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INVESTIGATIONS OF THE GRAVIMETRIC TIME SERIES OF THE GEODYNAMIC OBSERVATORY POTSDAM

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Contents

Preface

Temporal variations of gravity are an important tool for the investigation of geodynamic processes in different scales both in space and in time. More than eight years of continuous registrations of gravity variations performed at the Geodynamic Observatory Potsdam of the Central Institute for Physics of the Earth of the Academy of Sciences of the GDR were available both for the study of the whole tidal spectrum including the small constituents in the region of the diurnal resonance phenomenon and for the derivation of the parameters of the main tidal waves and its temporal characteristics and for special studies of the behaviour and the quality of the instrumentation. First impressions on the possible temporal variations of some tidal parameters could be obtained.

To get an idea on the possibilities for the determination of zonal tides and gravity variations with longer periods a detailed investigation was undertaken to find out the most reliable parameters for the fortnightly tide of the moon. By these studies clearly could be shown that the irregular parts in the instrumental drift of the gravimeter limit the possibilities for the detection of real gravity variations below an amplitude of about ten microgals and with periods longer than a few weeks. The influence of non-tidal processes is disturbing the zonal tidal variations too and required special investigations.

The stimulating support by the Director of the Central Institute for Physics of the Earth, Prof. Dr. habil. H. Kautzleben, and the helpful cooperation with the Computing Centre of the Central Institute for Astrophysics of the Academy of Sciences of the GDR are kindly acknowledged by the authors.

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Results of an eight years gravimetric earth tide registration series at Potsdam

by

HANS-JÜRGEN DITTFELD

Z u s a m m e n f a s s u n g

Eine gravimetrische Gezeitenmessung von März 1974 bis Februar 1982 mit dem Askania-Gravimeter GS 15 Nr. 222 lieferte 63 341 stündliche Meßwerte in 13 Datenblöcken. Zur Identifikation und Verbesserung von Fehlmessungen diente eine spezielle Methode, die auf dem Vergleich der Meßwerte mit denen einer synthetischen Gezeitenkurve beruht. Von insgesamt 45 Wellengruppen werden Gezeitenparameter mitgeteilt, deren Fehler nach CHOJNICKI aus dem Spektrum der Residuen nach der harmonischen Analyse berechnet wurden. Die Fehler für die Amplitudenfaktoren der Hauptwellen liegen bei ± 0.0002 bis ± 0.0008; die der zugehörigen Phasen zwischen ± 0.01° und ± 0.04°. Der Fehler des einzelnen Meßwertes beträgt ± 0.66 µGal = ± 6.6 nm s^{-2}.

Die abschließenden Resultate der Langzeitmessung werden mit entsprechenden Werten anderer Stationen verglichen, insbesondere im tagesperiodischen Teil des Gezeitenspektrums, wo der Resonanzeffekt des flüssigen Erdkerns die Amplituden beeinflußt. Gleitend verschobene Teilanalysen des Beobachtungsmaterials liefern zeitliche Variationen der Ergebnisparameter, unter anderem eine auffällige Korrelation der Amplitude der Hauptwelle O1 mit der Tageslänge.

S u m m a r y

Digital Earth tide records were carried out between March 1974 and February 1982 at the Gravimetric Observatory Potsdam using the Askania gravimeter GS 15 No. 222. Altogether 63 341 hourly readings subdivided in 13 data blocks were analyzed after the elimination of misreadings by comparison of measured values and predicted ones generated by the synthesis of the tidal effect using the measured parameters.

The inner accuracy of the main tidal amplitude factors amounts to ± 0.0002 ... ± 0.0008, those of the phases to ± 0°.01 ... ± 0°.04, calculated by the aid of CHOJNICKI's harmonic analysis containing the error estimation based on the spectrum of the residuals. The yielded mean square error was m_0 = ± 0.66 microgal.

The final result is compared with the mean values at other stations especially in the diurnal part of the tidal spectrum, where the resonance of the liquid outer core of the Earth influences the results. The temporal variations of the tidal parameters were determined by overlapping partial analyses. Each of them contained more than 10 000 readings and they succeded one another in steps of about 90 days. The results were com-

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pared with those of corresponding analyses of the series of two other Earth tide stations.

1. The measurement

The Askania gravimeter GS 15 No. 222 was installed on a special pillar in a cellar room of the main building of the Central Earth Physics Institute of the Academy of Sciences of the GDR in Potsdam. The room was air-conditioned and the temperature was kept constant to about ± 0.1 K, except during the very rare control measurements. No seasonal variations of the temperature were observed but the air-humidity have had fluctuations between 30% and 70%. All the registration devices were situated in another room and also the calibrations of the registration were done by remote control without entering the gravimeter room.

The RC-filtered output (τ = 18,778 sec.) was measured by means of a digital voltmeter with a scale value of about

0.0289 microgal/digit. *)

Triggered by the atomic clock of the time service of the institute, fourdigit readings are punched every five seconds during the last three minuts of every hour. The hourly values for the further processing are calculated from the 36 single readings by the aid of a smoothing parabola with errors almost smaller than ± 0.1 microgal. For a direct control of the measurement, an analogous recorder was connected parallel to the digital voltmeter.

From March 20, 1974 until February 15, 1982 there were only 12 interruptions of the registration, the longest one for 180 days because of joint measurements with other gravimeters at Pecny station/ČSSR and for 30 days because of special registrations in the laboratory.

Calibrations of the recording were carried out three times a month in the first years but later on monthly. Therefore the electromagnetic calibration device in the gravimeters was used. Its transforming coefficient of

$$k = (0.42335 \pm 0.00078) \text{ microgal/microampere}$$

was defined by comparison of the results of 39 spindulum calibrations with those of the simultaneous electromagnetic calibrations. The scale of the measuring screw of the gravimeter is known from measurements at the Czechoslovakian National Gravimetric Calibration Base in 1975 [7]. Because the error of these field calibrations is ± 0.03 % only the relative error $m_k / k = 1.84 \cdot 10^{-3}$ is mainly limiting the absolute accuracy of the final results.

The drift of the zero position of the gravimeter beam was about + 16.53 milligal during the total measuring period corresponding to an average of 5.5 microgal per day. But there have been parts of negative drift occurring in every winter during the minimum of the air humidity at the station. So the drift may be described by a polynom and a number of waves the biggest one of which shows a yearly period [4].

*) 1 microgal = 10^{-8} m . sec^{-2}

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2. Data processing

After elimination of the usual zero displacements caused by the drift the measured values were transformed to microgals using linear interpolated scale values between consecutive calibrations. So the occurring random variations of the scale of the registration are almost included in the errors of the tidal parameters calculated later on.

Differences were calculated between the measurement and the corresponding values of an Earth tide synthesis which was generated using the results (amplitude factors and phases) of former analyses. The curve of these differences contains the drift and all the non-tidal effects registrated yet. By the aid of these residual curves ('Restkurven') the misreadings, caused for instance by earthquakes, electric disturbances or calibrations, may successfully be detected and oorrected with an accuracy of better than 0.5 microgal. The Restkurve is useful also for the interpolation of interruptions of the registration up to several hours.

Finally the corrected series was analyzed by CHOJNICKI's programme A15H where the drift is eliminated by the PERTZEV filtration or by the zero point method in the case of analyses with longperiodic tidal waves. The error estimation in this programme basing on the spectrum of the residuals after the harmonic analysis yields to about the same error values like VENEDIKOV's M74-programme [8] and may be regarded equivalent. For the registration of the GS 15 No. 222 the errors of the main diurnal parameters were calculated about 2.25 times bigger than by the former 'classical' programme of CHOJNICKI, but the errors of the semidiurnal parameters are nearly the same because of the very small semidiurnal noise.

3. Analysis results

The final result for 45 wave groups is given in tables I and II. The additional column entitled 'corr.' contains the phases corrected because of the instrumental phase lag:

$$C = \arctan \omega\tau + \frac{360^\circ + \Delta t}{T}$$

τ = 18.778 sec.

ω - frequency of the tidal waves

T - period of the tidal waves

Δt = 18 sec., additional time shift not regarded during processing.

Averaged tidal parameters are often used for the calculation of tidal prognoses via a synthesis of the whole effect applying measured values for the different frequency bands. Weighted mean results for Potsdam are listed in table III. The weights were determined by the errors of the analysis on the one hand or by the amplitudes of the wave groups on the other. S1 parameters are not regarded in these mean values because of the big meteorologic influence. As to be seen it is inopportune in every case because of the increasing error to gather diurnal and semidiurnal parameters as a mean

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WYROWNANIE KONCOWE - FINAL ADJUSTMENT CHOJNICKI METHOD

KONCOWE WYNIKI OBLICZEN - OCENA DOKLADNOSCI NA PODSTAWIE RESIDUUM
FINAL RESULTS OF COMPUTATIONS - ESTIMATION OF ACCURACY BASED ON RESIDUAL

0.0 1978 731 0 0 1000111 52.3809 -13.0676 82 0.0 981.261400

STATION POTSDAM 0764 VERTICAL COMPONENT GERMAN DEMOCR.REPUBLIC

52 23 N 13 4 E H 82 M P 3 M D 250 KM
ZENTRALINSTITUT FUER PHYSIK DER ERDE POTSDAM H.J. DITTFELD
GRAVIMETER ASKANIA GS-15 NO. 222 DIGITAL
ELECTROMAGNETIC CALIBRATION SENSITIVITY 0.0289 MRCGAL/DIGIT
INSTALLATION H.J. DITTFELD
MAINTENANCE H.J. DITTFELD, W. ALTMANN

LEAST SQUARE ANALYSIS IN CLASSICAL MANNER (CHOJNICKI)
FILTRATION OF OBSERVATIONS / FILTER 51/ 1438
POTENTIAL CARTWRIGHT-EDDEN-(DOODSON) / COMPLETE EXPANSION
COMPUTATION - ZENTRALINSTITUT FUER ASTROPHYSIK.K.ARLT-POTSDAM - COMPUTER ES 1040

74 3 22 0- 74 4 13 21 / 74 4 18 0- 74 6 19 21 / 74 6 25 0- 74 7 13 21 / 74 7 19 0- 74 11 4
74 11 12 0- 74 12 14 21 / 75 6 22 0- 75 10 27 21 / 75 11 5 0- 76 1 6 21 / 76 1 12 0- 76 9 17
76 9 22 0- 76 11 1 21 / 76 12 2 0- 78 5 4 21 / 78 5 10 0- 81 2 9 22 / 81 2 14 21- 81 5 9
81 5 15 13- 82 2 14 4

TOTAL NUMBER OF DAYS 2886 62691 READINGS

WAVE GROUP ARGUMENT	N	SYMBOL	ESTIM. AMPL.	AMPLITUDE FACTOR VALUE	R.M.S.	corr.	PHASE DIFFERENCE VALUE	R.M.S.	SUM OF AMPLIT.
105.-11X.	14	SMQ1	0.1949	1.12065	0.05833	-3.85	-3.974	2.982	0.5334
124.-125.	7	2Q1	0.5886	1.23404	0.01807	+2.72	2.591	0.839	1.0204
126.-129.	14	SIG1	0.7176	1.16029	0.01480	-0.62	-0.756	0.731	1.2717
133.-135.	15	Q1	4.5624	1.15115	0.00230	-0.31	-0.445	0.114	7.3724
136.-139.	15	RO1	0.9185	1.15953	0.01197	-0.55	-0.687	0.591	1.5975
143.-146.	18	O1	24.4781	1.15212	0.00043	+0.00	-0.142	0.021	36.6771
147.-149.	8	TAU1	0.4768	1.15820	0.02364	+0.26	0.112	1.169	0.5713
152.-155.	15	M1	2.3456	1.13560	0.00421	+0.28	0.134	0.213	5.2269
156.-158.	7	CHI1	0.3523	1.13862	0.02806	+0.40	0.252	1.412	0.6162
161.-162.	3	PI1	0.7969	1.16790	0.01342	-0.14	-0.292	0.658	0.8605
163.-163.	7	P1	14.1969	1.18039	0.00079	+0.35	0.193	0.039	14.2105
164.-164.	3	S1	0.2399	0.81427	0.04670	97.38	97.224	3.287	0.4582
165.-165.	11	K1	37.3470	1.13970	0.00029	+0.20	0.042	0.014	49.1021
166.-166.	2	PSI1	0.3305	1.21009	0.03345	-0.69	-0.841	1.584	0.3432
167.-168.	7	PHI1	0.5654	1.22109	0.01817	+3.85	3.696	0.853	0.7072
172.-173.	6	THE1	0.3780	1.21510	0.02764	+0.26	0.097	1.303	0.5992
174.-177.	16	J1	1.8123	1.16824	0.00542	-0.41	-0.565	0.266	3.4588
181.-184.	8	SO1	0.2892	1.11888	0.03476	+2.54	2.380	1.780	0.5370
185.-186.	10	OO1	0.5512	1.15520	0.01317	+0.58	0.414	0.654	2.6169
191.-1E3.	19	NY1	0.1221	1.17328	0.06703	-1.05	-1.224	3.274	0.7871
207.-22X.	21	EPS2	0.2365	1.10466	0.02413	+3.14	2.857	1.252	0.5106
233.-236.	10	2N2	0.6447	1.17522	0.00758	+1.77	1.486	0.370	0.9611
237.-23X.	10	MY2	0.8403	1.15937	0.00614	+2.24	1.957	0.304	1.0177
243.-246.	17	N2	5.2046	1.17894	0.00097	+1.83	1.541	0.047	6.4211
247.-248.	7	NY2	0.9985	1.18364	0.00504	+1.40	1.106	0.244	1.1312
252.-258.	26	M2	29.0395	1.18406	0.00018	+1.20	0.907	0.009	29.7315
262.-264.	5	LMB2	0.2096	1.17393	0.02435	-0.53	-0.828	1.189	0.2347
265.-267.	12	L2	1.3542	1.14517	0.00506	+0.20	-0.099	0.253	1.8092
271.-272.	2	T2	0.7386	1.17304	0.00686	-0.87	-1.173	0.335	0.7980
273.-273.	4	S2	13.0467	1.18847	0.00040	+0.36	0.053	0.019	13.1179
274.-277.	12	K2	2.5554	1.18645	0.00181	+0.18	-0.124	0.087	5.0059
282.-285.	15	ETA2	0.1304	1.18074	0.03514	-0.25	-0.567	1.705	0.4496
292.-2X5.	14	2K2	0.0208	1.07422	0.12116	+4.65	4.331	6.461	0.2203
327.-375.	17	M3	0.2722	1.00808	0.00943	-0.55	-0.990	0.536	0.6127

R.M.S.ERROR M-ZERO 0.6572 MIKROGAL

R.M.S.ERROR FOR BANDS D 1.9584 SD 0.9294 TD 0.6245

O1/K1 1.0109 1-O1/1-K1 1.0889 M2/O1 1.0277

REFERENCE EPOCH 1978 7 31 0.0

Table I Final analysis result of the eight years tidal registration at Potsdam

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WYROWNANIE KONCOWE - FINAL ADJUSTMENT CHOJNICKI METHOD

KONCOWE WYNIKI OBLICZEN - OCENA DOKLADNOSCI NA PODSTAWIE RESIDUUM
FINAL RESULTS OF COMPUTATIONS - ESTIMATION OF ACCURACY BASED ON RESIDUAL

0,0 1978 731 0 0 1001111 52,3809 -1310676 82 0,0 981,261400

STATION POTSDAM 0764 VERTICAL COMPONENT GERMAN DEMOCR,REPUBLIC

52 23 N 13 4 E H 82 M P 3 M D 250 KM
ZENTRALINSTITUT FUER PHYSIK DER ERDE POTSDAM H,J, DITTFELD
GRAVIMETER ASKANIA GS-15 NO1 222 DIGITAL
ELECTROMAGNETIC CALIBRATION SENSITIVITY 0,0269 MKRGAL/DIGIT
INSTALLATION H,J, DITTFELD
MAINTENANCE H,J, DITTFELD, W, ALTMANN

LEAST SQUARE ANALYSIS IN CLASSICAL MANNER (CHOJNICKI)
DRIFT - ZERO POINT METHOD
POTENTIAL CARTWRIGHT-EDDEN-(DOODSON) / COMPLETE EXPANSION
COMPUTATION - ZENTRALINSTITUT FUER ASTROPHYSIK,K,ARLT-POTSDAM - COMPUTER ES 1040

74 4 21 8- 74 4 14 9 / 74 4 17 11- 74 6 20 5 / 74 6 24 17- 74 7 14 1 / 74 7 18 15- 74 11 5 6
74 11 11 17- 74 12 15 3 / 75 6 21 14- 75 10 27 22 / 75 11 4 19- 76 1 7 4 / 76 1 11 14- 76 9 18 0
76 9 21 12- 76 11 2 9 / 76 12 1 16- 78 5 5 5 / 78 5 9 17- 81 2 10 12 / 81 2 14 15- 81 5 9 8
81 5 14 18- 82 2 14 11

