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Key Points:

- We developed an analytical model for MS wave amplitude on location information, kp index, wave normalized frequency, and normal angle
- A drastic fluctuation of MS wave amplitude distribution on the kp index was observed when $kp > 5$
- A detailed global distribution of MS wave amplitude is presented

Supporting Information:

- Supporting Information S1

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Analytical Fast Magnetosonic Wave Model Based on Observations of Van Allen Probe

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Abstract Based on observations of Van Allen Probe-A during the period from 19 September 2012 to 28 February 2016, the relations of the fast magnetosonic (MS) wave amplitude B_w with kp index, the wave normal angle (WNA), and the wave normalized frequency ($norF$) are presented. Then, we establish an analytical regression model for MS wave amplitude as a function of geomagnetic storm activity (presented by kp index), L -shell (L), magnetic local time (MLT), magnetic latitude (λ), and the characteristics of MS wave, that is, wave $norF$ and WNA. From the analytical B_w models, we found MS wave amplitude B_w has a positive relation with the intensity of geomagnetic activities both inside and outside the plasmopause, while the B_w can reach higher values inside the plasmopause than it does outside the plasmopause as the kp index increases. The B_w distribution on the $norF$ demonstrates that most of the wave energies are concentrated on the lower harmonics part, which results from the excitation mechanism of MS waves. In addition, the B_w distribution on the WNA shows that the waves with larger normal angles have higher values of wave amplitude. Our analytic MS wave model agrees with the observed distribution in 3-D space of L , MLT , and λ well with high value of determine coefficient R^2 . The extended λ dimension will help us to calculate the more accurate bounced averaged diffusion coefficients during particles transit time.

1. Introduction

Fast magnetosonic (MS) waves, also named equatorial noise, is one of the universal magnetospheric waves, and the others include whistle mode chorus waves, hiss, and electromagnetic ion cyclotron (EMIC) waves. Different from the other waves, MS wave shows the unique properties of linear ellipticity and quasi-perpendicular propagation. The latitudinal extent of MS waves is mostly confined near the magnetospheric equator (Gurnett, 1976; Hrbáčková et al., 2015; Meredith et al., 2008; Russell et al., 1970; Santolík et al., 2004; Yuan et al., 2019; Zou et al., 2019). MS waves are thought to be generated at frequencies close to harmonics of the proton gyrofrequency and excited by proton ring distributions with positive gradient in the phase space density at energies of 10 keVs (Boardsen et al., 1992; Chen et al., 2010; B. Liu et al., 2018; Sun et al., 2016, 2017). Additionally, MS waves have a wide MLT coverage and can exist both inside and outside the plasmopause. Some previous works have put forward that MS wave plays a role in accelerating electrons from keVs to MeVs via Landau resonance (Horne et al., 2000, 2007; Ni et al., 2018; Shprits, 2016; Shprits et al., 2013; Yu et al., 2020), as well as in scattering ring current ions (Fu et al., 2016; Yu et al., 2019; Yuan et al., 2020). Therefore, MS waves become an essential part in the inner magnetospheric kinetic process. A precise MS wave model is necessary for the research on the effect of MS waves in the inner magnetosphere.

Over the past decades, many statistic works have demonstrated the basic properties of MS waves (Hrbáčková et al., 2015; Ma et al., 2013). Santolík et al. (2004) use the observations of Cluster spacecraft to define the absolute value of ellipticity of MS wave ≤ 0.2 and the magnetic field intensity of MS wave $> 10^{-6}$ nT²/Hz. Yuan et al. (2019) have analyzed the parameters distributions of MS wave ellipticity and normal angle and shown the global distribution in different absolute value ranges of geomagnetic latitude (0–5°, 5–15°, and 15–25°) based on observations from Van Allen Probe-A, demonstrating that MS wave can exist in some high-latitude regions, not just be limited near geomagnetic equator.

For the studies of MS wave model, Kim and Chen (2016) have developed a MS wave model with dependence on the solar wind dynamic pressure based on measurements of the Time History of Events and Macroscale Interactions during Substorms (THEMIS) spacecraft. They found that the solar wind dynamic pressure is

effective in enhancing wave intensity at the postnoon sector outside the plasmopause. Kim and Shprits (2017) reanalyzed the spatial and temporal dependences of wave amplitudes on the solar wind and AE index. They took both effects of solar wind activity and geomagnetic activity into account and found that the wave amplitudes outside/inside the plasmopause are well correlated with the preceding AE, southward interplanetary magnetic field (IMF B_S), and solar wind number density (N_{SW}) with time delays, each corresponding to 2–3/3–4, 4–5/3–4, and 2–3/8–9 hr. Ma et al. (2019) combined the waveform receiver and magnetometer on board Van Allen Probe to show more detailed lower frequency structures of MS waves. They have chosen the frequency range from f_{cHe} (gyrofrequency of helium) to f_{LHR} (lower hybrid resonance frequency) to establish a MS wave empirical model in different levels of AE index.

Though previous researches have carried out the MS wave model of geomagnetic activities, up to now, an analytical model for MS wave amplitude B_w , which depends on location information, kp index, wave normalized frequency ($norF$) and wave normal angle (WNA), has not been reported. In this study, we use the kp index to represent the geomagnetic activity. Then we develop an analytical root-mean-square (RMS) amplitude model for MS wave as a function of kp , L , MLT , λ , and its WNA and $norF$ normalized by local proton gyrofrequency. Dependences on kp , wave $norF$, and WNA are all separated from the dependence on spatial location variants as scaling factors (Wang et al., 2019).

