf{J}(:,t)}{||\mathbf{J}(:,t)||}\delta_z \right|$$

$(\delta_z \propto exp(-z))$

$\mathbf{r}_i^T$ → residual of (1)

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$$\underset{\triangle\mathbf{m}}{\text{minimize}} \quad ||\triangle\mathbf{m}||_0 \qquad \text{subject to} \quad \min_{\mathbf{m}} \phi^\beta(\mathbf{m},\mathbf{d})$$

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Compressed Sensing
Orthogonal matching pursuit

$$\min ||\mathbf{x}||_1 \;\; \text{st.} \;\; \mathbf{Ax} = \mathbf{b}$$

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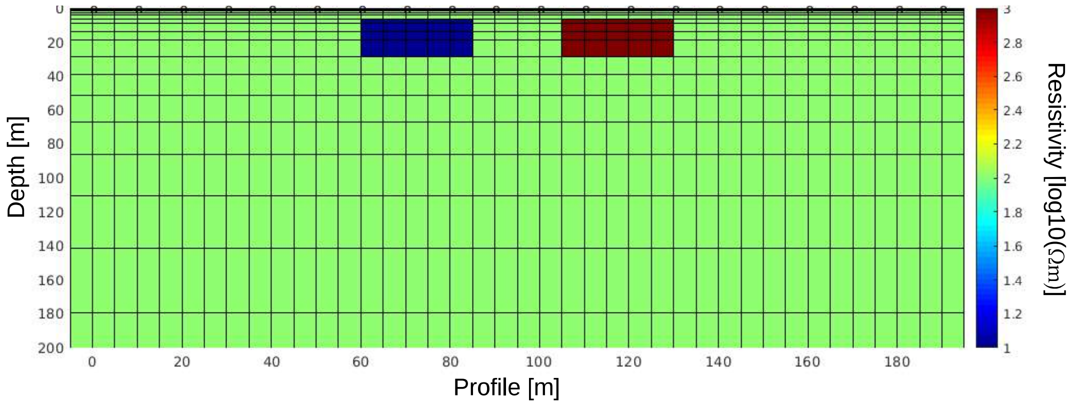
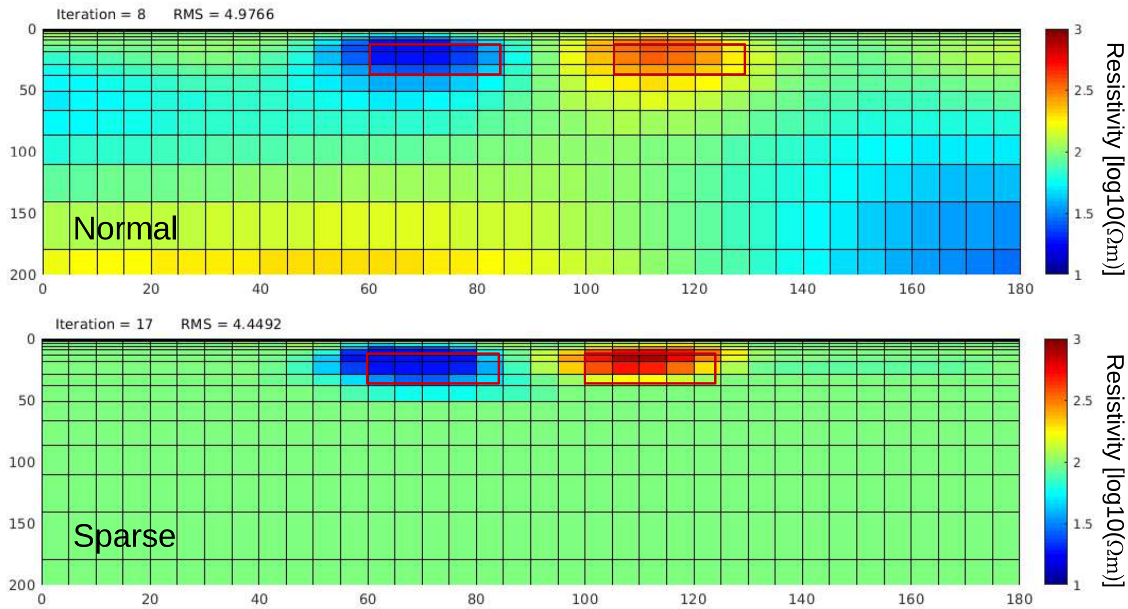
2d sparse inversion – model