TOTAL NUMBER OF DAYS 2887 62915 READINGS

WAVE GROUP ARGUMENT	N	SYMBOL	ESTIM. AMPL.	AMPLITUDE FACTOR VALUE	AMPLITUDE FACTOR R.M.S.	corr.	PHASE DIFFERENCE VALUE	PHASE DIFFERENCE R.M.S.	SUM OF AMPLIT.
055,-055,	4	LP	2,3753	1,17390	0,01853	-	2,204	3,507	2,3873
056,-056,	4	SA	0,4367	1,15731	0,03247	-0.85	-0,852	1,597	0,4452
057,-057,	7	SSA	2,6395	1,25038	0,00524	-0.26	-0,260	0,243	2,7371
058,-059,	3	STA	0,1502	1,21661	0,09012	+5.01	5,012	4,217	0,1618
062,-063,	8	MSM	0,6194	1,22867	0,02228	+0.80	0,794	1,036	0,6796
064,-068,	19	MM	3,4138	1,12542	0,00432	-0.33	-0,335	0,218	3,7969
071,-074,	13	MSF	0,4797	0,76602	0,02834	+2.54	2,527	2,117	0,6400
075,-077,	15	MF	3,5350	1,27679	0,00329	-0.42	-0,435	0,149	8,5177
080,-083,	12	MSTM	0,0900	1,42475	0,07725	-3.64	-3,652	3,107	0,4257
084,-086,	19	MTM	0,6616	1,22995	0,01735	-0.12	-0,141	0,809	1,6445
091,-093,	23	MSQM	0,1560	1,28009	0,08265	+5.08	5,061	3,691	0,6056
105,-11X,	14	SGQ1	0,1949	1,19166	0,05122	-3.23	-3,357	2,462	0,5334
124,-125,	7	2Q1	0,5886	1,21065	0,01660	+2.51	2,381	0,780	1,0204
126,-129,	14	SIG1	0,7176	1,19228	0,01367	-0.76	-0,888	0,657	1,2717
133,-135,	15	Q1	4,5624	1,14831	0,00219	-0.31	-0,442	0,109	7,3724
136,-139,	15	RO1	0,9185	1,15189	0,01145	-0.53	-0,673	0,570	1,5975
143,-146,	18	O1	24,4781	1,15055	0,00042	+0.01	-0,129	0,021	36,6771
147,-149,	8	TAU1	0,4768	1,16837	0,02312	+0.54	0,395	1,134	0,5713
152,-155,	15	M1	2,3456	1,13624	0,00416	+0.22	0,074	0,210	5,2269
156,-158,	7	CHI1	0,3523	1,12643	0,02767	+0.36	0,210	1,407	0,6162
161,-162,	3	PI1	0,7969	1,16109	0,01323	-0.30	-0,451	0,653	0,8605
163,-163,	7	P1	14,1969	1,15068	0,00077	+0.34	0,192	0,039	14,2105
164,-164,	3	S1	0,2390	0,82832	0,04600	96.87	96,718	3,182	0,4582
165,-165,	11	K1	57,3470	1,14152	0,00028	+0.19	0,032	0,014	69,1021
166,-166,	2	PSI1	0,3305	1,19632	0,03290	-1.07	-1,220	1,576	0,3432
167,-168,	7	PHI1	0,5654	1,27265	0,01786	+3.55	3,397	0,804	0,7072
172,-173,	6	THE1	0,3780	1,19736	0,02692	+0.23	0,067	1,288	0,5992
174,-177,	10	J1	1,8123	1,16628	0,00526	-0.35	-0,508	0,259	3,4588
181,-184,	8	SO1	0,2892	1,11470	0,03310	+2.29	2,126	1,702	0,5370
185,-186,	10	OO1	0,5512	1,21940	0,01249	+0.25	0,085	0,587	2,6169
171,-183,	19	NY1	0,1221	1,17395	0,06143	-1.23	-1,396	2,998	0,7871
207,-22X,	21	EPS2	0,2365	1,10254	0,02298	+2.77	2,494	1,194	0,5106
233,-236,	10	2N2	0,6447	1,17183	0,00742	+1.78	1,496	0,363	0,9611
237,-23X,	10	MY2	0,8403	1,16363	0,00604	+2.21	,921	0,297	1,0177
245,-246,	17	N2	5,2046	1,17899	0,00097	+1.84	1,554	0,047	6,4211
247,-248,	7	NY2	0,9985	1,18178	0,00506	+1.46	1,166	0,245	1,1312
252,-258,	26	M2	29,0395	1,18372	0,00018	+1.20	0,903	0,009	29,7315
262,-264,	5	LMB2	0,2096	1,17408	0,02478	-0.45	-0,752	1,209	0,2347
265,-267,	12	L2	1,3542	1,14391	0,00515	+0.26	-0,046	0,258	1,8092
271,-272,	2	T2	0,7386	1,17646	0,00694	-0.90	-1,204	0,338	0,7980
273,-273,	4	S2	13,0467	1,18848	0,00041	+0.36	0,053	0,020	13,1179
274,-277,	12	K2	2,5554	1,18353	0,00183	+0.18	-0,127	0,089	5,0059
282,-285,	15	ETA2	0,1304	1,18506	0,03483	-0.53	-0,843	1,684	0,4496
292,-2X5,	14	2K2	0,0208	1,02397	0,11689	+5.56	5,244	6,540	0,2203
327,-375,	17	M3	0,2722	1,01035	0,00957	-0.56	-1,002	0,543	0,6127

R.M.S.ERROR M-ZERO 0,8251 MIKROGAL

R.M.S.ERROR FOR BANDS LP 2,4667 D 1,9325 SD 0,9417 TD 0,6247

O1/K1 1,0079 1-O1/1-K1 1,0638 M2/O1 1,0288

REFERENCE EPOCH 1978 7 31 0,0

Table II Analysis result with longperiodic waves

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value for tidal calculations.

Table III. Mean tidal parameters 1974-1982
GS 15 No. 222 digital, Station Potsdam

	N	$p \sim m^{-2}$		$p \sim A^2$	
		δ	$\varkappa^\circ$	δ	$\varkappa^\circ$
diurnal waves	19	1.1443 ± .0015	+ 0.17 ± .03	1.1446 ± .0015	+ 0.13 ± .03
semidiurnal waves	13	1.1846 ± .0007	+ 1.05 ± .11	1.1844 ± .0010	+ 1.07 ± .11
diurnal and semidiurnal waves	32	1.1712 ± .0035	+ 0.60 ± .09	1.1533 ± .0031	+ 0.34 ± .08
longperiodic waves	11	1.2232 ± .0237	- 0.33 ± .10	1.2450 ± .0198	- 0.21 ± .23

N - number of averaged wave groups

The fully corrected main tidal parameters are given at table IV. Here the orientation towards the ellipsoid-normal (SKALSKI), the inertial correction (PARISKIJ) and the indirect effect of the oceans (PERTZEV) as published in [2] are included for the first two columns.

Table IV. Corrected main tidal parameters at Potsdam

analysis programme	CHOJNICKI, A15H		DUCARME	
indirect effect	PERTZEV		SCHWIDERSKI	
Wave	δ	$\varkappa^\circ$	δ	$\varkappa^\circ$
O1	1.1582 ± .0004	- 0.04 ± .02	1.1565 ± .0005	- 0.08 ± .03
P1	---	---	1.1473 ± .0009	+ 0.21 ± .05
K1	1.1416 ± .0003	+ 0.07 ± .01	1.1379 ± .0003	+ 0.07 ± .02
N2	---	---	1.1588 ± .0010	+ 0.05 ± .05
M2	1.1580 ± .0002	+ 0.42 ± .01	1.1566 ± .0002	- 0.20 ± .01
S2	1.1641 ± .0004	+ 0.70 ± .05	1.1633 ± .0004	- 0.06 ± .02

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Furthermore the corresponding values are listed, calculated at the ICET/Brussels by DUCARME's programme and corrected because of the ocean loading on the base of SCHWIDERSKI's investigations (column 3 and 4). There are significant deviations between the corrections but the coincidence of the corrected δO1 and δM2 is remarkable for both the results. It is a sign for a significant residual component in M2 that the most accurate measured phase lag of this wave (1°.20 ± 0°.01) does not reach zero with the different corrections.

4. Results concerning the resonance of the liquid outer core

Accurate tidal results may be discussed with respect to the influence of the resonance of the liquid outer core of the Earth on the diurnal spectrum of the tidal waves. But the accuracy of the amplitude factors is not yet sufficient for all the small constituents of the tidal potential to decide between different Earth models. Therefore we have to use very long series or a big quantity of results.

On figure 1 the result of the Potsdam series is compared with the Molodenski II model and with the corresponding results of other stations in Europe which are normalized on δO1 to avoid calibration problems. Firstly, for this purpose the weighted means of 14 results obtained at West European stations are used each with an observational series of more than one year [1;3;6]. Secondly, there are marked the corresponding results of a global analysis of 16 series at several stations of the CAPG, altogether 74 016 hourly readings [2,(p. 32 and 36)]. Except of the different accuracy of these results they mostly agree in the frame of the error bars with the Potsdam result. For the majority of the results the biggest deviations against the model are occuring in the case of φ1 and the best fitting is reached for K1 if the correction of the indirect effect of the oceans is applied. For a better clearness a selection of the parameters demonstrated on figure 1 is listed in tables Va and Vb.

Table Va. Results concerning the resonance of the liquid outer core with corrections of the indirect effect

	Model Mo II	Potsdam 1974/82	global analysis East-Europe	correction
$\frac{\delta P1}{\delta O1}$	0.9954	0.9920 ± .0009	---	SCHWIDERSKI
$\frac{\delta K1}{\delta O1}$	0.9823	0.9857 ± .0004	0.9874 ± .0010	PERTZEV
$\frac{\delta K1}{\delta O1}$	0.9823	0.9839 ± .0005	---	SCHWIDERSKI
δO1	1.1638	1.1582 ± .0004	1.1605 ± .0010	PERTZEV
δO1	1.1638	1.1565 ± .0005	---	SCHWIDERSKI
δO1-δK1	0.0206	0.0165 ± .0005	0.0146 ± .0012	PERTZEV
δO1-δK1	0.0206	0.0186 ± .0006	---	SCHWIDERSKI

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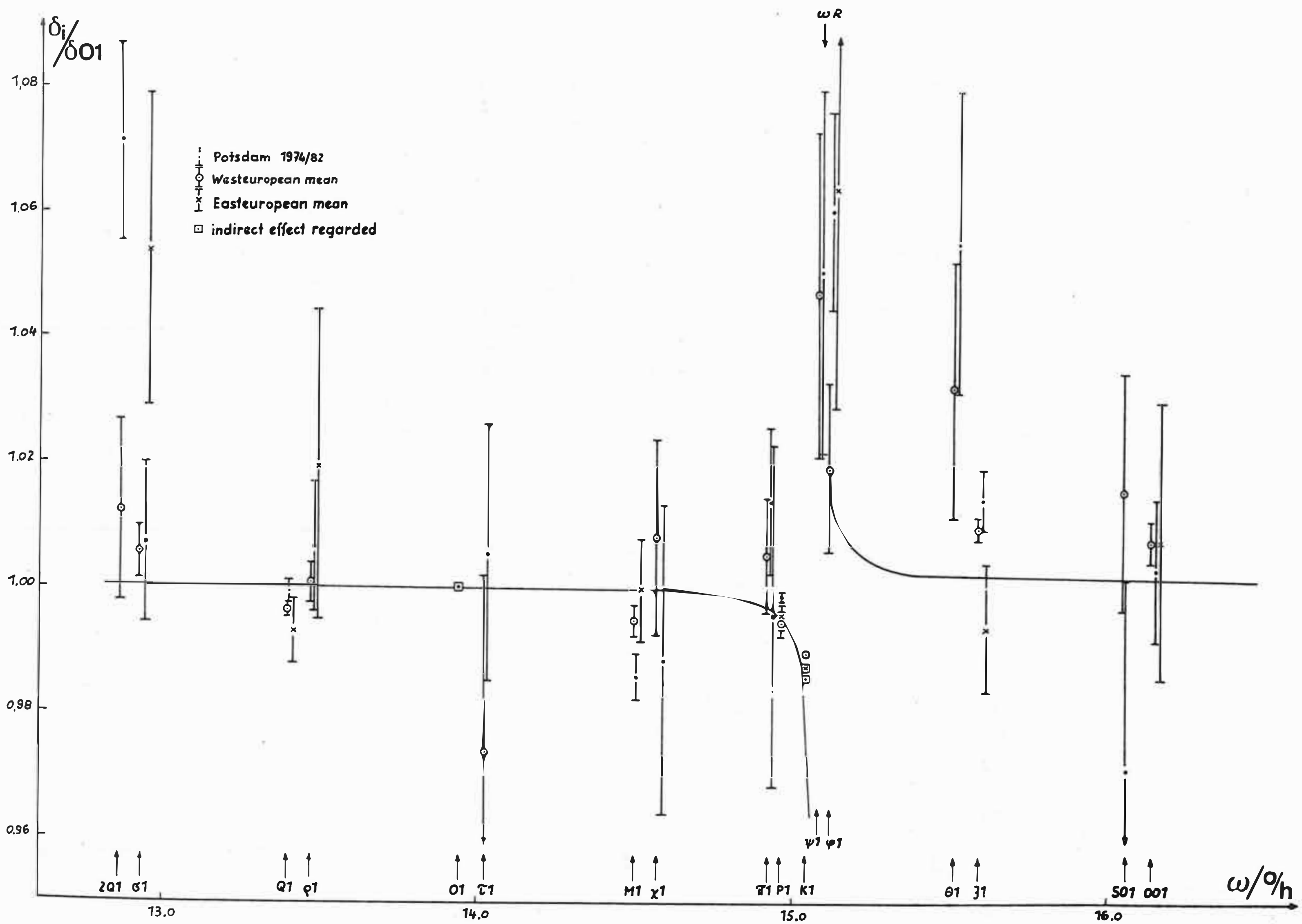

Fig. 1: Normalized δ - factor of the diurnal tidal spectrum

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Table Vb. Amplitude factors divided by δ01 as pictured at figure 1

Wave-group	ω °/h	Model Mo II	weighted mean West-Europe	Potsdam 1974/82	global analysis East-Europe
Q1	13.399	1.00017	0.9963 ± .0007	0.9992 ± .0020	0.9928 ± .0050
M1	14.497	0.99948	0.9947 ± .0024	0.9857 ± .0037	0.9995 ± .0082
π1	14.918	0.99673	1.0049 ± .0091	1.0137 ± .0117	0.9955 ± .0271
P1	14.959	0.99536	0.9943 ± .0020	0.9985 ± .0008	0.9953 ± .0018
K1	15.041	0.98230	0.9894 ± .0002	0.9892 ± .0004	0.9897 ± .0011
ψ1	15.082	1.06934	1.0467 ± .0258	1.0503 ± .0290	1.2193 ± .0670
φ1	15.123	1.01246	1.0190 ± .0134	1.0599 ± .0158	1.0634 ± .0349
OO1	16.139	1.00103	1.0072 ± .0034	1.0027 ± .0114	1.0075 ± .0220
δ01 obs.	13.943	1.1638	1.1501 ± .0018	1.1521 ± .0004	1.1534 ± .0010
(δ01--δK1) obs.	---	0.0206	0.0123 ± .0003	0.0124 ± .0005	0.0119 ± .0012

It seems to be fact that significant contributions for the discussion of different models may be obtainable at present from the big waves O1 , P1 and K1 only. But here also significant deviations against the model appear in the case of the unnormalized values of δ01 and D = δ01 - δK1. Using SCHWIDERSKIs corrections it must be noticed, that D is nearer to the model, but the deviation of δ01 itself against the model becomes about 0.2% bigger than in the case if PERTZEVs correction is applied. Both the corrections do not lead exactly to the model values. The comparable good agreement of $\delta\psi$1/δ01 at Potsdam seems to be accidental with respect to the error. Because of the fact that the better accordance with the model is preponderantly observed in eastern Europe we may conclude that an improved knowledge of the indirect effect would lead to a further progress in the discussion of the resonance.

5. Temporal variations of the results

The application of overlapping gliding harmonic analyses with a shift of about 90 days using CHOJNICKIs programme A15H leads to significant temporal variations of the calculated parameters at Potsdam. This is also valid for the main waves and especially for the parameters characterizing the resonance of the liquid core. The partial analyses are processed each with more than 10 000 hourly readings, the final error of an

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ordinate was below ± 0.7 microgal in every case and a resolution of 19 wave groups was chosen for this calculations.

The temporal variations of the most significant amplitude factors are compared with the length of the day [9] on figure 2. Remarkable is the big variation of δO1 between May 1979 and October 1980 which is surely not caused by an alteration of the calibration constants as to be seen from the stability of δM2. δO1 is increasing + 0.0054 or 0.47% during this period and mostly affecting the parameter of resonance D = δO1 - δK1 which is increasing when the yearly mean of the length of the day is decreasing. Outside this period D is mainly affected by δK1 and no clear correlation is visible to the length of the day.

The correlation between parameters characterizing the resonance of the liquid outer core and the length of the day is already mentioned by LECOLAZET [5] but using the δ-factors divided by δO1. Considering the trend of δO1 these normalization is not suitable for the Potsdam results because the variation of δO1 will affect the trend of all the quotients δ_i/ δO1 in the same way, simulating a common variation.
As a conclusion may be pointed out:

- The effect of the resonance in tidal results is influenced by temporal variations, partially connected with variations of the rotation of the Earth.
- Results of Earth tidal measurements at a well-established station are not valid for a very long time. Even if the duration of the measurements is longer than one and a half year the results of various series with the same instrument at the same site may differ also for the main tides outside the error borders.

With respect to the error bars the temporal variation seems to be significant. To assume for these variations a geophysical origin we have to look for similar alterations in the series of other Earth tidal stations. Therefore the series

- GS 15 No. 228, Pecny/ČSSR, March 1975 until August 1981
35 086 hourly readings, 85 data blocks
m_0 = ± 0.73 microgal
- LaCoste Romberg No. G318, Berlin (West), March 1978 until February 1981
23 652 hourly readings, 4 data blocks
m_0 = ± 1.00 microgal

are investigated with an identical processing of the 90 days shifted analyses almost with the same beginning and end of the sections as for the partial analyses of the GS 15 No. 222. Since absolute deviations are mostly caused by differences in the calibrations we shall discuss the temporal trend of the tidal parameters only. A strong correlation of the results of the three instruments is very seldom for one of the 38 parameters (δ, $\varkappa$ for 19 wave groups). Parallel trends are occurring during the period 1976 - 1981 almost for shorter sections but not for the whole period and more frequently for both the GS 15 than for one of them and the LaCoste Romberg gravimeter. But anticorrelations are observed, too. Some of the clear correlated sections are shown in figure 3.

The alteration of δO1 and δO1 - δK1 between 1979 and 1980 is very well confirmed by the Pecny measurement, but with regard to the other pictures may be concluded that for a significant detection of parameter variations with geophysical relevance there are needed tidal measurements with an accuracy and long time stability characterized by mean square error values of m_0 smaller than ± 0.7 microgal. In the case of a higher noise level in general other effects (different for the instruments at different stations) are overdrawing the variations caused by the Earth's behaviour.

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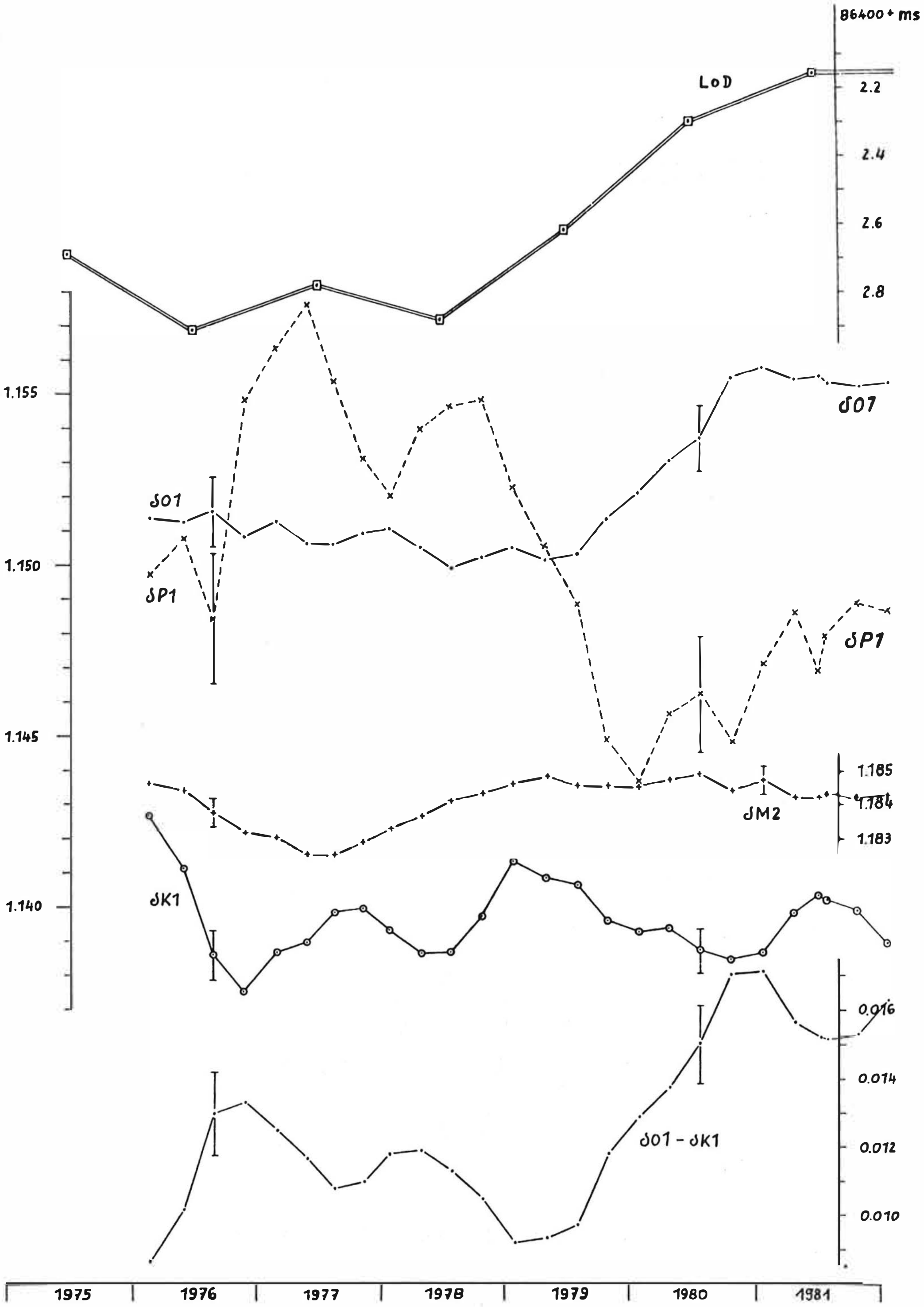

