3D Boundary Conditions in Finite-Element Electromagnetic Forward Modelling: First Results

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Basics

Goal: Develop a 3D inversion code for MT data, based on:

3D forward modelling code [1]
general inversion framework EMILIA [2]

Details of the forward modelling code:

- Edge-based FEM, unstructured tetrahedral mesh (Tetgen [3])
- Goal-oriented mesh refinement, guided by discontinuity of $\mathbf{J}$
- Curl-curl equation of total $\mathbf{E}$, direct LU-solver PARDISO [4]

The boundary value problem in frequency domain for MT:

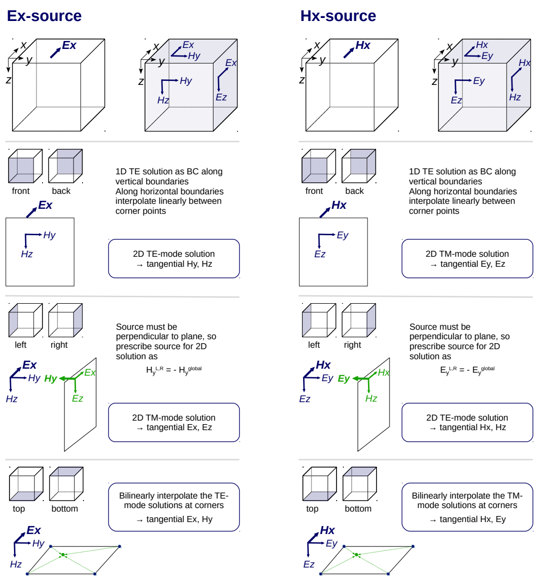
$$\begin{aligned} \nabla \times \frac{1}{\hat{z}} \nabla \times \mathbf{E} - \hat{y}\mathbf{E} &= \mathbf{0} \quad \text{in} \quad \Omega, \\ \hat{\mathbf{n}} \times \frac{1}{\hat{z}} \nabla \times \mathbf{E} &= \mathbf{g}_t \quad \text{on} \quad \partial\Omega, \end{aligned} \tag{1}$$

with $\hat{z} = -i\omega\mu$, $\hat{y} = \sigma - i\omega\epsilon$, $\mathbf{g}_t = \hat{\mathbf{n}} \times \mathbf{H}_0$ and $\mathbf{H}_0$ the plane wave solution for the background model.

What's new: $\mathbf{H}_0$ can be the plane wave solution for **3D background model**.

Boundary Conditions

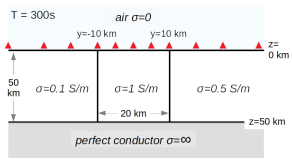
- calculate 2D solution for independent models at all 4 sides
- to obtain full solution, always solve for 2 source polarisations
- use only tangential fields as boundary conditions for 3D problem [5]



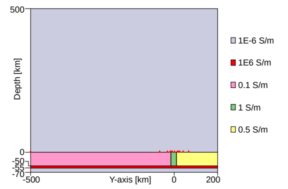
Then interpolate the recovered fields onto the boundary nodes of the 3D mesh to use as boundary conditions.

First Results

Validation: compare to semi-analytic solutions for a specific model [6], [7]

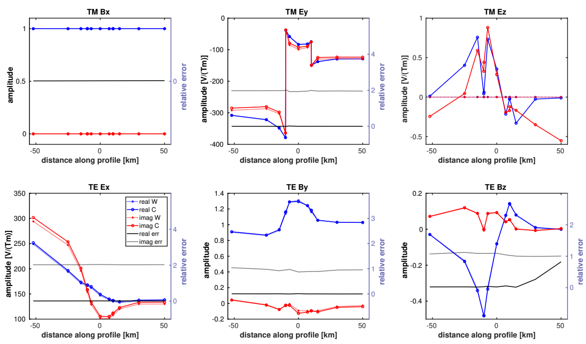


Model used by [6] and [7]



Our model in 2D, in 3D: model is elongated along third dimension

Compare 2D solution, that is used at the boundaries:



Comparison of Weaver's solution (dashed lines) to ours (solid lines), with relative error (black lines); Plotted fields are normalised by a reference B_0

Our 2D solution matches Weaver's solution, but our 3D solution still shows some differences (probably due to too coarse mesh, that we used so far).

Outlook

- Compare 3D model results of finer mesh to semi-analytic results
- Combine the 3D forward modelling code with inversion framework EMILIA [2]

References

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