- Radiomagnetotelluric (te-tm) / DCR Dipole-Dipole
- 20 stations, 10 m separation
- synthetically generated data (w/ 5% Gaussian noise)
- 8 RMT frequencies: 256000 Hz ... 500 Hz
- 840 model parameter

2d Sparse inversion vs. normal – single DC



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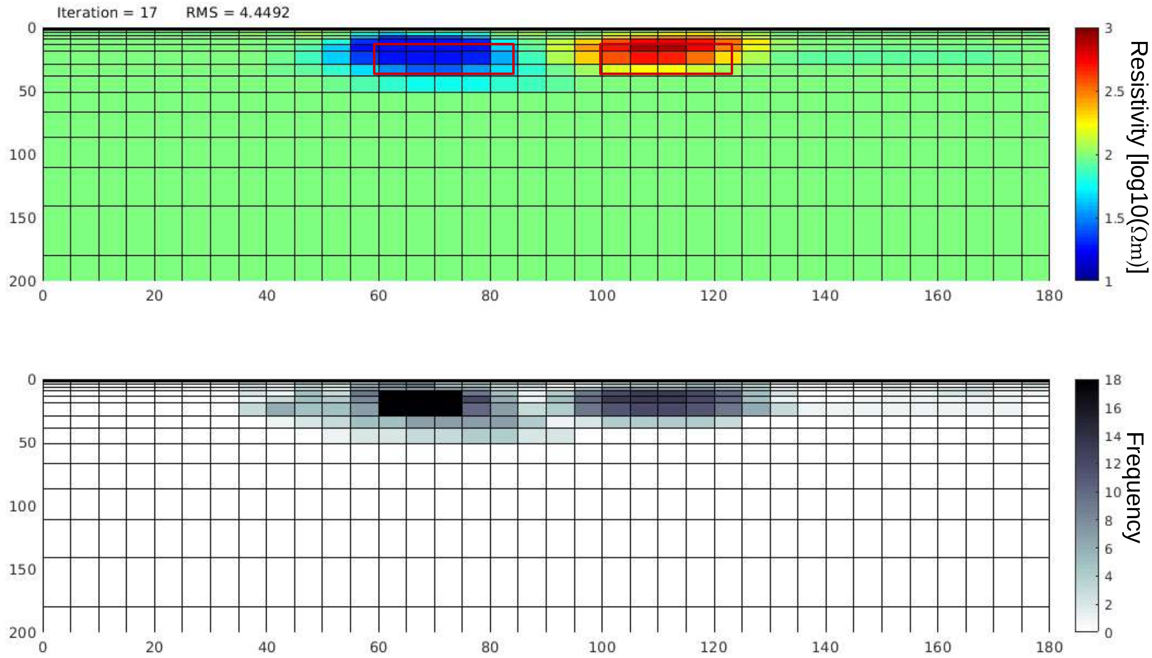

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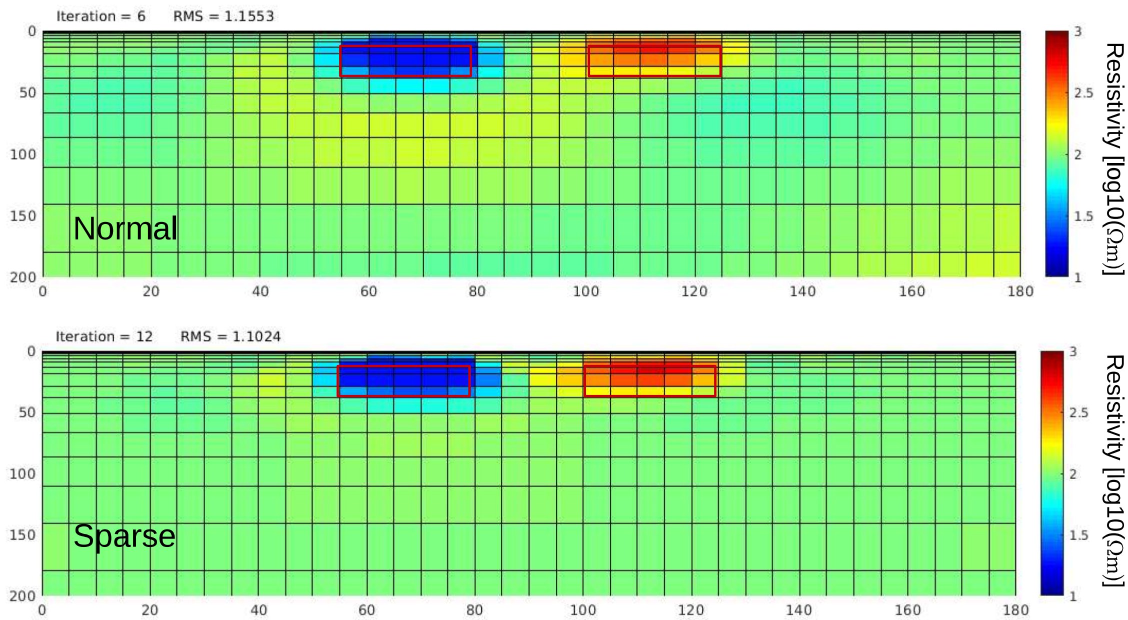
2d Sparse inversion vs. normal – single RMT