Fig. 2: δ - factors of the diurnal main waves and M_2 and the yearly mean value of the length of the day (LoD)

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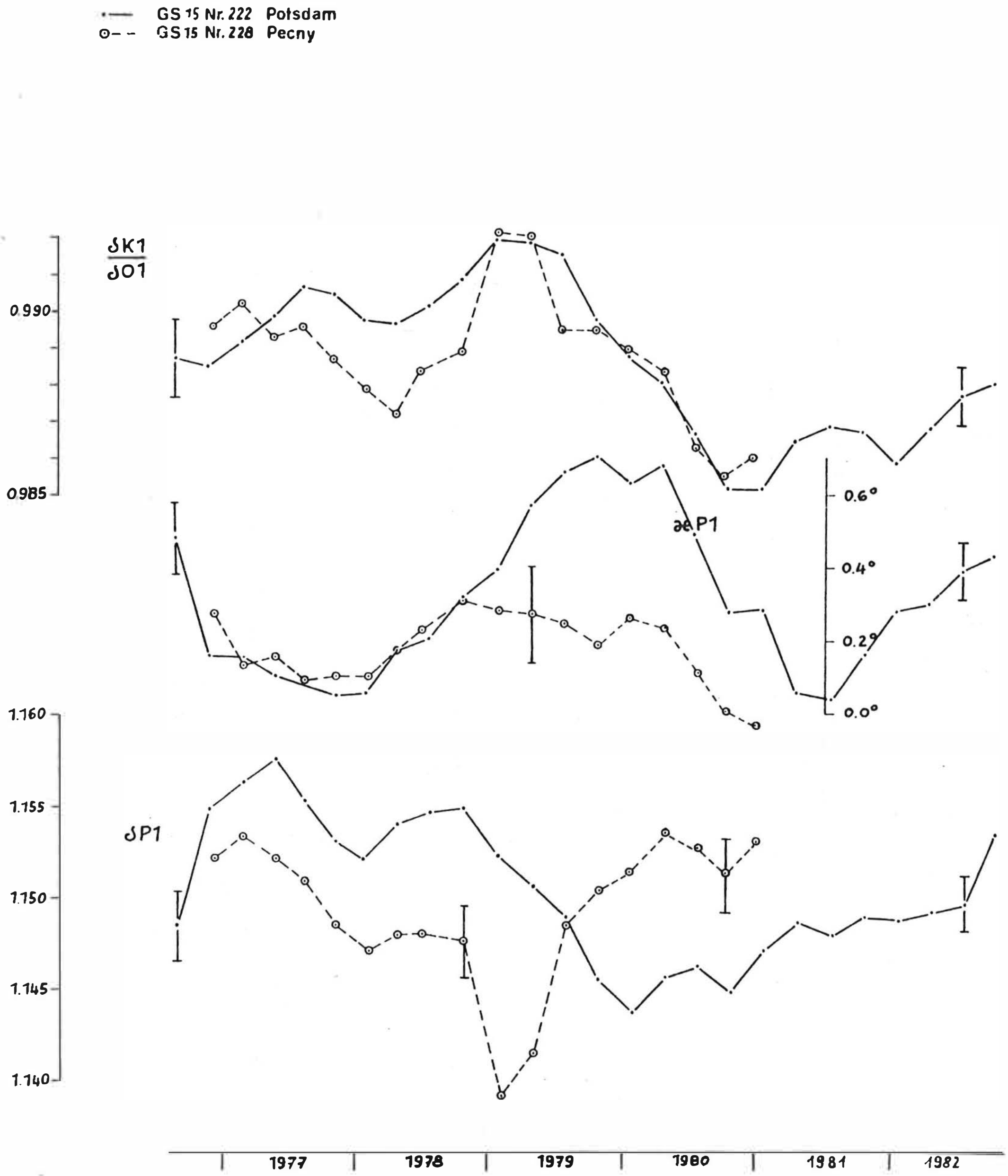

Fig. 3: Various correlations of Earth tidal parameters at two stations
·——— GS 15 No. 222 Potsdam
⊙– – – GS 15 No. 228 Pecny

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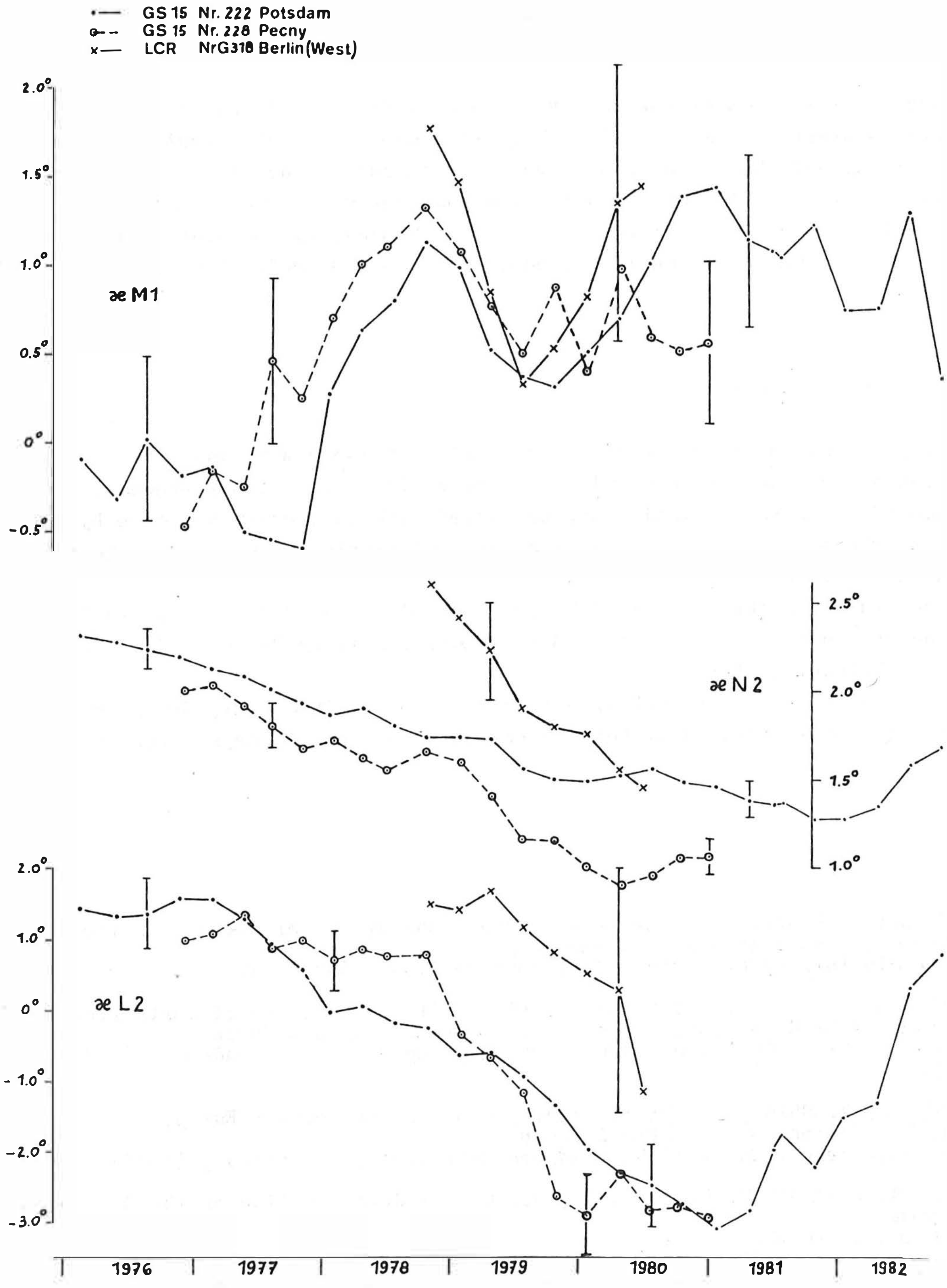

Fig. 4: Similar temporal trends at three different stations
•——— GS 15 No. 222 Potsdam
o- - - GS 15 No. 228 Pecny
x——— LCR No. G318 Berlin (West)

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Especially longperiodic correlations are to be found for the phase lags, the best ones for M1, 2N2, N2 and L2 as illustrated in figure 4. These are constituents of the tidal potential only caused by the moon and always containing the term p , the length of the perigeum of the moon.

For a further verification of the observed longperiodic variations of Earth tidal parameters more uninterrupted series of high quality are to be investigated. But it may be concluded already now that such kind of variations must be included in the discussions and applications of the results of Earth tidal measurements for the examination of more detailed geophysical models, especially if a very high accuracy is required like for the problem of the resonance of the liquid outer core and its reflection in the diurnal tidal spectrum.

A c k n o w l e d g e m e n t s

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On the determination of the gravimetric M_f tide at Potsdam

by

C. ELSTNER, M. HARNISCH and W. SCHWAHN

S u m m a r y

The determination of the gravimeter factor δ for the M_f-wave group demands the consideration of a few pecularities:

- the M_f-group consists of two large constituents (for Potsdam T_1 = 13.633 days, A_1 = 2.331 μGal, T_2 = 13.661 days, A_2 = 5.627 μGal) and few minor ones. This implies an amplitude variation between 3 and 8 μGals and a beat period of 18.6 years for the two largest waves and of 16.5 years for the whole wave group,
- the long-term homogeneity of the time series,
- the signal-to-noise ratio for this long period part of the spectrum.

Formulas are given for the computation of the instantaneous and the mean value both of the period and of the amplitude for the years 1974 through 1982.

By several methods (CHOJNICKI-analyses, generalized BUYS-BALLOT schema, FOURIER-analyses, statistical estimates) we get an empirical δ-factor of δ = 1.026 and a phase lag of $\varkappa = 7\overset{\circ}{.}7$. The consideration of the ocean tide influence improves this value to δ = 1.073, $\varkappa = 7\overset{\circ}{.}4$. Both by the spectral distribution of the gravity time series in the M_f-range and by a further splitting up of the M_f-wave group in the CHOJNICKI-procedure one can conclude that non-tidal disturbances exist. Their mean amplitude is estimated to about 0.75 μGal. Assuming a zero phase lag $\varkappa = 0\overset{\circ}{.}0$ we find out an undisturbed gravimeter factor

$$\delta = 1.176 \pm 0.04 .$$

From the spectra of local air pressure and the observed gravity variations in the range of the M_f-periods it is impossible to get a close connection between the above noted non-tidal disturbance and the air pressure.

Z u s a m m e n f a s s u n g

Die Bestimmung des Gravimeterfaktors δ für die M_f-Wellengruppe verlangt die Beachtung einiger Besonderheiten:

- Die M_f-Wellengruppe besteht aus zwei großen Wellen (für Potsdam T_1 = 13.633 Tage, A_1 = 2.331 μGal, T_2 = 13.661 Tage, A_2 = 5.627 μGal) und einigen kleineren. Daraus resultiert eine Amplitudenschwankung zwischen 3 und 8 μGal und eine Schwebungsperiode von 18.6 Jahren für die beiden größten Wellen und von 16.5 Jahren für die ganze Wellengruppe.
- Gute Langzeithomogenität der Zeitreihe wird vorausgesetzt.
- Gesonderte Untersuchung des Signal-Rausch-Verhältnisses für diesen langperiodischen Teil des Spektrums.

Die momentane und die mittlere Periode und Amplitude für den Zeitraum von 1974 bis 1982 werden berechnet.

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Mit Hilfe verschiedener Methoden (CHOJNICKI-Analysen, verallgemeinertes BUYS-BALLOT-Schema, FOURIER-Analysen, statistische Schätzungen) erhalten wir einen empirischen δ-Faktor von $\delta = 1.026$ und eine Phasenverschiebung von $\varkappa = 7\overset{\circ}{.}7$. Die Berücksichtigung des Meeresgezeiteneinflusses verbessert diesen Wert auf $\delta = 1.073$, $\varkappa = 7\overset{\circ}{.}4$. Aus der Spektralverteilung der Schwerevariationen im M_f-Bereich und einer weitergehenden Aufspaltung der M_f-Gruppe beim CHOJNICKI-Verfahren kann man auf die Existenz einer nicht gezeitenbedingten Störung schließen. Ihre mittlere Amplitude wird zu 0.75 µGal geschätzt. Unter der Annahme der Phasendifferenz $\varkappa = 0\overset{\circ}{.}0$ finden wir einen ungestörten Gravimeterfaktor

$$\delta = 1.176 \pm 0.04 .$$

Aus den Spektren des Luftdruckes und den beobachteten Schwerevariationen im M_f-Bereich ist ein gesicherter Zusammenhang der nicht gezeitenbedingten Störung mit dem Luftdruck nicht ableitbar.

1. Introduction

In the Physics of the Earth the tidal forces are an important tool for the estimation of the mechanical properties of the Globe. These outer forces have several frequency bands in space and time, well seperated from each other, mainly the terdiurnal, semidiurnal, diurnal and the longperiodic ones. Each of them serves for a specific physical interpretation.

The longperiodic part occuring in the form of zonal harmonics only is most interesting to fill the gap between the diurnal phenomena and the long - term rheology obtained by the analysis of the CHANDLER - wobble. This type of tidal deformations directly influences the rate of rotation of the Earth and affects the dynamical response of the earth (WAHR et al. (1981)). Within the longperiodic tides the M_f-wave group has the largest amplitude. It enters as well into the astronomical (for instance PILNICK (1970, 1976), LAMBECK (1980), MERRIAM (1980), YODER et. al. (1981), HEFTY (1982)) as into the gravimetric observational series (for instance LECOLAZET and STEINMETZ (1966), VENEDIKOV (1981)). It means on the one hand that not only from the theoretical point of view but also from the point of observational techniques two different methods can be used and are able to contribute to the same aim. On the other hand, from the point of view of data analysis with respect to the presence of noise, this wave group is the most favoured one.

Its determination needs a drift-free observation series. This is a drastic restriction for the most types of tidal gravimeters, which often show a yearly drift of a few mGal (10^{-5} m s^{-2}). A careful treatment of the predominate and complicated drift must be carried out under a special attention to the fact that its elimination do not influence the M_f-range. There are also some difficulties in the case of observations with a small and stable drift, for instance for the observations of the LCR 058 with respect to the estimation of the M_f-tide (VENEDIKOV 1981; VENEDIKOV & DUCARME 1979). The new generation of gravity meters basing on the principle of a super-conducting sensor seems to be nearly drift-free (GOODKIND 1979; RICHTER, BREIN et al. 1982). Nevertheless it seems reasonable to look for informations about the long - periodic tides in each long - time series, because they are rare up to now.
This paper deals with the results of the attempt to estimate the parameters of the M_f-group by several different methods on the basis of a long gravimetric series in Potsdam.

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2. The computation of the theoretical temporal mean parameters by analytic methods

The analysis of gravimetric time series extended over more than half a year also gives the parameters (amplitude relations and phase differences) for the M_f-group consisting of 15 partial waves. For all these partial waves one and the same pair of parameters will be assumed to be valid. For the evaluation of the results of the different methods of analysis an analytic presentation of the sum of the most important waves of this group is useful.

2.1 Analytic representation of the sum of tidal waves

With an accuracy of a few percent we may substitute the 15 waves of the M_f-group by their two greatest ones and we get for the temporal variation $\delta g(t)$ of the instantaneous gravity value due to these two waves

$$(2.1.1) \qquad \delta g_{M_f}(t) = A_1 \cos(2s + N' + \pi) + A_2 \cos(2s + \pi),$$

where s and N' design the mean longitude of the Moon and the negative longitude of its ascending node respectively. At Potsdam the amplitudes $A_{1,2}$ are:

$A_1 = 2.331$ µgal
$A_2 = 5.627$ µgal.

Instead of (2.1.1) we may write:

$$(2.1.2) \qquad \delta g_{M_f}(t) = A_1 \cos(\nu_1 t + \varphi_1) + A_2 \cos(\nu_2 t + \varphi_2),$$

where $\nu_1 = \frac{d}{dt}(2s + N') = 1.10023945\ ^\circ/h$,

$\nu_2 = \frac{d}{dt}(2s) = 1.09803304\ ^\circ/h$

and $\varphi_{1,2}$ mean the phase angles for $t=0$.

Using the relation

$$(2.1.3) \qquad A_1 \cos\alpha + A_2 \cos\beta = (A_1 + A_2)\cos\left(\frac{\alpha - \beta}{2}\right)\cos\left(\frac{\alpha + \beta}{2}\right) - (A_1 - A_2)\sin\left(\frac{\alpha - \beta}{2}\right) \times \sin\left(\frac{\alpha + \beta}{2}\right)$$

and the abbreviations

$$\frac{\nu_1 + \nu_2}{2} = \bar{\nu}, \quad \nu_1 - \nu_2 = \Delta\nu, \quad \frac{\varphi_1 + \varphi_2}{2} = \bar{\varphi}, \quad \varphi_1 - \varphi_2 = \Delta\varphi$$

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we calculate from (2.1.2) the gravity variation

(2.1.4) $$\delta g_{M_{f2}}(t) = R_2(t)\cos(\bar{\nu} t + \bar{\varphi} - P_2(t)),$$

(2.1.5) where $$R_2(t) = (A_1^2 + 2A_1A_2\cos(\Delta\nu\, t + \Delta\varphi))^{\frac{1}{2}}$$ and

(2.1.6) $$P_2(t) = -\arctan \frac{A_1 - A_2}{A_1 + A_2} \tan(\frac{\Delta\nu}{2}t + \frac{\Delta\varphi}{2}).$$

If a more accurate representation is desired further constituents of the M_f-group may be added to (2.1.4). For three constituents hold the following relations:

(2.1.7) $$\delta g_{M_{f3}}(t) = \sum_{i=1}^{3} A_i\cos(\nu_i t + \varphi_i) =$$

$$= R_2\cos(\bar{\nu} t + \bar{\varphi} - P_2) + A_3\cos(\nu_3 t + \varphi_3).$$

By the aid of theorem (2.1.3) we get:

(2.1.8) $$\delta g_{M_{f3}}(t) = R_3\cos\left(\frac{\bar{\nu} + \nu_3}{2} t + \frac{\bar{\varphi} + \varphi_3}{2} - P_3(t)\right),$$

with the amplitude and phase functions:

(2.1.9) $$R_3(t) = (R_2^2 + A_3^2 + 2R_2A_3\cos((\bar{\nu} - \nu_3)t + \bar{\varphi} - \varphi_3 - P_2(t)))^{\frac{1}{2}},$$

(2.1.10) $$P_3(t) = \frac{P_2}{2} - \arctan\left(\frac{R_2 - A_3}{R_2 + A_3} \tan\left(\frac{(\bar{\nu} - \nu_3)}{2}t + \frac{\bar{\varphi} - \varphi_3 - P_2}{2}\right)\right).$$

In the same manner we can derive formulas for the computation of amplitudes and phase angles if we wanted to include further waves.

