Adaptive finite element modelling for 3D MT/RMT problems in anisotropic media

Huang Chen[1,2], Zhengyong Ren[1], Thomas Kalscheuer[2], and Jingtian Tang[1]
[1]School of Geosciences and Info-Physics, Central South University, Changsha, China
[2]Department of Earth Sciences, Uppsala University, Uppsala, Sweden
csuchenhuang@csu.edu.cn

UPPSALA UNIVERSITET

Introduction

Isotropic inversions in an anisotropic medium can result in severe artifacts and lead to erroneous interpretations. Hence, to achieve appropriate interpretation for realistic Earth models, forward algorithms have to allow for an anisotropic subsurface.

Recently, the adaptive finite element method (FEM) has been used in solving anisotropic MT forward problems [1]. However, to our best knowledge, it has not been used in solving anisotropic RMT problems allowing for both anisotropic conductivity and permittivity yet. Consequently, based on the previous work in isotropic case [2], we develop a goal-oriented finite element approach for MT and RMT problems allowing for anisotropic distribution of both conductivity and permittivity.

Methods

Boundary value problem for MT and RMT problems

The boundary value problem of electric field is given as

$$\begin{aligned} \nabla \times \frac{1}{\hat{z}} \nabla \times \mathbf{E} - \hat{\mathbf{y}}\mathbf{E} = \mathbf{0} \quad &\text{in} \quad \Omega, \\ \hat{\mathbf{n}} \times \frac{1}{\hat{z}} \nabla \times \mathbf{E} = \mathbf{g}_t \quad &\text{on} \quad \partial\Omega, \end{aligned} \tag{1}$$

where $\hat{z} = -i\omega\mu$ is the impedivity, $\hat{\boldsymbol{y}} = \boldsymbol{\sigma} - i\omega\epsilon_0\boldsymbol{\epsilon_r}$ is the admittivity **tensor** and $\mathbf{g}_t = \hat{\mathbf{n}} \times \mathbf{H}_0$, $\mathbf{H}_0$ is the analytical solution of a 1D layered Earth model.

Edge-based FEM approximation with unstructured mesh

By using tetrahedral mesh and edge-based Galerkin FEM, the boundary value problem in Eq. (1) is discretized into a system of linear equations

$$\mathbf{KE} = \mathbf{F}, \tag{2}$$

where $\mathbf{E} = \{E_i\}, i = 1, 2, ..., N_e$, N_e is the number of total edges in the discretized tetrahedral mesh, and

$$K_{ij} = \iiint_{\Omega} \nabla \times \mathbf{N}_i \cdot \frac{1}{\hat{z}} \nabla \times \mathbf{N}_j dv - \iiint_{\Omega} \mathbf{N}_i \cdot \hat{\mathbf{y}}\mathbf{N}_j dv, \tag{3}$$

$$F_i = \iint_{\partial\Omega} \mathbf{N}_i \cdot \mathbf{g}_t ds. \tag{4}$$

To guarantee the accuracy, the direct LU solver is used to solve Eq. (2).

Adaptive mesh refinement

Two a-posterior error estimators based on the discontinuities of fields are chosen to guide the mesh refinement. One is $[\eta_e^{\mathbf{J}}]$ named error estimator J:

$$[\eta_e^{\mathbf{J}}]^2 = \sum_{i=1}^{4} \frac{1}{2} \iint_{F_i} |\hat{\mathbf{n}} \cdot (\hat{\mathbf{y}}_- \mathbf{E}_- - \hat{\mathbf{y}}_+ \mathbf{E}_+)|^2 ds, \tag{5}$$

and another one is $[\eta_e^{\mathbf{H}}]$ named error estimator H:

$$[\eta_e^{\mathbf{H}}]^2 = \sum_{i=1}^{4} \frac{1}{2} \iint_{F_i} \left| \hat{\mathbf{n}} \times (\frac{1}{z_-} \nabla \times \mathbf{E}_- - \frac{1}{z_+} \nabla \times \mathbf{E}_+) \right|^2 ds, \tag{6}$$

where F_i is the $i-th$ triangular face enclosing the tetrahedral element e. To avoid unnecessary refinement, the goal-oriented strategy [2] is enforced.

Results

Algorithm validation and performance

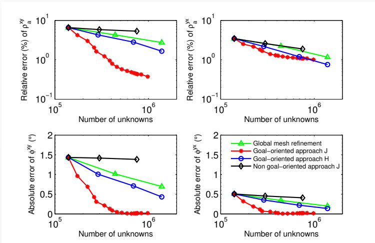

Figure 1: Errors of ρ_a and ϕ responses in terms of different mesh refinement strategies.

A horizontal anisotropic half-space model is used.

- $f = 10\,Hz$
- $\rho_x/\rho_y/\rho_z/ = 10/100/100\,\Omega m$
- $\Omega = [-10\,km, 10\,km]^3$

The results are compared with closed-form solutions: $\rho_a^{xy} = 10\,\Omega m$, $\rho_a^{yx} = 100\,\Omega m$, $\phi^{xy} = -45°$ and $\phi^{yx} = 135°$.

Effect of anisotropic permittivity on RMT response

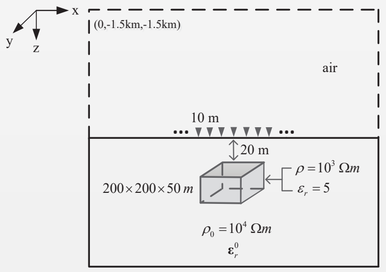

Figure 2: Geometry of a conductive rectangular prism buried in a resistive half-space with permittivity changed from isotropic to horizontal anisotropic.

- $f = 100\,kHz$
- $\epsilon_r^0 = 5 \rightarrow 5/20/5$
- $\Omega = [-1.5\,km, 1.5\,km]^3$
- Sites located in: $[-300\,m,\ 300\,m]$
- Unknowns: 1296640

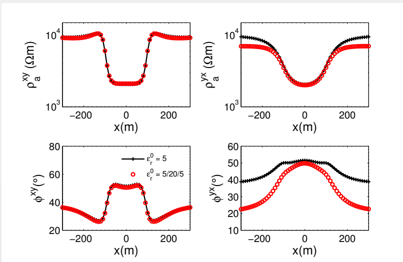

Figure 3: Comparison of apparent resistivities and phases.

The deviation between isotropic and anisotropic model is clearly visible in the yx-mode. The reason is that the responses of yx-mode are mainly influenced by the components of E_y which is changed with $\epsilon_{r,y}^0$ varied from 5 to 20.

Conclusion

We have developed a goal-oriented finite element approach for 3D MT and RMT problems allowing for anisotropic distributions of both conductivity and permittivity. The anisotropic half-space model for MT problem validates the presented algorithm and shows the performances of different mesh refinement strategies. The rectangular prism model with varied permittivity shows different effects of anisotropic permittivity on RMT responses in different modes.

References

[1] Ying Liu, Zhenhuan Xu, and Yuguo Li. Adaptive finite element modelling of three-dimensional magnetotelluric fields in general anisotropic media. *Journal of Applied Geophysics*, 151:113 – 124, 2018.

[2] Zhengyong Ren, Thomas Kalscheuer, Stewart Greenhalgh, and Hansruedi Maurer. A goal-oriented adaptive finite-element approach for plane wave 3-D electromagnetic modelling. *Geophysical Journal International*, 194(2):700–718, 05 2013.

©Po-Cheng Tang