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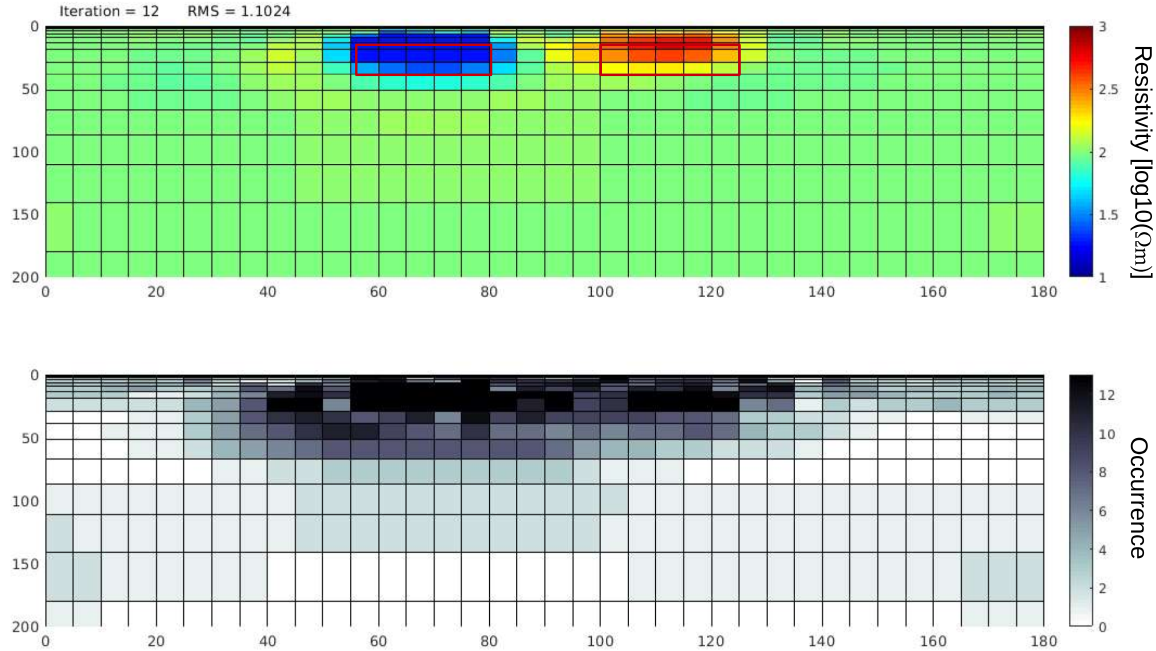
2d Sparse inversion – single RMT