The instantaneous period $T_i(t)$ is easily calculated in the following manner ($i = 2,3$):

(2.1.11) $$\bar{\nu}_i t + \varphi - P_i(t) = \bar{\nu}_i(t + T) + \varphi - P_i(t + T) - 2\pi.$$

Expressing $P_i(t + T)$ by a Taylor-series and using $\dot{P} = dP/dt$ we get:

(2.1.12) $$T_i(t) = \frac{2\pi}{\bar{\nu}_i - \dot{P}_i(t)}$$

wherby $P_2(t)$ resp. $P_3(t)$ were calculated by the aid of (2.1.6) resp. (2.1.10).

Fig. 1 shows the functions $R_2(t)$ and $R_3(t)$ as well as $T_2(t)$ and $T_3(t)$ between 1974 and 1982 for the geographical latitude 52.°3809 (Potsdam). Amplitudes R_2 are oscillating with a period of 18.6 years between 7.96 and 3.30 µGal without any terrestrial tidal response (δ = 1.0). The minimum of R_2 in July 1978 is followed by a maximum in November 1987. The more accurate amplitude function R_4 shows slightly changed data

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depending on time. At present its period amounts to 16.5 years and the extremas reach 3.00 μGal (April, 1978) and 8.24 μGal (January, 1987) respectively. The complete representation of the instantaneous parameters of the M_f-group, R_{M_f} and T_{M_f} easily can be received by numerical vector addition, but for our purposes analytical formulas are more useful.

For different methods of analysis mean values of the tidal parameters are needed. The arithmetic mean values of amplitudes and periods for March, 1974 up to Febr., 1982 are summarized in table 1.

Table 1
Arithmetic mean values of amplitudes and periods (3, 1974 - 2, 1982)

R_2 = 4,24 μGal	T_2 = 13.669 days
R_3 = 4,26 μGal	T_3 = 13.669 days
R_4 = 4.27 μGal	T_4 = 13.670 days

Because of the very small deviations between the mean values of R and T for the given interval of analysis these functions may be represented by R_2 and T_2 with a sufficient degree of accuracy.

For the synchronization method and the Fourier-analysis we need integrated mean values, signed by a bar, for the parameters of the M_f-group. Therefore we first of all determine

$$(2.1.13)\qquad \bar{R}_2(\vartheta) = \frac{1}{\vartheta}\int_{t_A}^{t_E} R_2(t')\,dt' .$$

$t_{A,E}$ denote the dates of the begin and the end of the analysis, $t_E - t_A = \vartheta$. Substituting the integrand of (2.1.13) by (2.1.5) and using the abbreviations $a = A_1^2 + A_2^2$, $b = 2A_1A_2$ and $\Delta\varphi = \varphi_1 - \varphi_2$ we find the following general expression for $\bar{R}_2$:

$$(2.1.14)\qquad \bar{R}_2(\vartheta) = \frac{\sqrt{a}}{\Delta\nu\vartheta} \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{2k} \left(\frac{b}{a}\right)^{2k} \left[\frac{1}{2^{2k}}\binom{2k}{k}\left(\Delta\nu\vartheta + \right.\right.$$

$$\left.\left. + \frac{1}{2^{2k-1}} \sum_{l=0}^{k-1} \binom{2k}{l} \cdot \frac{\sin((2k-2l)(\Delta\nu\, t_E + \Delta\varphi)) - \sin((2k-2l)(\Delta\nu\, t_A + \Delta\varphi))}{2k-2l}\right)\right]$$

$$+ \sum_{k=0}^{\infty} \binom{\frac{1}{2}}{2k+1} \left(\frac{b}{a}\right)^{2k+1} \cdot$$

$$\cdot\left[\frac{1}{2^{2k}} \sum_{l=0}^{k} \binom{2k+1}{l} \cdot \frac{\sin((2k-2l+1)(\Delta\nu\, t_E + \Delta\varphi)) - \sin((2k-2l+1)(\Delta\nu\, t_A + \Delta\varphi))}{2k-2l+1}\right]$$

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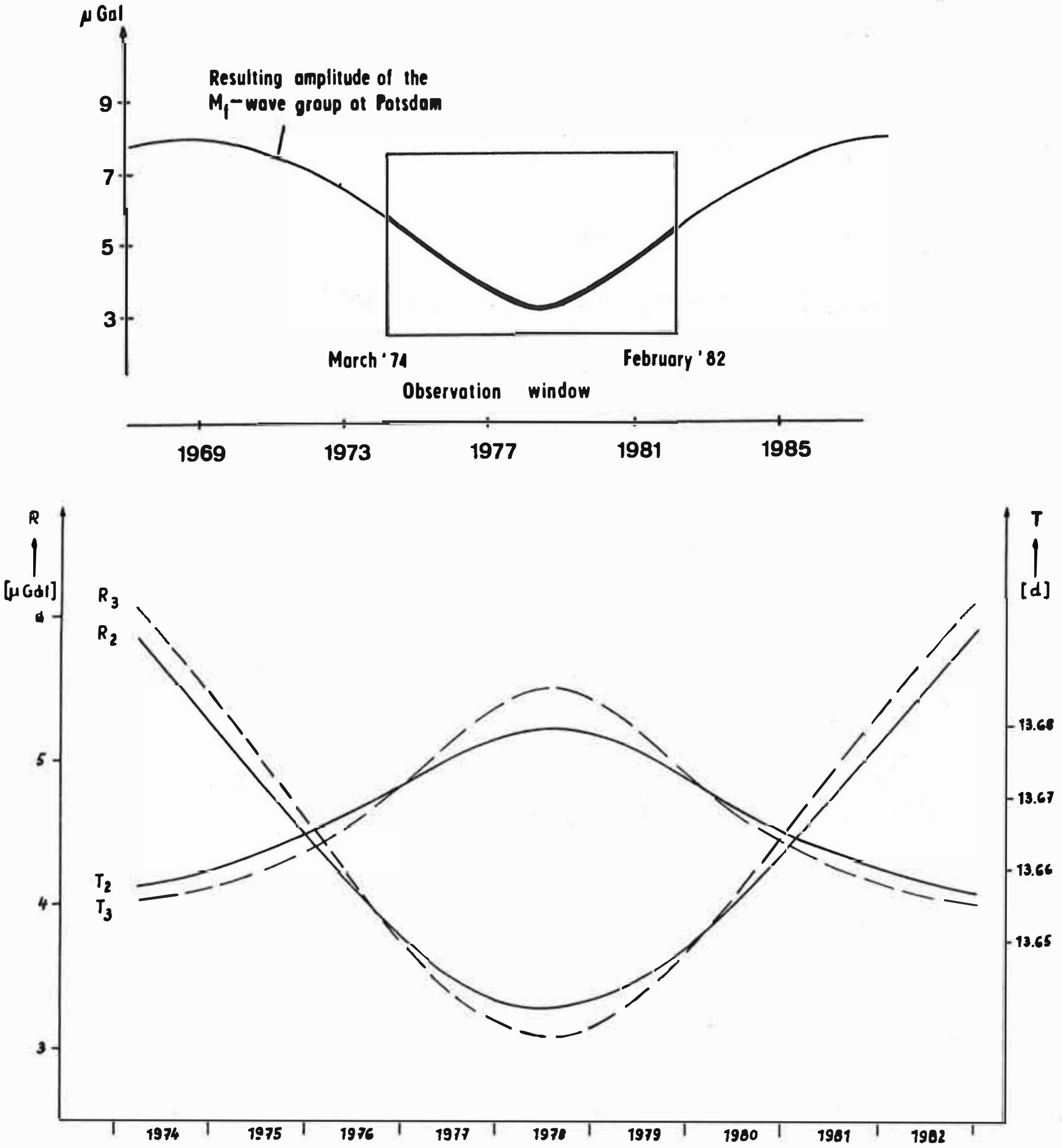

Fig. 1: General view of the temporal variation of the amplitude of the M_f-wave group in the observation window of the gravimetric time series at Potsdam (above). Below amplitudes R_i and periods T_i following the superposition of the two (i=2) or three (i=3) largest single theoretical waves inside the M_f group

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For practical calculations we may limit the infinite series in (2.1.20) in dependence on the desired accuracy. If we assume $\vartheta > 365$ days and $10^{-3}\,\mu$Gal for the uncertainty of $\bar{R}$ we find the following relation:

$$(2.1.15) \quad \bar{R}_2(\vartheta) = c_0 + \frac{1}{\vartheta} \sum_{l=1}^{5} c_l \left(\sin l\,(\Delta\nu\, t_E + \Delta\varphi) - \sin l\,(\Delta\nu\, t_A + \Delta\varphi)\right).$$

At Potsdam the following numbers are valid:

$$\sqrt{a} = 6.0908\ \mu\text{Gal},\quad b/a = 0.7072,\quad \Delta\nu = 2.20641 \cdot 10^{-3}\ {}^{\circ}/\text{h}$$

$$\begin{aligned} c_0 &= 5.8732\ \mu\text{Gal} \\ c_1 &= 59155.2754\ \mu\text{Gal/h} \\ c_2 &= -2953.8615 \\ c_3 &= 395.8016 \\ c_4 &= -67.3322 \\ c_5 &= 14.4999 \end{aligned}$$

Using (2.1.12) we calculate the instantaneous period T_2. Differentiation of equation (2.1.6) yields

$$(2.1.16) \quad \dot{P}_2 = \frac{-(A_1^2 - A_2^2)\,\frac{\Delta\nu}{2}}{A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta\nu\, t + \Delta\varphi)} = \frac{-(A_1^2 - A_2^2)\,\Delta\nu}{2R_2^2}$$

Substituting (2.1.16) in (2.1.12) we get

$$(2.1.17) \quad T_2(t) = \frac{4\pi\, R_2^2(t)}{2R_2^2\bar{\nu} + \Delta\nu\,(A_1^2 - A_2^2)} .$$

and the instantaneous frequency

$$(2.1.17a) \quad \omega_2(t) = \frac{2\pi}{T_2(t)} = \bar{\nu} + \frac{\Delta\nu}{2R_2^2}(A_1^2 - A_2^2) = \bar{\nu} - \dot{P}_2 .$$

The temporal mean frequency is given by integration over the interval :

$$(2.1.18) \quad \bar{\omega}_2(\vartheta) = \frac{1}{\vartheta}\int_{t_A}^{t_E} (\bar{\nu} - \dot{P}_2)\,dt = \bar{\nu} - \frac{1}{\vartheta}\left(P_2(t_E) - P_2(t_A)\right) .$$

By the aid of (2.1.6) we finally get:

$$(2.1.18a) \quad \bar{\omega}_2(\vartheta) = \bar{\nu} + \frac{1}{\vartheta}\left[\arctan\left(\frac{A_1 - A_2}{A_1 + A_2}\tan\left(\frac{\Delta\nu}{2}t_E + \frac{\Delta\varphi}{2}\right)\right) - \arctan\left(\frac{A_1 - A_2}{A_1 + A_2}\tan\left(\frac{\Delta\nu}{2}t_A + \frac{\Delta\varphi}{2}\right)\right)\right] .$$

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To receive unique numerical results in (2.1.18a) and also in the expressions for the phase functions P(t) the π-periodic tan-functions should be substituted by

(2.1.18b) $$\tan\frac{\varphi}{2} = \frac{1-\cos\varphi}{\sin\varphi}$$

in numerical calculations.

Using equations (2.1.15) and (2.1.18a) we calculated for our observational interval March, 22, 1974 to Feb., 14, 1982 the following values:

(2.1.19) $$\bar{R}_2(\vartheta) = 4.198\ \mu\mathrm{Gal}$$
$$\bar{\omega}_2(\vartheta) = 1.097342\ ^\circ/\mathrm{h}$$
$$\bar{T}_2(\vartheta) = 13.6694\ \mathrm{d}$$

2.2. Representation by Fourier-analysis

The function y(t) for the two main waves with the parameters (A_i, ν_i, φ_i), i = 1,2

(2.2.1) $$y(t) = A_1\cos(\nu_1 t + \varphi_1) + A_2\cos(\nu_2 t + \varphi_2)$$

may be transformed in the frequency domain by a finite Fourier-transformation limited to the time interval $t_A \leq t = t_B$,
where $\bar{t} = (t_B - t_A)/2$ and φ_i design the phase angle for $t = t_A$.

Then the approximation of y(t) is given by f(t):

(2.2.2) $$f(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos\frac{k\pi t}{\bar{t}} + b_k \sin\frac{k\pi t}{\bar{t}} ,$$

where

(2.2.3) $$a_0 = \frac{1}{2\bar{t}}\int_{-\bar{t}}^{+\bar{t}} y(t)dt,$$

$$a_k = \frac{1}{\bar{t}}\int_{-\bar{t}}^{+\bar{t}} y(t)\cos\frac{k\pi t}{\bar{t}}dt \quad \text{and} \quad b_k = \frac{1}{\bar{t}}\int_{-\bar{t}}^{+\bar{t}} y(t)\sin\frac{k\pi t}{\bar{t}}dt .$$

The preposition of the FOURIER-analyses consists in the assumption that only these waves exist, whose wavelengths are an integer part of the interval $2\bar{t}$. If we confine our consideration on a finite time interval the so-called"leakage phenomenon"takes place. It means in the conception of the FOURIER-transforms the broadening of a line in the spectrum according to the convolution of this line by the spectral function of the time window, the sinx/x-function. This broadening implies in the notions of the FOURIER-analyses that we obtain a contribution to the amplitudes of periods in the FOURIER-representation, where in fact no waves exist, if the wavelength of the given wave is not an integer part of $2\bar{t}$. It is a very strong restriction for all these cases where either the wavelength is not exactly known or an ensemble of wavelengths, not related in a manifold of the interval $2\bar{t}$ to each other must be considered. If the relation

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(2.2.3) $\frac{\tau}{\pi}(\nu_1-\nu_2) < 1$

is valid the two waves cannot be separated by the FOURIER-analysis. It means that in the case of an integer relation for the one wavelength T_1 and the length of the interval 2τ one cannot find an integer value for $2\tau/T_2$. In chapter 4.2.1 this problem will be considered in a general discussion. Here we give the results only: For a real number $\varrho_i, \varrho_i < 3.5 \ll N$, N an integer number $N \approx 2\tau/T_i$, the relation

(2.2.4) $$\varrho_i = \frac{2\tau - NT_i}{T_i}$$

describes the difference between the actual length for the wave k = N and the length of the time interval in terms of the wave under consideration. Then holds for the amplitude R of the k th harmonic wave with $k = N \pm j$, j is the integer part of ϱ_i,

(2.2.5) $$R_i(k) = A_i \frac{\sin \pi\varrho_i}{\varrho_i} .$$

In the case of the superposition of two waves (i = 1,2) we must consider the amplitude of the vector summation for each k. One obtain

(2.2.6) $$R_{F_2}(k) = \left[\left(A_1 \frac{\sin \pi\varrho_1}{\varrho_1}\right)^2 + \left(A_2 \frac{\sin \pi\varrho_2}{\varrho_2}\right)^2 + 2A_1 A_2 \frac{\sin \pi\varrho_1 \sin \pi\varrho_2}{\varrho_1 \varrho_2} \cos(\varphi_1 - \varphi_2)\right]^{1/2}$$

On the basis of N = 210 waves of the mean wavelength $\overline{T}_2 = 13.67$ (according (2.1.19)) the parameters in (2.2.4) and (2.2.6) are $\tau = 34448$ h, N = 210, $\varrho_1 = 0.5614$, $\varrho_2 = 0.1298$ and the numerical calculation of R_{F_2} resulted to the following value for the interval t_A = March, 22, 1974, 01^{00} UT, t_E = January, 29, 1982, 15^{00} UT

(2.2.7) $$R_{F_2}(210) = 4.164 \ \mu\text{Gal} .$$

The Fourier-representation of the theoretical tidal data including all 15 waves of the same interval on the basis of hourly "readings" yielded the values

(2.2.8) $$\overline{R}_{M_f} = 4.206 \pm 0.2 \ \mu\text{Gal}$$
$$\overline{T}_{M_f} = 13.669 \ \text{d.}$$

From the comparison of both the results may be seen, that the representation of the M_f-group by their two greatest partial waves is quite sufficient for the purposes of Fourier-analyses and naturally also for the handling of the data by the synchronization method.

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3. Description of the long time drift by polynomials and trigonometric functions

The continuous recordings of the temporal variations of gravity performed since 1974 at our gravimetric observatory at Potsdam resulted in high accurate parameters ($\pm 2 \cdot 10^{-4}$ in the gravimeter factors δ, $\pm 0\overset{\circ}{.}02$ in the phase lags) of the diurnal and semidiurnal main tidal waves (see DITTFELD, H.-J.: "Results of an eight years' gravimetric earth tide registration series at Potsdam", table 1, page 6 of this issue, and also DITTFELD and VARGA (1983)). Also the reliability of the small tidal waves Q_1 and K_2 was confirmed in the framework of the transworld tidal gravity measurements (MELCHIOR et al., 1983).

Since there are only 12 interruptions, caused a loss of only 4.85% (140 days) of the 2886 registration days the material seems to be appropriate for an investigation of the M_f-tide. The drift of the gravimeter Gs 15 Nr. 222 in the course of eight years shows an unique shape and calls for a description by an analytical function (fig. 2). But we must keep in mind that during the whole eight years the drift of the zero position was about 16.5 milligals (corresponding to an average of 5.5 microgals per day), whereby the wanted phenomenon has a total range of 8 microgals in 14 days!

There are the measured values (MV(t)) diminished by the diurnal, semidiurnal and terdiurnal tides (TD(t)), obtained by a synthesis of the results of a CHOJNICKI-analysis. Practically we subtracted the residuals (RES(t)) of this tidal analysis from the results of the PERCEV-filtration (PF(t)) of the observed data:

$$(3.1) \quad RK(t) = MV(t) - TD(t) = PF(t) - RES(TD,t)$$

The registrations are interrupted by some gaps which separate the whole interval into data blocks in an arbitrary manner. Formula 3.1 is valid for a single data block only. The further treatment of the data, particularly the determination of an analytic expression for the drift behaviour over the whole interval of eight years, requires the correction of the arbitrary ordinate differences between the single data blocks and the interpolation of the missing restcurve values inside the gaps. For this purpose single pairs of consecutive blocks were adjusted by a polynomial of the third order. For data blocks of more than two month the data set was restricted to an interval of two month only, either at the end or at the beginning of the blocks, because the fitting of the data at the borders of the gaps is better, when shorter time intervals were used. Thus a level correction for each block except for the first one and a set of polynomial coefficients were obtained, which were used to interpolate the missing values inside the gaps between the data blocks. To close the gap caused by the investigations at the Pecny station, the restcurve data obtained by the analysis of the measurements at Pecny itself were used, because they yielded a better fitting than the interpolated data in this gap.

When the level corrections were added and the gaps were filled up with the interpolated values, the restcurve was used as the basis for the computation of the drift function by least squares method. A good first approximation to the restcurve is given by a polynomial of the third order and an additional harmonic wave with a period T_y in the order of one year. The monotonous decrease of the amplitude of the trigonometric terms was described by a linear dependence on time.

