3D CSEM Forward Modelling

Key features and first results of our recently developed finite-element code

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Introduction

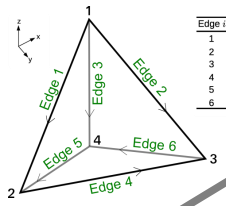
- **3D controlled-source electromagnetic modelling** code in **frequency domain**
- Using **unstructured tetrahedral meshes** of **finite-elements**
- First order **Nédélec basis functions** - vector edge interpolation functions [1]
- **Dirichlet boundary conditions**
- Model parameters: **electric resistivity** and **magnetic permeability**
- Calculation of the electric fields in an edge-based manner
- **Curl-Curl-Equation** for the electric field (**E**) with time dependency of $e^{i\omega t}$:

$$\nabla \times \frac{1}{\mu} \nabla \times \mathbf{E} - i\omega \frac{1}{\rho} \mathbf{E} = i\omega \mathbf{J}_{source} \quad (1)$$

- Planned to be incorporated in the inversion framework EMILIA [2] as a 3D module

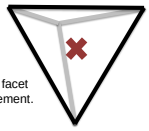
Achievements

Key Features



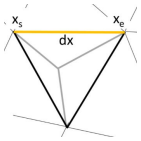
Edge i	Node i_1	Node i_2
1	1	2
2	1	3
3	1	4
4	2	3
5	4	2
6	3	4

Receiver implementation



The receivers are located in the middle of the air-earth facet of the receiver-earth element.

Source implementation
Example: HED in x-direction



Extended HEDs and magnetic loop sources are implemented following the same concept.

$$b_i^e = i\omega \iiint_V \mathbf{N}_i^e \cdot \mathbf{J}_{source}\, dV \quad (2)$$

Using: $\mathbf{J}_{source} = \vec{x}\, I \mathscr{H}(x_s, x_e)\delta(y_s)\delta(z_s)$ (3)

$$b_i^e = i\omega \iiint_V \mathbf{N}_i^e \cdot \vec{x}\, I \mathscr{H}(x_s, x_e)\delta(y_s)\delta(z_s)\, dV \quad (4)$$

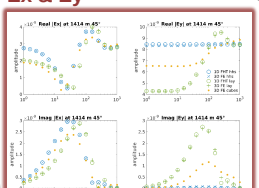
Using: $\iiint_V \mathbf{F} \cdot \vec{x}\, \mathscr{H}(x_s, x_e)\delta(y_s)\delta(z_s)\, dV = \int_{x_s}^{x_e} \mathbf{F}\vec{x}(x, y_s, z_s)\, dx$ (5)

$$b_i^e = i\omega I \int_{x_s}^{x_e} \mathbf{N}_i^e \vec{x}(x, y_s, z_s)\, dx \quad (6)$$

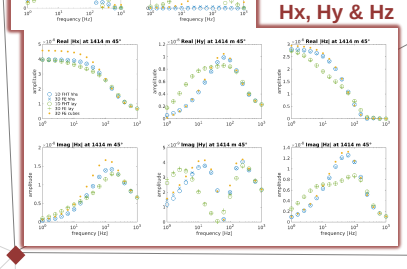
similar to [3]

The 3D forward modelling is stable over three frequency decades using the same mesh.

Ex & Ey

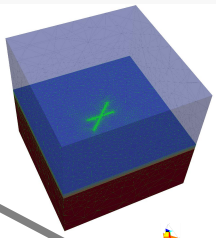


Hx, Hy & Hz



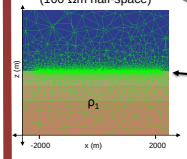
The relative errors of all five field components are below 4 % at 1414 m distance and 45° angle to the HED source.

Testing Model



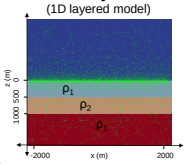
TetGen

"hom"
(100 Ωm half-space)



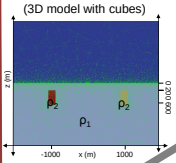
38 inline receivers

"lay"
(1D layered model)



$\rho_1 = 100\ \Omega m$
$\rho_2 = 10\ \Omega m$

"cubes"
(3D model with cubes)



refined mesh around source, receivers and anomalies

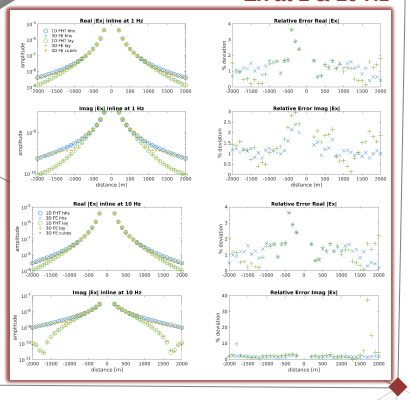
Model dimension:
10 x 10 x 10 km

Mesh [4]:
~60000 dof

The open source solver PARDISO [5] is applied to solve the system of equations.

A semi-analytic solution utilising the Fast Hankel transform (FHT) from EMILIA [2] is used for comparison.

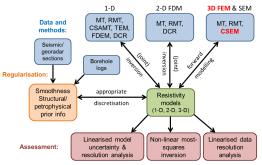
Ex at 1 & 10 Hz



45°

inline

implement into EMILIA [2] framework



Code validation

Code validation

Outlook

References

[1] J. Jin. The Finite Element Method in Electromagnetics. Wiley-IEE Press, 3th edition, 2014.
[2] T. Kalscheuer, M. d. l. A. Garcia Juanatey, N. Meqbel, L. B. Pedersen. Non-linear model error and resolution properties from two-dimensional single and joint inversions of direct current resistivity and radiomagnetotelluric data. Geophys. J. Int., 182(3):1174-1188, 2010.
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Acknowledgements

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