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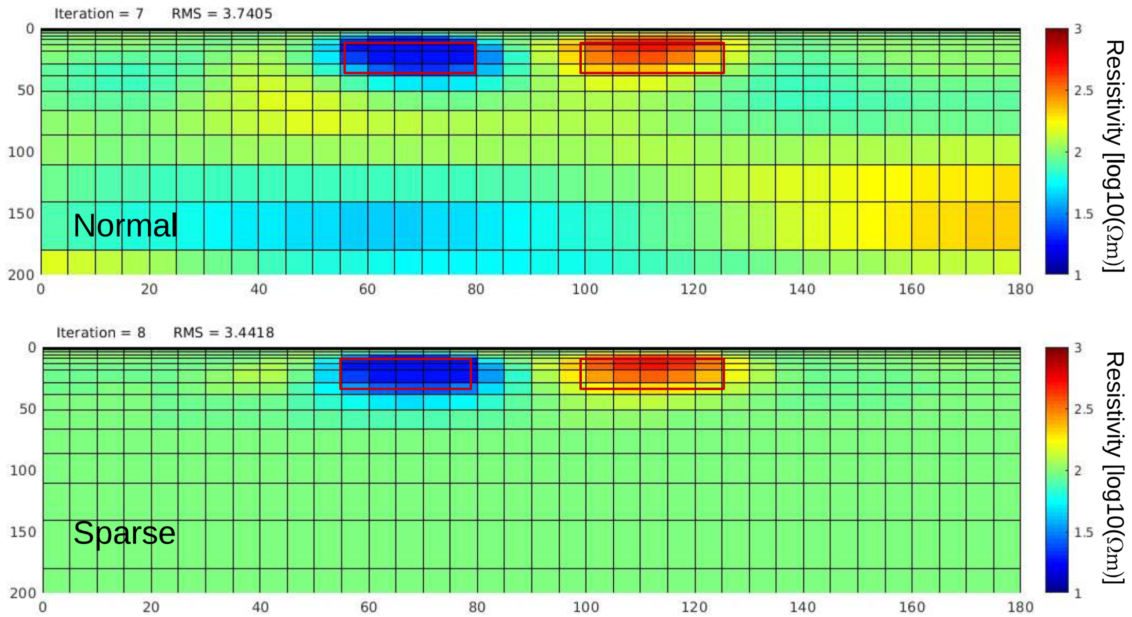
2d Sparse inversion vs. normal – **joint**



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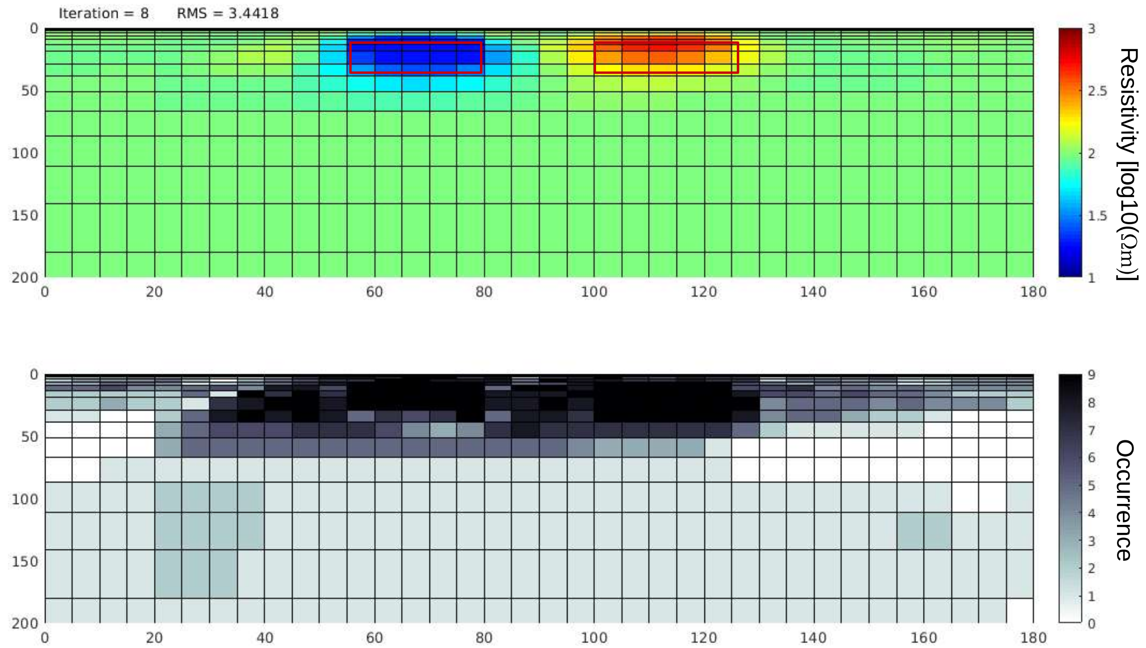
2d Sparse inversion – **joint**



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Conclusion - Outlook

- developed a two-dimensional inversion scheme for sparse joint inversions
- greedy approach, works with simple heuristics
- convergent, also with noisy data ⟶ enhances inversion quality
- gives promising results for simple synthetic models using only a fraction of full Jacobian ⟶ also for joint applications

- developing two-dimensional meshless DC/CSEM simulation algorithms
  - work in progress ...
- developing a two-dimensional (meshless) joint inversion algorithm
  - different coupling between methods ⟶ row and column selection
  - more studies, other omp implementations ...

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Thank You!

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