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The drift function is then

$$(3.2)\quad D_y(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + (a_4 + a_6 t)\cos\frac{2\pi}{T_y}t + (a_5 + a_7 t)\sin\frac{2\pi}{T_y}t \;,$$

where the trigonometric terms describe the superposition of a wave of a constant amplitude D_{y0} and phase lag φ_{y0} with a second one of the same period but with a linear time-dependent amplitude D_{yt} and phase lag ψ_y

$$(3.3)\quad D_y(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + D_{y0}\cos\left(\frac{2\pi}{T_y}t + \varphi_{y0}\right) + D_{yt}\cos\left(\frac{2\pi}{T_y}t + \psi_y\right)$$

with

$$D_{y0} = \sqrt{a_4^2 + a_5^2} \qquad \varphi_{y0} = -\arctan\frac{a_5}{a_4}$$

$$D_{yt} = t\sqrt{a_6^2 + a_7^2} \qquad \psi_y = -\arctan\frac{a_7}{a_6}$$

The period T_y was determined iteratively altering the period in steps of a tenth of a day. When the mean square error reached a minimum, the corresponding period and coefficients were assumed to be the right.

The first constituent of the drift function obtained in such a way contains a large linear term characterizing the rise of the data, and a quadratic and a cubic term as a feature of the steadily decreasing slope of the curve. The period amounts to 366.1 days; in the course of eight years the amplitude decreases from about 1000 µGal in March 1974 to 230 µGal in February 1982. The mean amplitude is 587.4 ± 4.8 µGal valid in March 1978 (central day of the adjustment).

When this first and predominate part was eliminated, the restcurve still showed a periodic character (fig. 3). Therefore further consecutive adjustments were performed in a similar manner using the same model and the same decision rule.
Only the periods were varied. The sequence of them was determined by the values of the corresponding amplitudes by turn of decreasing values so that the maximum diminuation of the mean square error in each step was attained. Step by step a series of further eight periods was obtained (tab. 2, e.g. fig. 3 a, b), all of them between 100 and 1000 days, outside the range of the interesting long-periodic tides M_f and M_m. Generally the coefficients of the adjoined polynomials were notequal zero but very small in comparison with those of the first adjustment. These terms are of purely trigonometric character, but the amplitudes are much smaller than that of the yearly term. Their time dependence is quite different according to the periods as one can see in the last columns of table 2 (amplitude in the central day, in the beginning and in the end of the observation interval).

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This consecutive determination in the manner of a "pre-whitening" leads to a stepwise diminuation of the range of the ordinates in the order of 10^2 and therefore to a higher resolution in the wavelength with regard to the terms of small amplitude, particularly if the principle of maximum diminuation of mean square errors was used consequently. As one can see in the last restcurve (fig. 3c) the elimination of the long time drift in the range above 100 days by this procedure yields a data set suitable for further analysis concerning the M_f - and M_m - tides.

The procedure described above is justified by numerical considerations only. There are several causes, that the attempt to determine the complete drift function consisting in a polynomial and nine trigonometric terms of different periods and amplitudes in the form

$$(3.4) \quad D(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \sum_{i=1}^{9} (a_{4i} + a_{6i} t) \cos\frac{2\pi}{T_i} t + (a_{5i} + a_{7i} t) \sin\frac{2\pi}{T_i} t$$

by a single adjustment was not sucessful and yielded an unstable system of normal equations. At first there is the large range of ordinate values of the original restcurve from 500 to 15000 μGal. Secondly the amplitudes adjoined to these trigonometric terms are very different also and moreover time dependent in different scales (tab. 2, figs. 2-3). Thirdly the time series is not long enough that it would be possible to separate waves with periods of some hundred days differing by a few days only.

The physical mechanisms causing the single longperiodic terms in the drift function are at present not yet known. Probably most of them are not connected with geophysical phenomena. For example the big deviation which occured in 1978 (fig. 2 and remarkable peaks in fig. 3a,b) may be produced by some additional experiments performed by DITTFELD. He gives the following explanation: This disturbance might be caused by an artificial jump of the air humidity in the station from 36.5% to 71.0% for some days at the end of March 1978. Also the natural periodic variations of the air humidity in the station seem to be influencing the drift rate. A regular yearly variation of the humidity in the station between 1974 and 1979 with an amplitude of 10.72% and a period of 362 days was observed. This value agrees very well with the yearly drift term of the gravimeter, and every year the minimum of the drift curve lies nearly 100 days later than the minimum of the humidity. It means that not only the temperature but also the humidity should be very constant inside a Earth tide station.

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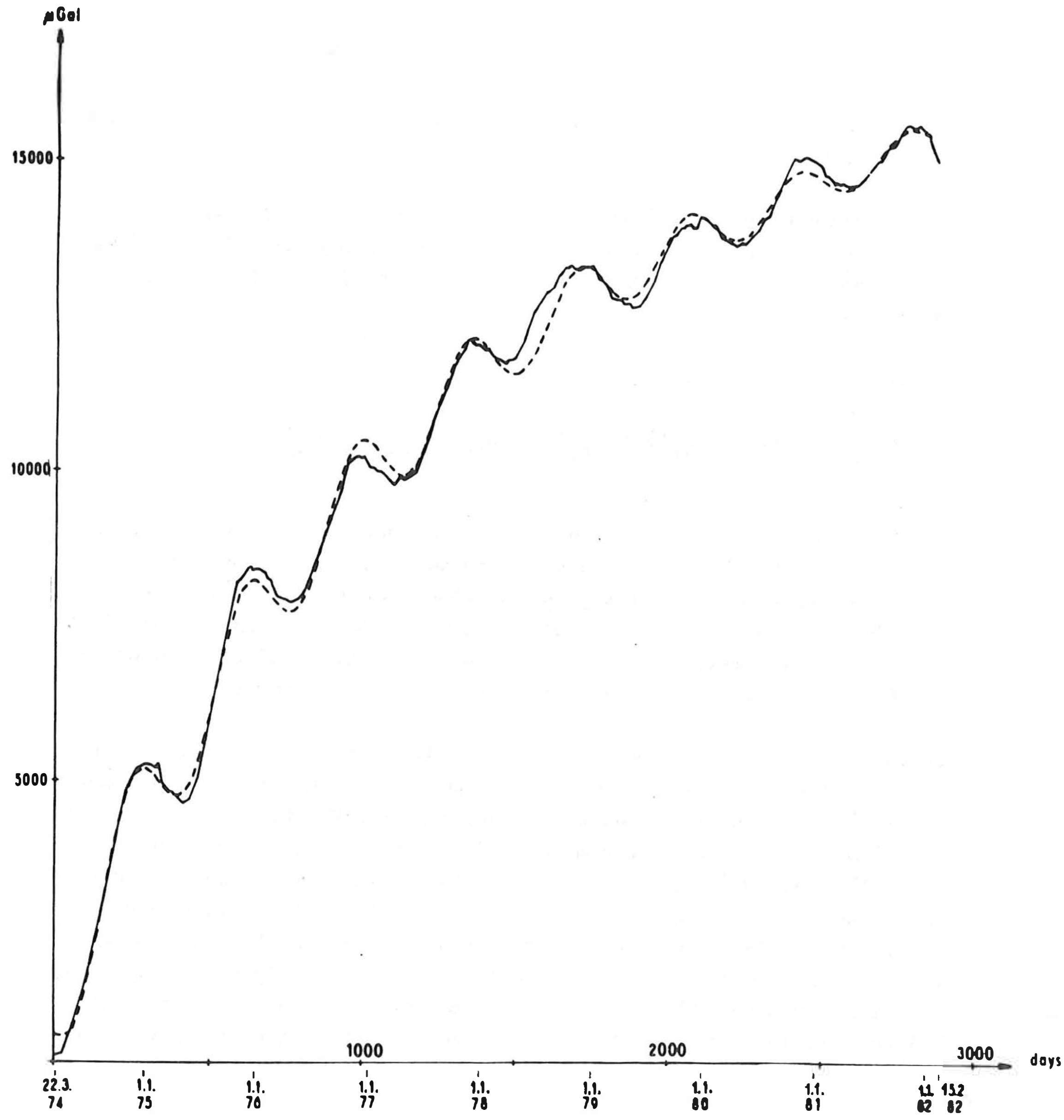

Fig. 2: Restcurve for the gravimeter GS 15 Nr. 222 at Potsdam after jump corrections and interpolation of the gaps (full line) and the main drift constituents: polynom of third order and a yearly period (broken line).

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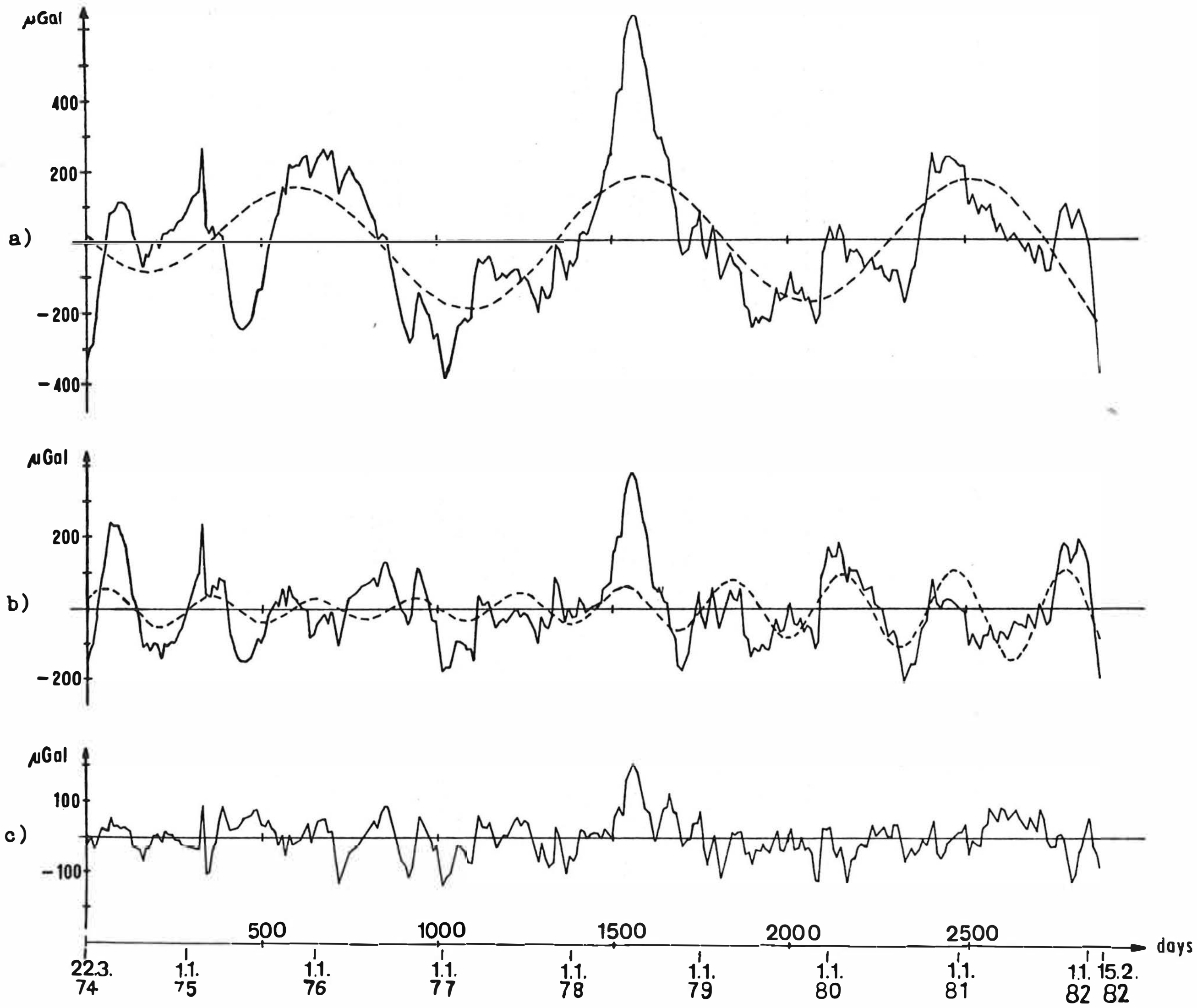

Fig. 3, a-c: Representation of the sequence of the drift elimination. The data set in fig. 3c designated by RRK8GL9 is the result after elimination of the 9th harmonic constituent. This data set is the basis for further investigations.

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Table 2: Constituents of the analytic drift function determined by the adjustment of the restcurve after formula 3.2 (central day March the 2nd 1978) in the order of their computation

Period [d]	m_o [μGal]	a_o [μGal]	a_1 [μGal/d]	a_2 [μGal/d^2]	a_3 [μGal/d^3]	a_4 [μGal]	a_5 [μGal]	a_6 [μGal/d]	a_7 [μGal/d]	D_o [μGal]	φ_o [rad]	D_{-1440} [μGal]	D_{+1440} [μGal]
366.1	179.56	11892.141 ± 5.038	3.7695 $\pm 0.1008\cdot10^{-1}$	$-0.1656\cdot10^{-2}$ $\pm 0.5514\cdot10^{-5}$	$0.5321\cdot10^{-6}$ $\pm 0.7444\cdot10^{-8}$	-143.860 ± 4.766	-569.536 ± 4.773	$-0.2171\cdot10^{-1}$ $\pm 0.5800\cdot10^{-2}$	0.2909 $\pm 0.5726\cdot10^{-2}$	587.42	1.323	994.82	231.00
966.0	134.24	- 7.8032 ± 3.8995	$0.5385\cdot10^{-1}$ $\pm 0.8378\cdot10^{-2}$	$0.5694\cdot10^{-5}$ $\pm 0.4852\cdot10^{-5}$	$-0.6435\cdot10^{-7}$ $\pm 0.6563\cdot10^{-8}$	114.319 ± 3.566	132.292 ± 3.925	$0.1528\cdot10^{-1}$ $\pm 0.4759\cdot10^{-2}$	$0.1009\cdot10^{-1}$ $\pm 0.5300\cdot10^{-2}$	174.84	0.858	149.63	200.35
465.0	102.65	- 1.0017 ± 2.8812	$0.1446\cdot10^{-1}$ $\pm 0.5855\cdot10^{-2}$	$0.9262\cdot10^{-6}$ $\pm 0.3166\cdot10^{-5}$	$-0.1245\cdot10^{-7}$ $\pm 0.4392\cdot10^{-8}$	59.105 ± 2.689	78.405 ± 2.773	$0.8136\cdot10^{-1}$ $\pm 0.3274\cdot10^{-2}$	$-0.3285\cdot10^{-1}$ $\pm 0.3464\cdot10^{-2}$	98.19	-0.925	216.50	65.86
317.0	87.81	4.3239 ± 2.4604	$0.8542\cdot10^{-2}$ $\pm 0.4931\cdot10^{-2}$	$-0.1044\cdot10^{-4}$ $\pm 0.2681\cdot10^{-5}$	$-0.1026\cdot10^{-7}$ $\pm 0.3639\cdot10^{-8}$	- 13.666 ± 2.308	50.074 ± 2.345	$0.1926\cdot10^{-1}$ $\pm 0.2764\cdot10^{-2}$	$0.6512\cdot10^{-1}$ $\pm 0.2861\cdot10^{-2}$	51.91	-1.304	60.20	144.53
661.1	75.35	6.0759 ± 2.1162	$0.3200\cdot10^{-2}$ $\pm 0.4457\cdot10^{-2}$	$-0.1362\cdot10^{-4}$ $\pm 0.2324\cdot10^{-5}$	$-0.1665\cdot10^{-8}$ $\pm 0.3493\cdot10^{-8}$	54.933 ± 2.022	32.053 ± 2.020	$0.4446\cdot10^{-2}$ $\pm 0.2716\cdot10^{-2}$	$0.7315\cdot10^{-2}$ $\pm 0.2492\cdot10^{-2}$	63.60	0.528	53.09	74.67
207.0	64.96	0.2786 ± 1.8162	$0.4585\cdot10^{-2}$ $\pm 0.3636\cdot10^{-2}$	$-0.7529\cdot10^{-6}$ $\pm 0.1961\cdot10^{-6}$	$-0.4898\cdot10^{-8}$ $\pm 0.2675\cdot10^{-8}$	- 31.242 ± 1.721	- 44.062 ± 1.708	$0.3611\cdot10^{-2}$ $\pm 0.2094\cdot10^{-2}$	$0.4607\cdot10^{-2}$ $\pm 0.2035\cdot10^{-2}$	54.01	0.954	62.43	45.60
250.1	57.06	0.7921 ± 1.5943	$-0.4591\cdot10^{-2}$ $\pm 0.3207\cdot10^{-2}$	$-0.2025\cdot10^{-5}$ $\pm 0.1718\cdot10^{-5}$	$0.5273\cdot10^{-8}$ $\pm 0.2374\cdot10^{-8}$	- 34.830 ± 1.512	- 14.933 ± 1.500	$0.3131\cdot10^{-1}$ $\pm 0.1853\cdot10^{-2}$	$0.1140\cdot10^{-1}$ $\pm 0.1795\cdot10^{-2}$	37.90	0.405	85.85	10.36
165.8	55.20	0.0649 ± 1.5417	$-0.6564\cdot10^{-3}$ $\pm 0.3092\cdot10^{-2}$	$-0.2320\cdot10^{-6}$ $\pm 0.1658\cdot10^{-5}$	$0.7340\cdot10^{-9}$ $\pm 0.2277\cdot10^{-8}$	3.895 ± 1.452	0.7366 ± 1.457	$0.4442\cdot10^{-2}$ $\pm 0.1749\cdot10^{-2}$	$0.2397\cdot10^{-1}$ $\pm 0.1758\cdot10^{-2}$	3.96	0.0396	33.87	36.72
133.3	53.51	- 0.2566 ± 1.4943	$-0.2962\cdot10^{-3}$ $\pm 0.2993\cdot10^{-2}$	$0.5746\cdot10^{-6}$ $\pm 0.1607\cdot10^{-5}$	$0.3199\cdot10^{-9}$ $\pm 0.2200\cdot10^{-8}$	- 2.789 ± 1.414	7.098 ± 1.405	$0.1157\cdot10^{-2}$ $\pm 0.1715\cdot10^{-2}$	$0.2077\cdot10^{-1}$ $\pm 0.1678\cdot10^{-2}$	7.63	-1.196	23.24	37.02
$\sum_{i=1}^{9}$		11894.615	3.8486	$-1.6759\cdot10^{-3}$	$4.4480\cdot10^{-7}$								

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4. Parameter estimation for the M_f-group by several numerical methods

The methods which will be applied in chapter 4 might be subdivided by several points of view: either the consideration of spectral properties or the order of mathematical operations.

On the one hand (chapter 4.1) here procedures will be used which need absolutely the temporal variations of gravity due to the theoretical development of the tide generating potential. It means that they are uniquely restricted on those periodicities which also occur in the development. On the other hand (chapter 4.2) there methods exist which consider the amplitudes of the gravity variation in the whole M_f-range and its surroundings. Here we are unable to decide which of the frequencies should be used for the determination of the δ-factor. Only by considerations of the theoretical time series either by analytical computations (see chapter 2) or by numerical procedures (chapter 4.2) we get the suitable frequency.

To evaluate the validity of the results it is important to know whether completely different mathematical treatments were used. Each of these methods must include the estimation of a mean value (due to the presence of noise in our observation series) and a determination of the relation between theoretical and empirical values (due to the fact that δ is itself a relation between these quantities).

In chapter 4.1 at first we undertake an assignment between the theoretical and the observed values and then the computation of the mean value is carried out. In chapter 4.2 this order is changed. By the CHOJNICKI-procedure in chapter 4.1 the allocation of these both values in the temporal domain is taken by the error equations and the summation is carried out in the formalism for the normal equations. By the BUYS-BALLOT-schema in chapter 4.1 the assignment is undertaken in the phase domain and the following summation for the mean value too.

In chapter 4.2 the estimation either for the Fourier-series representation and for the estimation of the autocovariance function is taken place in the temporal domain at first, whereas the calculation of the relationship follows in the frequency domain.

4.1 Methods in the temporal domain

4.1.1 Chojnicki-analyses

The Chojnicki analysis (CHOJNICKI 1973) is one of the standard procedures which determine the parameters of the tides by the principle of the least squares rule. It requires an elimination of the instrumental drift either by the aid of several filters usuable in the case of the short-periodic tides or by other methods in the case of long-periodic zonal tides. In the computation of the zonal tides we have used as well the zero point method after LASSOVSKI as the analytic drift function described in chapter 3. This function includes only periods longer than 100 days, and therefore zonal tides not longer than one month remain undisturbed.

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The results obtained by the two kinds of treatment are considerably different. The zero point method yields the largest value for the δ factor of the M_f tide but the smallest mean square error (see the paper by DITTFELD, H.-J., this issue, page 7)

(4.1.1.1) $\delta_{M_f} = 1.2768 \pm 0.0033$ $\varkappa_{M_f} = -0^\circ.420 \pm 0.149$

The analysis of the drift-free restcurve (RRK8GL9, fig. 3c) results

(4.1.1.2) $\delta_{M_f} = 1.035 \pm 0,449$ $\varkappa_{M_f} = -10^\circ.60 \pm 24.86$

In the same manner we obtained the parameters of the M_m tide, but its mean square errors are still larger than those of the M_f tide due to the smaller amplitude of this tide. The results given in the same order as those of the M_f tide are

(4.1.1.3) $\delta_{M_m} = 1.1254 \pm 0.0043$ $\varkappa_{M_m} = -0^\circ.335 \pm 0.218$

(4.1.1.4) $\delta_{M_m} = 1.209 \pm 0.597$ $\varkappa_{M_m} = 19^\circ.34 \pm 28.26$

The remarkably higher inner accuracy of the parameters of the first treatment is caused by the very close fitting of the zero point-drift to the observations. But the differences between the parameters of both the treatments indicate a significant character, which must be cleared by further investigations.

4.1.2. Determination of the tides M_f (and M_m) by the aid of a modified Buys-Ballot-method

The synchronization method may be applied to the determination of tides, because their periods are well known. Therefore it is possible to attribute to each date of observation a distinct value of the phase angle χ always situated between 0 and 2π. The connection between χ and the time is given by:

(4.1.2.1) $$2\pi k + \chi = 2\pi \frac{t}{T},$$

where T is the wavelength of the tide and k = 0, 1, 2 ... .
In the case of a superposition of two waves with slightly different periods this expression must be changed, since T is now a function of t as considered in chapter 2 (2.1.4, 2.1.12). The time - dependent phase may then be written in the form

(4.1.2.2) $$\bar{\nu} t + \bar{\varphi} - P_2(t) = 2\pi k + \chi(t)$$

where $0 \leq \chi(t) \leq 2\pi$. If $P_2(t)$ is not a linear function of t, then the length of the interval 2π of the phase angle corresponds to the time-dependent period T_2 and for the whole time interval t_A to t_E 2π must be assigned to the mean period $\bar{T}_2$.

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WYROWNANIE KONCOWE - FINAL ADJUSTMENT CHOJNICKI METHOD

KONCOWE WYNIKI OBLICZEN - OCENA DOKLADNOSCI NA PODSTAWIE RESIDUUM
FINAL RESULTS OF COMPUTATIONS - ESTIMATION OF ACCURACY BASED ON RESIDUAL

0.0 1978 731 0 0 1001111 52.3809 -13.0676 820 0.0 981.261200

DATEI RRK8GL9

STATION POTSDAM, GRAVIMETER NR.222, 1974-1982
SPRUNG- UND GANGBEFREIT VON HARNISCH (SPRUENGE NACH SPOLYN, LANGZEIT-
GANG NACH ZAGANG IN 9 ITERATIVEN ANSAETZEN MIT ZEITABHAENGIGEN AMPLITUDEN
RESTKURVE
VOLLSTAENDIGE CHOJNICKI-ANALYSE

LEAST SQUARE ANALYSIS IN CLASSICAL MANNER (CHOJNICKI)

POTENTIAL CARTWRIGHT-EDDEN-(DOODSON) / COMPLETE EXPANSION
COMPUTATION - ZENTRALINSTITUT FUER ASTROPHYSIK,KIARLT-POTSDAM - COMPUTER ES 1040

74 3 22 0- 82 2 14 3

TOTAL NUMBER OF DAYS 2886 69268 READINGS

WAVE ARGUMENT	GROUP N	SYMBOL	ESTIM. AMPL.	AMPLITUDE FACTOR VALUE	AMPLITUDE FACTOR R.M.S.	PHASE DIFFERENCE VALUE	PHASE DIFFERENCE R.M.S.	SUM OF AMPLIT.
055.-055.	4	LP	2.3756	20.83273	10.29202	94.108	8.645	2.3876
056.-056.	4	SA	0.4368	73.80747	4.44287	-179.164	3.443	0.4453
057.-057.	7	SSA	2.6398	5.56004	0.72699	-161.113	7.480	2.7374
058.-059.	3	STA	0.1503	16.74540	12.37128	89.947	42.529	0.1619
062.-063.	8	MSM	0.6195	2.89938	3.07592	68.366	60.778	0.6797
064.-068.	19	MM	3.4142	1.20869	0.59602	19.336	28.263	3.7974
071.-074.	13	MSF	0.4708	0.84476	3.90296	-80.677	264.711	0.6400
075.-077.	15	MF	3.3354	1.03521	0.44877	-10.601	24.855	3.5187
080.-083.	12	MSTM	0.0900	3.32137	10.31327	36.018	177.927	0.4258
084.-086.	19	MTM	0.6617	1.00351	2.36122	-27.178	134.778	1.6446
091.-0E3.	23	MSQM	0.1561	2.12174	11.11713	-96.332	300.195	0.6057
105.-11X.	14	SM01	0.1949	0.06992	0.08344	-101.786	68.380	0.5335
124.-125.	7	2Q1	0.5887	0.12364	0.02728	-176.665	12.642	1.0205
126.-129.	14	SIG1	0.7177	0.08070	0.02246	-157.379	15.947	1.2718
133.-135.	15	Q1	4.5629	0.01883	0.00360	166.836	10.956	7.3733
136.-139.	15	RO1	0.9186	0.04173	0.01886	-179.081	25.894	1.5977
143.-146.	18	O1	24.4809	0.00563	0.00069	158.873	6.999	36.6814
147.-149.	8	TAU1	0.4769	0.05600	0.03847	39.828	39.359	0.5714
152.-155.	15	M1	2.3459	0.01492	0.00682	-127.552	26.173	5.2276
156.-158.	7	CHI1	0.3524	0.06370	0.04541	-49.627	40.842	0.6163
161.-162.	3	PI1	0.7970	0.03564	0.02189	135.059	35.197	0.8606
163.-163.	7	P1	14.1986	0.00142	0.00128	-0.296	51.763	14.2122
164.-164.	3	S1	0.2399	0.14157	0.07616	-106.466	30.826	0.4582
165.-165.	11	K1	37.3513	0.00065	0.00046	35.171	40.653	49.1078
166.-166.	2	PSI1	0.3305	0.08526	0.05440	66.839	36.559	0.3433
167.-168.	7	PHI1	0.5654	0.01017	0.02952	-65.943	166.286	0.7073
172.-173.	6	THE1	0.3780	0.06047	0.04438	-57.907	42.055	0.5993
174.-177.	16	J1	1.8125	0.01941	0.00865	169.725	25.535	3.4593
181.-184.	6	SO1	0.2893	0.13044	0.05439	164.521	23.892	0.5370
185.-186.	10	OO1	0.5513	0.02101	0.02011	117.852	54.836	2.6172
191.-1E3.	19	NY1	0.1221	0.14033	0.09877	81.579	40.324	0.7872
207.-22X.	21	EPS1	0.2365	0.06776	0.03723	-87.657	31.481	0.5107
233.-236.	10	2N2	0.6448	0.02331	0.01212	-30.039	29.799	0.9613
237.-23X.	10	MY2	0.8404	0.03355	0.00989	-46.155	16.895	1.0178
243.-246.	17	N2	5.2052	0.00428	0.00160	-72.000	21.344	6.4219
247.-248.	7	NY2	0.9986	0.00572	0.00830	130.260	83.158	1.1313
252.-258.	26	M2	29.0429	0.00276	0.00030	114.048	6.268	29.7350
262.-264.	5	LMB2	0.2096	0.06644	0.04062	-98.824	35.032	0.2347
265.-267.	12	L2	1.3544	0.01995	0.00862	122.519	24.747	1.8095
271.-272.	2	T2	0.7387	0.01798	0.01136	-133.045	56.203	0.7980
273.-273.	4	S2	13.0482	0.00106	0.00067	41.462	36.211	13.1195
274.-277.	12	K2	2.5557	0.01032	0.00298	9.000	16.543	5.0065
282.-285.	15	ETA2	0.1304	0.02235	0.05593	134.238	143.404	0.4496
292.-2X5.	14	2K2	0.0208	0.21858	0.18580	-165.414	48.701	0.2204
327.-375.	17	M3	0.2723	0.02423	0.01605	-103.246	37.952	0.6128

R.M.S.ERROR M-ZERO 47.0783 MIKROGAL

R.M.S.ERROR FOR BANDS LP 357.6504 D 3.3654 SD 1.6220 TD 1.1004

O1/K1 8.6227 1-O1/1-K1 0.9950 M2/O1 0.4899

REFERENCE EPOCH 1978 7 31 0.0

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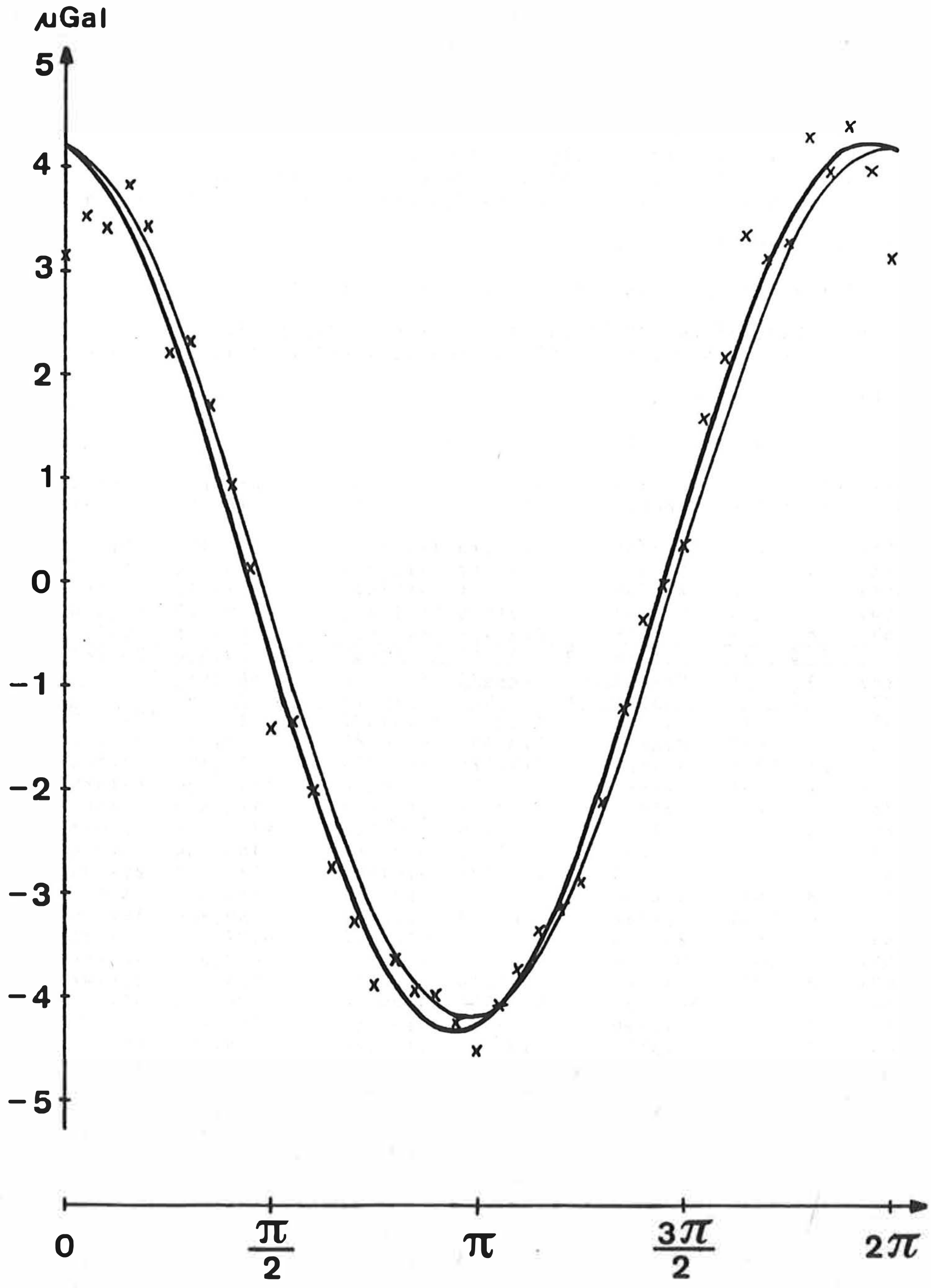

Fig. 4: Mean values of the theoretic gravity variations due to the largest two M_f waves (thin line) and of the empiric drift-free restcurve RHK8GL9 (crosses) for fourty sections in the interval between 0 and 2π according a Buys-Ballot-scheme. The mean values for the empiric data were then approximated by a trigonometric function (thick line).

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For the practical computations the whole range of 2π is subdivided into 40 sections of equal length of arc. For each date of observation the corresponding section was determined in dependence on $\chi(t)$, and both the theoretic values and the empiric ones were arranged in the forty sections. Finally, for each section the mean values were computed (fig. 4). Due to the averaging properties in the empiric values a large part of the accidental errors is removed and the function shows a similar shape as the theoretic one in accordance to formula (2.1.4). Therefore the amplitudes and phases for both the time series were determined by an adjustment by a trigonometric function with the wavelength $\overline{T}_2$ (2.1.19). So we get the amplitude $\overline{R}_2$ and the phase angle $\overline{\alpha}_2$ for the theoretical series and $\overline{R}_e$ and $\overline{\alpha}_e$ for the empiric one (measured data), and the gravimeter factor $\delta_{M_f} = \overline{R}_e/\overline{R}_2$ and the phase lag $\varkappa_{M_f} = \overline{\alpha}_e$.

Using the data of the drift-free restcurve (RRK8GL9, fig.3c) we obtained $\overline{R}_e = 4.295\,\mu$Gal and $\overline{\alpha}_e = 5\overset{\circ}{.}396$. From the theoretic data we found $\overline{R}_2 = 4.186\,\mu$Gal. With these values the parameters

$$(4.1.2.3) \qquad \delta_{M_f} = 1.026 \pm 0.020 \qquad\qquad \varkappa_{M_f} = 5\overset{\circ}{.}396 \pm 1.130$$

were calculated.

Using these parameters the single values of the two main tides of the M_f group were computed and subtracted from the restcurve. So we got a new restcurve, which was applied for the determination of the parameters of the M_m tide in quite the same manner, but using the relations mentioned above for a single wave only. The resulting curves are to be seen in fig. 5. The adjustment by a trigonometric function was carried out and yields the parameters $R_e = 3.623\,\mu$Gal, $R_t = 2.9656\,\mu$Gal, $\alpha_e = 26\overset{\circ}{.}08$.
The results are then

$$(4.1.2.4) \qquad \delta_{M_m} = 1.222 \pm 0.061 \qquad\qquad \varkappa_{M_m} = 26\overset{\circ}{.}08 \pm 3.50\,.$$

4.2. Methods in spectral domain

4.2.1. Parameterestimation under the influence of noise

The use of the least squares rule for the estimation of a mean value by averaging over a sample requires a white noise in the frequencies under consideration. This is a strong preposition, and we looked for a method which is unsensitive about the character of the noise.

From a general point of view our empirical gravimetric restcurve might be considered in the M_f region as the sum of a few harmonic constituents (f.i. see equ. (2.1.7)) and an additional noise $W(t) = W(t, \nu_i)$ in dependence on the frequency ν_i.

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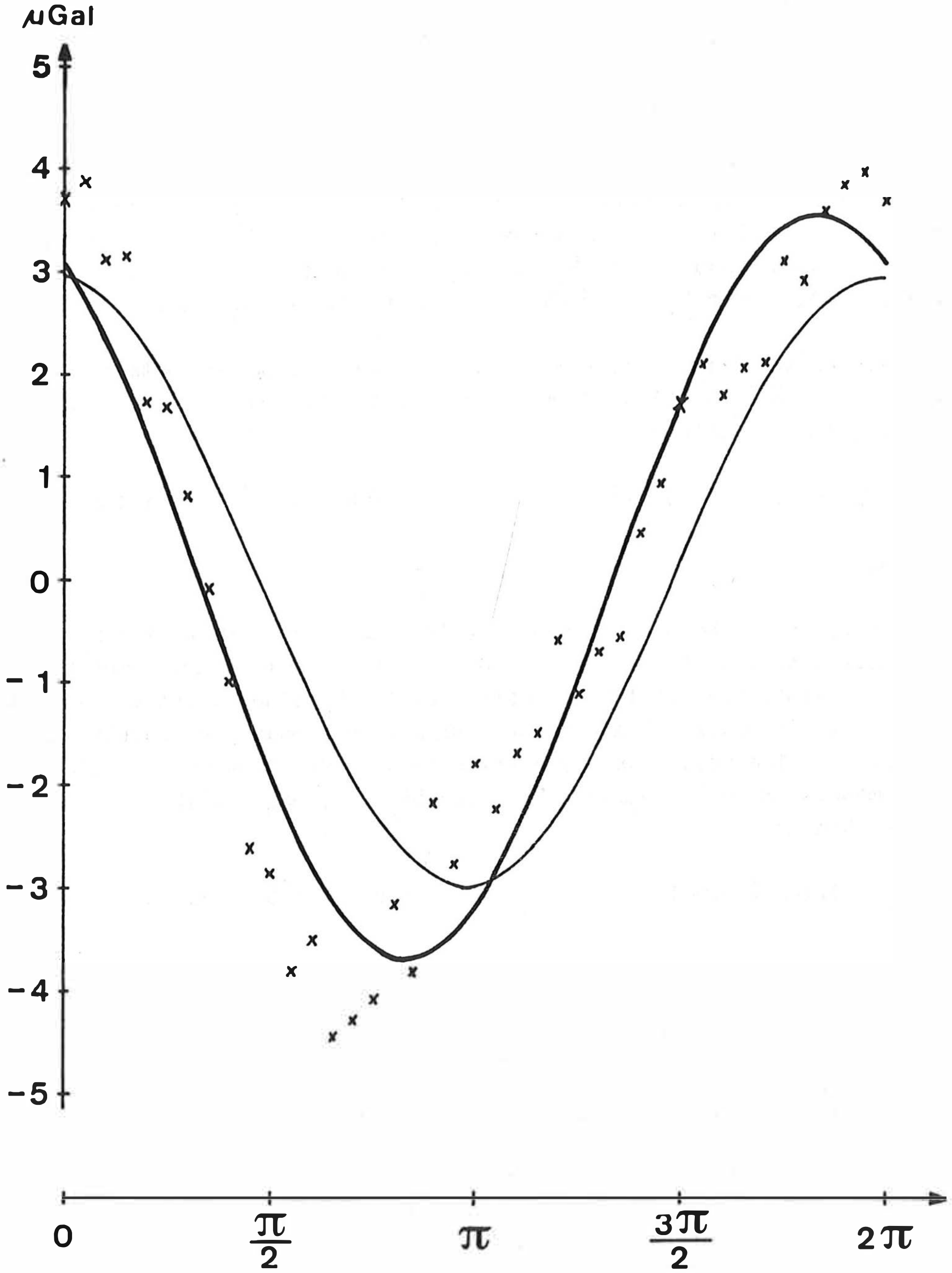

Fig. 5: Theoretic gravity variation due to the M_m wave (thin line) and mean values of the empiric drift-free restcurve RHK8GL9 (crosses) for fourty sections in the interval between 0 and 2π. Mean values for the empiric data were then approximated by a trigonometric function (thick line).

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.Therefore the stochastic model

(4.2.1.1) $$\Gamma(t) = \sum_{i=1}^{m} (A_i \cos(\nu_i t + \varphi_i) + W(t, \nu_i))$$

for the gravity variations may be supposed. For the M_f-wave group holds m = 15.

KURTHS (1982) describes algorithms based either on the temporal domain or on the estimation of autocovariance functions. The parameter estimation A_i takes place by the least squares error rule using the Marquardt method of regularization, whereby ν_i is approximately known. The regularizator $\lambda_{i,opt}$ for the i-th component is given by

(4.2.1.2) $$\lambda_{i,opt} = \frac{2}{(A_i)^2} \sum_{\tau=0}^{T_{ACF}} \bar{C}_{uu}(\tau) \cos \nu_i \tau = \frac{2W_i}{(A_i)^2}$$

whereas $\lambda_{i,opt}$ gives the (unknown) relation between the noise W_i to the amplitude A_i and follows from

(4.2.1.3) $$\sum_{\tau=0}^{T_{ACF}} \left(\frac{A_i^2}{2} \cos \nu_i \tau - \frac{\bar{A}_{i,\,reg}^2}{2} \cos \nu_i \tau \right)^2 \longrightarrow \text{Min.}$$

and $\bar{C}_{uu}$ is the empirical ACF of the noise and the other waves, T_{ACF} is the length of the ACF.

Without modelling the noise explicitely it can be stated that the signal is optimum described by this way. The Marquardt algorithm realizes a generalized pre-whitening, i.e. a minimum white noise on the frequency ν_i. In practical determination the exact value of ν_i is determined by a few iterations; the corresponding $\bar{A}_i$ is then valid.

The data set used for this method consists in the daily mean values of the data set RRK8GL9. The autocovariance function shows a considerable long-term contribution without a remarkable frequency ν_i. Therefore the temporal domain gives the best results. We obtain

$$T^E_{M_f} = 13.676 \text{ d} \qquad T^T_{M_f} = 13.671 \text{ d}$$

(4.2.1.4) $$R^E_{M_f} = 4.251\,\mu\text{Gal} \qquad R^T_{M_f} = 4.172\,\mu\text{Gal}$$

$$\delta_{M_f} = 1.019$$

From this result (difference to the other ones < 2%) we may conclude that in the several methods a white noise contribution (noise in the pure statistical sense) does not falsify the results. This gives the hint at the fact that the distortion of the δ - value must be caused by a non-tidal disturbance of deterministic character. To exclude possible numerical influences caused by strong spectral contributions in the farther neighbourhood of the M_f-wave we applied at next a band-pass filter.

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4.2.2 Filter methods

As in the preceeding chapter noted the autocovariance function of the daily mean values shows a slow decrease for increasing lags. It means that besides the trend for periods longer than 100 days was eliminated, long term constituents in the range of 20 days are important. Therefore a band-pass filter of the ORMSBY-type after JENTZSCH (1978) was used. Its parameter were (Tab. 3):

Table 3: Parameters for the ORMSBY-band-pass

Filter	roll-off frequency	rectangle	roll-off frequency	length of operator
M_f	0.0029976 cph (13.9 days)	0.0030193 cph (13.8 days) 0.0032808 (12.7 days)	0.0033068 cph (12.6 days)	± 1000

To avoid systematic errors of the transmission properties of the filter, both the empiric as well as the theoretic time series were treated. For the gravimeter factor one yields by FOURIER-analyses (see next chapter):

(4.2.2.1) $\delta_{M_f} = 1.019$ $\quad \varkappa_{M_f} = 6\overset{\circ}{.}3$ $\quad T_{M_f} = 13.67$ days .

The results of (4.1.1.2, δ = 1.035), (4.1.2.3, δ = 1.026), (4.2.1.4, δ = 1.019) and (4.2.2.1, δ = 1.019) differ systematically from the expected value in the order of 10 %. To get some hints for the causes of such a deviation we must look for a method which does not consider the tidal periods only.

4.2.1 Fourier-analyses

The aim of the FOURIER analyses in this chapter consists in the determination of the spectral distribution of amplitude and phase in the frequency domain of the M_f group and in its neighbourhood. The "philosophy" bases on the following consideration: The δ-values obtained in the preceeding chapters are falsified by a non-tidal disturbance. If we can reach a knowledge on the spectral constituents around the M_f spectral range then we can extrapolate from these values the disturbation for the tidal waves itself. In a second step it should be possible to find out on this basis a procedure for the correction of the δ-factor.

The method was applied both for theoretic time series for long-period waves on the basis of the CARTWRIGHT-TAYLER-EDDEN development and for the drift-free empirical data described in chapter 3 (data set RRK8GL9). For an interpretation of possible influences of the air pressure on the empiric gravity spectrum the series of local air pressure at Potsdam is also considered.

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The Fourier analysis of the theoretical series gives the expected values and serves as a "reference curve" to estimate

- the gravimeter factor δ_{M_f}

$$\delta_{M_f} = \frac{R^E_{M_f}}{R^T_{M_f}} = \frac{\sqrt{a^{E\,2}_{k_{Mf}} + b^{E\,2}_{k_{Mf}}}}{\sqrt{a^{T\,2}_{k_{Mf}} + b^{T\,2}_{k_{Mf}}}} \qquad (4.2.3.0)$$

for the k-th harmonic k_{Mf} of the wavelength 13.669 days (E means the empirical values, T the theoretical ones),

- the contribution of other influences (drift behaviour of the gravimeter, unknown correlations to the air pressure, humidity etc.) in the regions inside and outside the M_f group.

To get a quasi-continuous representation in the spectral domain we carry out several single Fourier analyses. Then their results are compiled in a common curve in dependence on the wavelength (fig. 6). Here are some explanations to understand what happens, if the length of the data interval does not contain an integer number of the wave to be determined.

At first we consider a wave $y(t) = A_0\cos(\nu_0 t + \psi_0)$ characterized by the unknowns A_0 and ψ_0. At first the time interval 2τ should contain an integer number N of periods with the frequency

$$\nu_0 = \frac{2\pi}{T_0}\ \text{, i.e.}\quad NT_0 = 2\tau,\ \nu_0 = \frac{N\pi}{\tau}\ . \qquad (4.2.3.1)$$

Inserting $y(t) = A_0\cos(\frac{N\pi}{\tau}t + \psi_0)$ into (2.2.3) the equation for the coefficients a_k and b_k may be written in the form

$$\begin{Bmatrix} a_k \\ b_k \end{Bmatrix} = \frac{1}{\tau}\int_{-\tau}^{+\tau}\left\{A_0\cos\psi_0\cos\frac{N\pi}{\tau}t\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\frac{k\pi}{\tau}t - A_0\sin\psi_0\sin\frac{N\pi}{\tau}t\begin{Bmatrix}\cos\\ \sin\end{Bmatrix}\frac{k\pi}{\tau}t\right\}dt\ . \qquad (4.2.3.2)$$

For the case $k = N = \frac{2\tau}{T_0}$, i.e. the single wave $A_0\cos(\nu_0 t + \psi_0)$ occurs N - times in the time interval 2τ, we obtain the well known result

$$\begin{aligned} &a_k = A_0\cos\psi_0 \qquad && b_k = -A_0\sin\psi_0 \qquad && \text{for } k = N \\ &a_k = 0 && b_k = 0 && \text{for } k \neq N\ . \end{aligned} \qquad (4.2.3.3)$$

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To look for gravity variations in the surroundings of the M_f - wave group we change slightly the length $2\overline{t}$ of the interval to get new Fourier coefficients a_{k_n}, b_{k_n} for a new period T_{k_n} in such a way that the M_f - wave does not contain an integer number N of periods in $2\overline{t}$, but $N+\varepsilon$, $\varepsilon<1$ and the following relations are valid:

$$(4.2.3.4) \qquad 2\overline{t} = NT_0 + \varepsilon T_0 = T_0(N+\varepsilon), \quad T_0 = \frac{2\overline{t}}{(N+\varepsilon)}, \quad \varepsilon = \frac{2\overline{t} - NT_0}{T_0}$$

and

$$(4.2.3.5) \qquad \nu_0 = \frac{2\pi}{T_0} = \frac{2\pi(N+\varepsilon)}{2\overline{t}} = \frac{\pi(N+\varepsilon)}{\overline{t}}$$

In this context we consider not only the wavenumber $k = N$, but also the wave numbers $k = N \pm j$, $j = 1,2,3$. It means that we use a set of $2j + 1$ coefficients for the spectral representation in the neighbourhood of the M_f wave group.

Inserting $\nu_0 = \frac{\pi(N+\varepsilon)}{\overline{t}}$ and $k=N\pm j$ in the equation (4.2.3.2) and integrating with the help of the well known indefinite integral relations we get:

$$(4.2.3.6) \qquad a_{N\pm j} = \frac{\overline{t}A_0\cos\psi_0}{\pi^2((N+\varepsilon)^2-(N\pm j)^2)} \cdot \left[\frac{\pi(N+\varepsilon)}{\overline{t}}\sin\frac{(N+\varepsilon)\pi}{\overline{t}}t\,\cos\frac{(N\pm j)\pi}{\overline{t}}t - \right.$$
$$\left. - \frac{\pi(N\pm j)}{\overline{t}}\cos\frac{(N+\varepsilon)\pi}{\overline{t}}t\,\sin\frac{(N\pm j)\pi}{\overline{t}}t\right]\Bigg|_{-\overline{t}}^{+\overline{t}} + \frac{\overline{t}A_0\sin\psi_0}{\pi^2((N+\varepsilon)^2-(N\pm j)^2)} \cdot$$
$$\cdot \left[\frac{\pi(N+\varepsilon)}{\overline{t}}\cos\frac{(N+\varepsilon)\pi}{\overline{t}}t\,\cos\frac{(N\pm j)\pi}{\overline{t}}t - \frac{(N\pm j)\pi}{\overline{t}}\sin\frac{(N+\varepsilon)\pi}{\overline{t}}t \cdot \right.$$
$$\left. \cdot \sin\frac{(N\pm j)\pi}{\overline{t}}t\right]\Bigg|_{-\overline{t}}^{+\overline{t}}$$

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The second term vanishes since $\cos(-x) = \cos x$ and $\sin\pi(N\pm j) = 0$, $\sin(-x) = -\sin x$ and also in the first item the second term vanishes for this reason.
For the first term we obtain under consideration of

$$\cos(N\pm j)\pi = \cos N\pi \cos j\pi = \begin{cases} +1 & \text{for N and j even or odd} \\ -1 & \text{for N or j odd} \end{cases}$$

and

$$\sin(N+\varepsilon) = \cos N\pi \sin\varepsilon\pi = \pm \sin\varepsilon\pi \quad \text{for} \quad N: \begin{cases} \text{even} \\ \text{odd} \end{cases}$$

the following expression

$$(4.2.3.7) \qquad a_{N\pm j} = \frac{2\tau A_0\cos\psi_0}{\pi^2((N+\varepsilon)^2-(N\pm j)^2)} \cdot \left(\frac{\pi(N+\varepsilon)}{\tau} \cos N\pi \sin\varepsilon\pi \cos N\pi \cos j\pi\right)$$

$$= \frac{2A_0\cos\psi_0(N+\varepsilon)\sin\varepsilon\pi}{\pi((N+\varepsilon)^2-(N\pm j)^2)} \cdot (-1)^j \quad .$$

The denominator may be transformed into

$$(4.2.3.8) \qquad 2N\varepsilon + \varepsilon^2 \mp 2Nj - j^2 = N\varepsilon + \varepsilon^2 \mp 2Nj - j^2 + N\varepsilon$$

$$= (N+\varepsilon)\left(\varepsilon + \frac{\mp j\,(2N\pm j) + N\varepsilon}{N+\varepsilon}\right)$$

and for $N \ll \varepsilon < 1$ we get

$$(4.2.3.9) \qquad 2N\varepsilon + \varepsilon^2 \mp 2Nj - j^2 \approx 2(N+\varepsilon)\left(\varepsilon + \frac{\mp j(2N\pm j)}{2N}\right)$$

$$= 2(N+\varepsilon)\left(\varepsilon \mp j \pm \frac{j^2}{2N}\right) .$$

Finally we obtain the Fourier coefficients

$$(4.2.3.10) \qquad a_{N\pm j} = \frac{A_0\cos\psi_0\sin\varepsilon\pi}{\pi\left(\varepsilon\mp j \pm \frac{j^2}{2N}\right)} \cdot (-1)^j$$

and by a similar way

$$(4.2.3.11) \qquad b_{N\pm j} = \frac{A_0\sin\psi_0\sin\varepsilon\pi}{\pi\left(\varepsilon\mp j \pm \frac{j^2}{2N}\right)} \cdot (-1)^{j+1} \quad ,$$

so that for the amplitude $R_k = \sqrt{a_k^2 + b_k^2}$ and the phase φ_0 $\left(-\tan\varphi_0 = \frac{b_k}{a_k}\right)$ we get the relations

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$$(4.2.3.12) \quad R_{N\pm j} = A_0 \left| \frac{\sin \varepsilon\pi}{\pi(\varepsilon \mp j \pm \frac{j^2}{2N})} \right|$$

and

$$(4.2.3.13) \quad \tan \varphi_{N\pm j} = \tan \psi_0 \, .$$

The phase value ψ_0 of the single wave $y(t)$ is not changed by the Fourier representation.

Neglecting in (4.2.3.12) the term $\pm \frac{j^2}{2N}$ (which is justified for $N>100$ and $j \leq 3$ for an accuracy of 1% of the amplitudes) and introducing $\varepsilon \mp j = \varrho$ we obtain

$$(4.2.3.14) \quad R_{N\pm j} = A_0 \left| \frac{\sin(\varrho \pm j)\pi}{\pi\varrho} \right| = A_0 \left| \frac{\sin \pi\varrho}{\pi\varrho} \right|$$

It means that the spectral representation of a single wave $y(t) = A_0 \cos(\nu_0 t + \psi_0)$ on the basis of the finite time interval 2τ does not give a δ- function, but a sinx/x - function.

If the interval used for the Fourier representation of a single harmonic wave does not contain exactly an integer number of periods of this wave a certain scattering of energy to the Fourier coefficients in the surrounding of the wave takes place.

The periods $T_{N\pm j}^{M_j}$ at which the maximum falsification appear and the influence on the Fourier parameters can be determined by the extrema M_j of the sin x / x - function according to the transcendental equation

$$(4.2.3.15) \quad \pi\varrho_{M_j} = \tan \pi\varrho_{M_j} \, .$$

The corresponding Fourier periods $T_{N\pm j}^{M_j}$ are

$$(4.2.3.16) \quad T_{N\pm j}^{M_j} = \frac{T_0(N \pm \varrho_{M_j})}{N} = \frac{T_0(N + \varepsilon_{M_j} \mp j)}{N}$$

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For the first (j = 1) and second (j=2) maximum side lobe we obtain

(4.2.3.17)
$$\varrho_{M_1} = \pm 1.4303 \qquad \varrho_{M_2} = \pm 2.4590$$
$$\varepsilon_{M_1} = \pm 0.4303 \qquad \varepsilon_{M_2} = \pm 0.4590$$
$$R_N = 0.7221\ A_0 \qquad R_N = 0.6877\ A_0$$
$$R_{N\pm1} = 0.2172\ A_0 \qquad R_{N\pm2} = 0.1284\ A_0$$

It means for the determination of the amplitude A_0 for instance for $j=1$: The data interval $2\overline{\tau}$ covers instead of NT_0 the length $NT_0 \pm 0.4303\ T_0$. Then we obtain the maximum of the first side lobes and for the coefficients $a_{k=N}$ and $b_{k=N}$ which should describe the true amplitude A_0 the distorted value $0.7221\ A_0$.

For the superposition of two waves which are not separable by Fourier analyses the Fourier amplitudes R_{F_2} are expressed by formula (2.2.6) instead of A_0.

For the determination of the surrounding amplitudes we must substitute in (2.2.6)

$$\varepsilon_i = \varrho_i \pm j , \quad i = 1,2 .$$

(Naturally also the spectra of more than two waves may be calculated in this way).

For the two main waves of the M_f group the function R_{F_2} is given in figure 6 , upper curve. The second wave is situated at $T = 13.633$ d , and its amplitude amounts 40 percent of that of the main wave at $T = 13.661$ d . We see that the influence of the second wave generates an asymmetric pattern with respect to the maximum of the amplitudes especially in the region of the left-hand side first secondary maximum (j = +1). The first minimum on the right-hand side reaches the zero value whereas on the left-hand side a contribution of 0.25 μGal is generated by the second wave.

Inserting the mean period $\overline{T}_2 = 13.669$ (equ. (2.1.19) in formula (4.2.3.16) we obtain for $N = 210$

(4.2.3.18)
$$T_{211}^{M_1} = 13.5763\ d \qquad T_{212}^{M_2} = 13.5093\ d$$
$$T_{209}^{M_1} = 13.7625\ d \qquad T_{208}^{M_2} = 13.8295\ d .$$

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Now we consider practical computations for our data series. The gravimetric registrations at Potsdam yield to a restcurve, which begins on March 22, 1974, 1^{00} (t_A, for all Fourier analyses the same date) and terminates on February 14, 1982, 4^{00}UT, i.e. 69268 hourly readings. Using the mean value $\overline{T}_2$ = 13.669 days = 328.06 hours it means that 211.14 periods are contained in this time interval. In order to change ε in the range ± 0.5 we must choose N = 210. To get an impression on the numerical (inner) reliability of the computation we vary ε in the range $+0.5122 \geqq \varepsilon \geqq -1.02$, i.e. the interval length $2\overline{\tau}$ varies from 69048 through 68544. $2\overline{\tau}$ in the case $\varepsilon = 0$ amounts 68896 (i.e. the time interval March 22, 1974, 1^{00}through January 29, 1982, 15^{00} UT). The variation of the ε value takes place in steps $\Delta\varepsilon \approx 0.077$. To cover the above noted range 21 single Fourier analyses were carried out. By other words: the Fourier coefficient for N = 210 is allocated to the periods 13.70, 13.695, 13.690,..., 13.60 in the order of the analyses. From different analyses we obtain for the same period the amplitudes and we can remark that they differ in the range 2.5 ... 5 % of the maximum amplitude, i.e. 0.1 ... 0.2 µGal for the gravity or 0.02 ... 0.04 mbar for the local air pressure.

The results of the 21 Fourier analyses, compiled to a quasi-continuous spectral representation for different data series are given in figure 6. The second curve (counted from the upper one) shows the distribution of the Fourier amplitudes calculated from the data series for all the long-periodic waves in the Carthwright-Tayler-Edden development, using the gravimeter factor $\delta = 1.0$. The agreement between the first and the second curve is quite well, i.e. the analytic Fourier representation on the basis of the two main waves gives the principal feature of the computed ones on the basis of the hourly data series.

The Fourier analyses on the basis of the restcurve RRK8GL9 (chapter 3, see also fig. 3c) show the following characteristics:

- We find an excellent agreement of the position $T^E_{M_f}$ = 13.669 for the maximum amplitude $R^E_{M_f}$ = 4.389 µGal and of the position T^E = 13.61 and its amplitude (expected 0.3 µGal, obtained 0.4 µGal) for the first left-hand minimum. We obtain from the values for the maximum according to equ. (4.2.3.0) the tidal parameters

(4.2.3.19) $$\delta_{M_f} = \frac{4.389 \pm 0.2}{4.206 \pm 0.2} = 1.044 \pm 0.1$$

$$\varkappa_{M_f} = 7\overset{\circ}{.}7 \pm 1\overset{\circ}{.}0$$

This values confirm the results in chapter 4.1.2.

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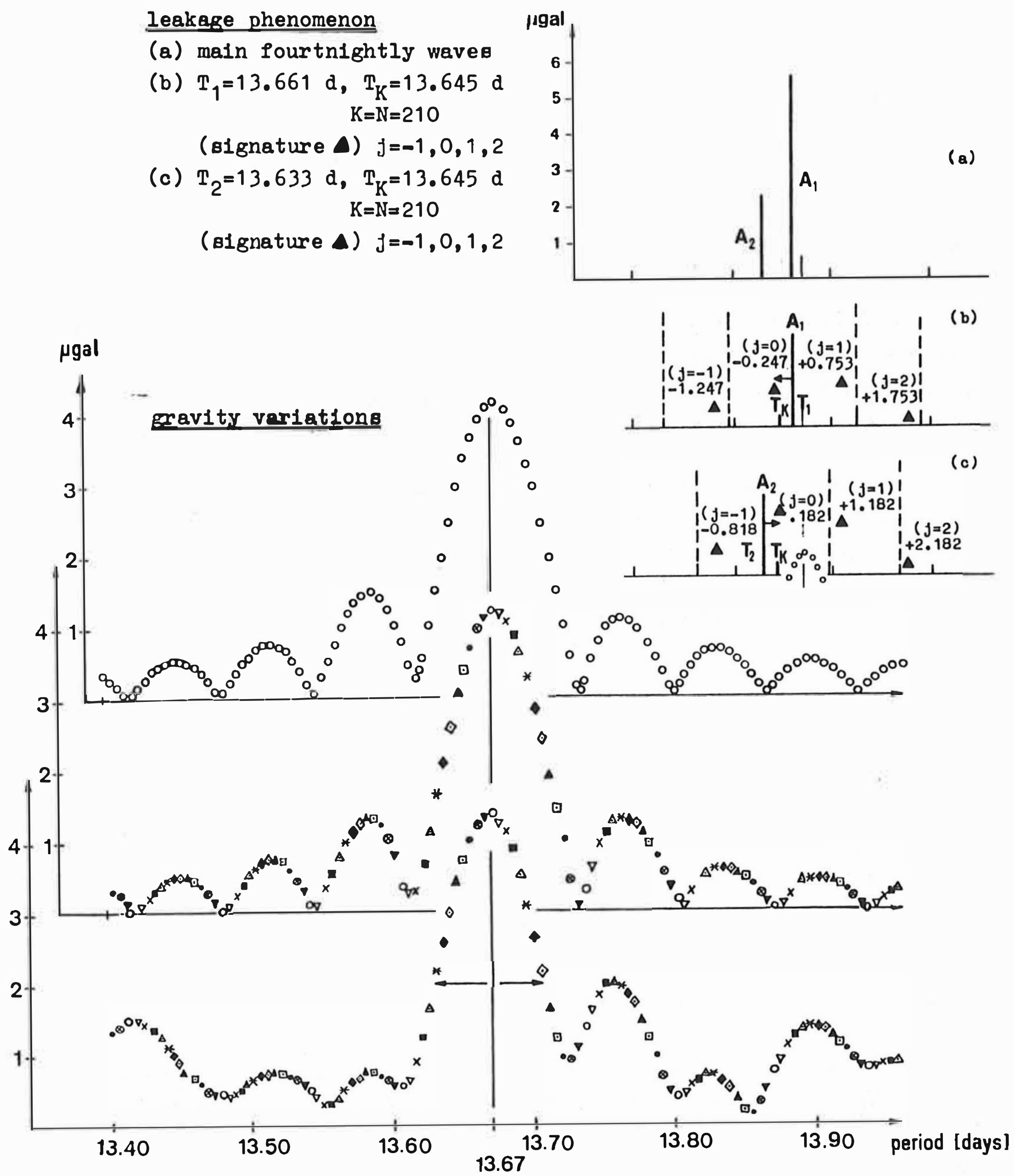

Fig. 6: Spectral representation (FOURIER-analyses) of the gravity variations in the time interval March 1974 - February 1982 (lower part of the figure) and some explanations concerning the leakage phenomenon (upper part).
The upper curve in the lower part consists of the values due to formula (2.2.6) for the two main waves, the middle curve represents the results of the FOURIER-analyses on the basis of an hourly data set due to all waves within the M_f -group, δ = 1.0; and the lower curve the results on the basis of the hourly digital empiric data set after drift elimination up to the ninth harmonic constituent. Note the nearly symmetric shape of the two first mentioned curves in contrast to the lower one as an indication for non-tidal constituents in the empiric data set.

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- We remark a few pecularities in the Fourier representation of the empirical gravimetric series :
- - the asymmetric shape in the range of the maximum (in contrary to the symmetric shape of the upper curve)
- - the remarkable diminuation of the left-hand side first maximum (expected 1.5 μGal, obtained 0.7 μGal)
- - the remarkable amplification of the right-hand side first maximum at the period 13.76 (expected 1.5 μGal, obtained 2.0 μGal)
- - at the first right-hand zero coefficient a significant contribution of 1.0 μGal in the empirical series is to be seen.

These facts lead to the conclusion that in the M_f range a non-tidal disturbance exists which can be characterized by an amplitude $R^S_{M_f}$ of approximately 0.7 ... 0.8 μGal.

To proof whether this disturbance might be seen in connection with the local air pressure we analysed this data series for the same time interval. The amplitudes vary from 0.1 through 0.8 mbar, for the periods 13,40 through 14.00d. At T_{M_f} we obtain

an amplitude $R^P_{M_f}$ = 0.666 mbar, $\varphi^P_{M_f}$ = 312 °.

To provide a detailed insight in the relation between the lot of results for the periods 13.70 through 13.635 days in polar diagrams the empiric (thick line), the theoretic (thin line) and the barometric (brocken line) amplitudes and phases are drawn (Fig. 7). One can remark on the one side the increase of the empiric amplitude with respect to the theoretic one from longer to shorter periods, on the other hand in the period range 13.680 through 13.640 days neither the air pressure amplitude increases nor the phase angle between empiric gravity and air pressure (approx. 67°) changes. Therefore we can conclude that the amplitude variation of the empirical vector cannot be produced by the local air pressure, but a non-tidal disturbance of other origin causes the above noted increase and prevents a stable gravimeter factor for the periods in the range of 13.70 ... 13.635 days.

To confirm and generalize this statement we split up the M_f-wave group into three groups M_{f1}, M_{f2}, M_{f3} (see table 4). Using the CHOJNICKI procedure we analyse the drift-free data set RRK8GL9 again. The gravimeter factors for the three groups differ considerably, but a common explanation may be found if we accept the above noted non-tidal constituent S_N in the order of 0.7 through 0.8 μGals. We consider the following equation

$$M_{fi} = \frac{(1.165 * A_i) \pm S_N}{A_i}, \quad i = 1,2,3; \qquad (4.2.3.20)$$

+ = in phase
− = out of phase

whereby the factor 1.165 is the value after WAHR (1981). In comparing columns 5, 6, 7 and 8 one can remark the good agreement in the magnitude of the gravimeter factors under the assumption of the existence of a non-tidal constituent.

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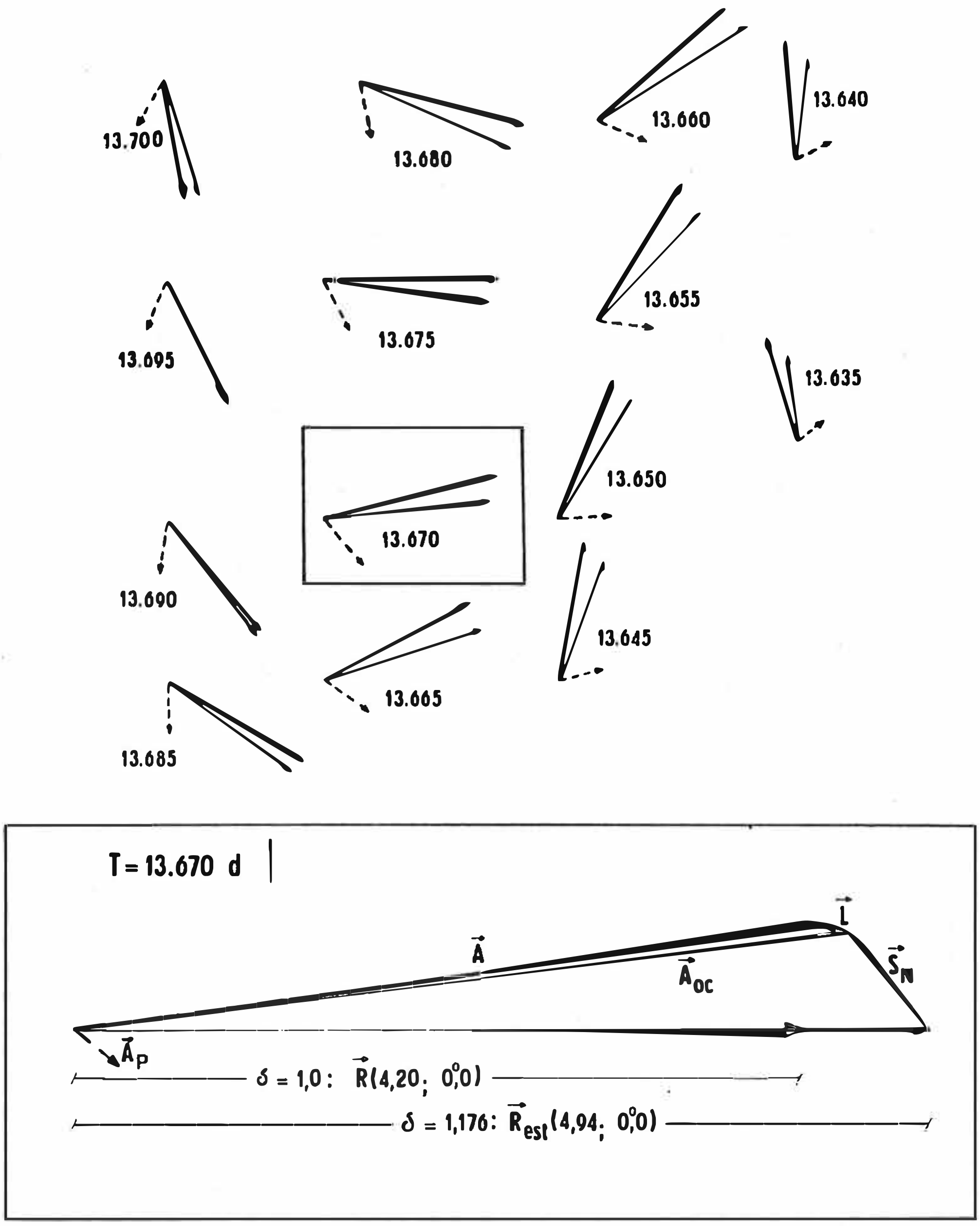

Fig. 7: Vectorial representation of the results of the FOURIER-analyses in the region of the M_f-tide (see fig. 6). For each period (in days) the empiric (thick line) and the theoretic (thin line) tidal gravity vector as well as the empiric local air pressure vector (broken line) are shown.
At the period of 13.670 days we obtained the maximum lengths of the tidal gravity vectors. A detailed vectorial diagram in the lower part explains the empiric ($\vec{A}$) and theoretic ($\vec{R}$) tidal vectors, the ocean loading vector $\vec{L}$, the ocean loading corrected vector $\vec{A}_{oc}$, the disturbing vector $\vec{S}_N$ and the local air pressure vector $\vec{A}_p$. Neglecting a phase lag we obtain a reasonable gravity factor using the estimated empiric tidal vector $\vec{R}_{est}$ under consideration of the vector $\vec{S}_N$. The amount of the vector $\vec{S}_N$ was derived from the non-tidal influences in fig. 6.

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Table 4: Gravimeter factors for three groups M_{f1}, M_{f2}, M_{f3} within the M_f-wave group

column 4 - maximum amplitude of the resulting main wave

column 5, 6, 7 - theoretical results on the basis of a non-tidal constituent S_N of 0.75 μGal in phase (col. 5), out-of-phase (col. 6) and their mean value (col. 7)

column 8 - CHOJNICKI-analysis, data set RRK8GL9

1 designation	2 DOODSON-arguments	3 periods (days)	4 max. Ampl.A (μGal)	5 $\frac{(1.165 * A)+S_N}{A}$	6 $\frac{(1.165 * A)-S_N}{A}$	7 $\frac{5+6}{2}$	8 CHOJNICKI analyses
M_{f1}	075.345- 075.365	13.805- 13.749	0.243	4.251	1.921	3.086	3.376 ± 7.55
M_{f2}	075.455- 075.585	13.719- 13.579	4.20	1.295	0.9379	1.116	1.013 ± 0.47
M_{f3}	076.354- 077.575	13.276- 12.663	0.019	40. 64	38.31	39.47	49.11 ± 92.4

5. Conclusions and interpretation

For a summary we compile in table 5 the results and consider the simple mean value. We neglect the mean square error since the methods are quite different and a comparison is not justified.

Table 5: Gravimeter factors and phase lags due to different methods of analysis, data set RRK8GL9, 69 200 hourly values, 2883 daily mean values resp.

	δ	$\varkappa$[°]
CHOJNICKI-analyses		
- total M_f	1.035	10.60
- M_{f2}	1.013	8.47
BUYS-BALLOT-schema	1.026	5.4
KURTHS method	1.019	—
FOURIER-analyses		
- unfiltered	1.044	7.7
- band-pass-filtered	1.019	6.3
mean value	1.026	7.7

The results show a sufficient coincidence and we can conclude that the single methods used for the analysis of our 8 years data set do not produce a systematic bias.

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Therefore the difference to WAHRs Earth model ($\delta^{th}_{M_f}$ = 1,1655, $\varkappa$ = 0°) must be caused by other influences.

The FOURIER-analyses provide the most detailed information on the spectral distribution in the M_f-region as a base for the wanted explanation. Therefore the further investigations base on their results. Using the unit µGal and the notation introduced by MELCHIOR (1983) we consider the vectors (for the values see chapter 4.2.3)

$\vec{A}$ (A, α) observed amplitude and phase (A = $R^{E}_{M_f}$ = 4.389 ± 0.05, α = $\varkappa$ = 7°.7 ± 1.5)

$\vec{R}$ (R, 0°) theoretical amplitude and phase due to the earth model (R = $R^{T}_{M_f}$ (δ = 1.0)= = 4.20 ± 0.05, 0.0)

$\vec{L}$ (L, λ) load vector due to the M_f-ocean tide

$\vec{X}$ (X, χ) final residual.

For an undisturbed observation the equation

(5.1) $\vec{A} = \vec{R} + \vec{L} + \vec{X}$

is valid, but we derived in chapter 4.2.3 a non-tidal disturbance $\vec{S}_N$ in the neighbourhood of the M_f-wave, described by (S_N = 0.75 ± 0.05, σ_N). Now we assume that this disturbance is acting on the M_f-waves itself. Therefore in (5.1) a further term $\vec{S}_N$ must be regarded.

(5.2) $A = \vec{R} + \vec{L} + \vec{X} + \vec{S}_N$.

In introducing $\vec{S}_N$ in equation (5.2) we are unable to estimate the influence of the lithosperic heterogeneities $\vec{X}$. Here we choose to simplify matters $\vec{X} = \vec{0}$.

On the basis of our gravimetric time series we look for a reasonable $\vec{R}_{est}$ which gives the estimated values for $\vec{R}$,

(5.3)
$$\vec{R}_{est} = \vec{A} - \vec{L} - \vec{S}_N = \vec{A}_{oc} - \vec{S}_N ,$$

whereby the ocean tide corrected vector $\vec{A}_{oc}$ already shows the deviation from the earth model. MELCHIOR and DUCARME (1983) submitted us kindly the values for $\vec{L}$ (L = 0.118 (±0.01), -2°.83 (177°.17 ± 2°)) calculated by the well known FARELL procedure based on GREENs functions on the basis of SCHWIDERSKI cotidal and corange maps for M_f. We add 180° to the phase angle in order to yield the same phase like our results (since the amplitudes of the zonal tides are negative for $\varphi >$ 35°.16). Furthermore we add raw error limits to estimate the accuracy of the final results.

The ocean tide corrected $\vec{A}_{oc}$(4.506 ± 0.05, 7°.42 ± 1°.5) differs considerably from the value of the WAHR-model for Potsdam (4.20 $\times$ 1.1655 = 4.89, 0°.0). In order to obtain

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$\vec{R}_{est}$ now we introduce the assumption that the phase lag $\varkappa$ for $\vec{R}_{est}$ vanishes, i.e. $\vec{R}_{est}$ coincides with the true $\vec{R}$ in the direction and the phase lag of $7.^{\circ}42$ in $\vec{A}_{oc}$ is attributed to the vector $\vec{S}_N$. Then (5.3) yields the form

$$(5.4) \qquad \vec{R}_{est}\ (R_{est},\ \alpha = 0) = \vec{A}_{oc} - \vec{S}_N = (4.94 \pm 0.16,\ 0).$$

Using the value of 4.20 μGals for the rigid Earth we get finally the gravimeter factor for the M_f-wave group at Potsdam:

$$(5.5) \qquad \boxed{\delta^{est}_{M_f} = 1.176 \pm 0.04}$$

The error of this estimation is produced mainly by the uncertainties in the vector of the disturbance. Inside the error limits the result agrees now with the value of the WAHR-model. We can remark that the consideration of a disturbing vector $\vec{S}_N$ derived from FOURIER-analyses results to a very sufficient coincidence with the predicted value. For other parameter sets (table 5) we obtain in the same manner values for $\delta^{est}_{M_f}$ which are within the error limits of ±0.04 (with the exception of ($\delta = 1.035,\ 10.^{\circ}60$), where $\vec{S}_N$ must have the amplitude of 0.8 μGal for a solution).

Independently we can also estimate the phase of the disturbance (again under the assumption of the angular coincidence of $\vec{R}_{est}$ and $\vec{R}$)

$$(5.6) \qquad \sigma_N = 129.^{\circ}0 \pm 15^{\circ}\ .$$

The vector of the local air pressure $\vec{P}$ for the mean M_f-period (13.67 days) amounts (0.666 mbar, 312°), i.e. the phase angle is about ($\sigma_N + 180^{\circ}$). On this basis we can derive a regression factor r

$$(5.7) \qquad r = -\ 1.14\ \mu\text{Gal/mbar}.$$

But this result is valid for this wavelength only (see chapter 4.2.3) and up to now it is impossible to find out in our material a significant and constant correlation between the variations of air pressure and gravity.

The results of our investigations on the basis of the gravimetric time series of the GS 15 222 shows that special considerations are required to estimate the character of the non-tidal disturbances in the range of the M_f-wave group.

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