In section 2, we mainly describe the data from Van Allen Probe and the method to identify MS wave and extract the wave structure variants. In section 3, we present the wave amplitude B_w regression model with the dependence of kp , wave $norF$, WNA , and the spatial location variants. The summary and conclusion are presented in section 4.

2. Data and Method

The Van Allen Probes, also named Radiation Belt Storm Probes (RBSP), were launched on 30 August 2012 and into the highly elliptical, low inclination ($\sim 11^\circ$) orbits (Mauk et al., 2013). The highest λ that Van Allen Probes can reach is up to $\sim 25^\circ$. We use the Waveform Receiver (WFR) of the Electric and Magnetic Field Instrument and Integrated Science (EMFISIS) suite (Kletzing et al., 2013) to obtain the wave matrix in 65 logarithmically spaced bins between 2 Hz and 12 kHz with a cadence of 6 s. The electron density (n_e) can be derived from the upper hybrid resonance frequency measured by High-frequency Receiver (HFR) of EMFISIS suite (Kurth et al., 2014; Zhelavskaya et al., 2016).

We have chosen the time interval from 19 September 2012 to 28 February 2016, ensuring enough data to reproduce the details of MS wave global distribution and establish a precise wave model. To identify the MS wave, we use the wave ellipticity and WNA derived from the SVD method (Santolík et al., 2003). To be noted, the accuracy of this method depends in large part on the magnetic field intensity of plasma waves and the background magnetic field. For a precise research of MS wave normal angles, Sun et al. (2020) have investigated the exact WNA distribution of MS waves using 2-D PIC simulation. However, based the satellite observations, we have to ignore the errors brought by SVD method. After picking out the waves with intensities larger than background level, we use the criteria of $|ellipticity| \leq 0.2$, $WNA > 70^\circ$ with frequency range of 2 Hz $\sim 1.5f_{LHR}$ (lower hybrid resonance frequency) to identify MS waves (Yuan et al., 2019). To be noted, we use the weighted mean of wave magnetic field power to determine the value of WNA and $norF$ of MS waves at every moment. We transfer the resolution of wave matrix from 6 s into 1 min. In other words, all of MS waves detected automatically are divided into samples in minute, and each sample contains the wave amplitude, the power weighted WNA and $norF$, location information, and the maximum kp value in the preceding 24 hr (kp^*). The events of MS wave amplitudes > 700 pT only account for less than 1%. The specific count of events > 700 pT is 69, out of the total 105,302 inside the PP and 46 out of the total 117,571. Most of MS wave events are at low intensity (1–100 pT), accounting for 95% of the total. So the samples whose wave amplitudes are too large (> 700 pT) or too small (< 1 pT) will be eliminated to ensure the universality of our analytical model.

In order to investigate the different properties of MS waves inside and outside the plasmopause, we should mark these samples. As for the location of plasmopause, we use the data from plasmopause test particle (PTP) simulation (Goldstein et al., 2014) to identify the plasmopause location preliminarily; then if the PTP judges the satellite position is inside the plasmopause but the measured electron density

Figure 1. An event of MS wave detected by RBSP-A on 27 January 2016. (a) Local electron density; (b) power spectra density of magnetic field; (c) the ellipticity; (d) WNA; (e) wave amplitude; (f) power-weighted WNA; (g) power-weighted *norF*; and (h) flag of the region inside or outside the plasmopause (black solid line) and *kp** index (red solid line).

$n_e < 50 \text{ cm}^{-3}$, it will be redressed outside the plasmopause. On the contrary, if the PTP judges the satellite position is outside the plasmopause but $n_e > 100 \text{ cm}^{-3}$, it will be redressed inside the plasmopause (Sheeley et al., 2001; Yuan et al., 2018).

As Figure 1 shown, we give an example during 11:00–15:30 UT on 27 January 2016 to explain how we deal with the satellite data. Figure 1a presents the local electron density, which can assist us to determine the location of plasmopause. Figure 1b shows the wave magnetic field power spectra. Here, we can see a typical MS wave from 12:00 to 14:45 UT between the lower hybrid resonance frequency (denoted by white solid line) and proton gyrofrequency (denoted by black solid line). After eliminating the background, we just make the ellipticity and WNA of MS waves detected by the method of Yuan et al. (2019) visible in Figures 1c and 1d. Figure 1e is the wave amplitude of MS waves by integrating the power spectra density of MS wave magnetic field and acquiring the RMS value of wave power. According to the power spectra density distribution of wave magnetic field, we show the power-weighted WNA and *norF* of MS wave, shown in Equation 1, in Figures 1f and 1g for each moment. In Equation 1, the f_{max} and f_{min} denote the upper and lower limits of

Figure 2. Distributions of the MS wave B_w on the kp^* index. (a) Inside the plasmopause and (b) outside the plasmopause. The gray solid line represents the observation and the red solid line represents the fitting scaling factor.

the wave frequency, respectively. B_{psd} means the power spectra density of wave magnetic field. The parameter X is the object to be weighted, which can be the WNA or $norF$. The last panel shows the flag of regions outside the plasmopause or inside denoted by solid black line corresponding to the left-hand axis Y and the kp^* value denoted by solid red line corresponding to the right-hand axis Y . After these preparations, the statistic and fitting work will be done in the next section.

$$WNA = \frac{\int_{f_{min}}^{f_{max}} B_{psd} \cdot X df}{\int_{f_{min}}^{f_{max}} B_{psd} df} \quad (1)$$

3. B_w Regression Model

In order to simplify the future computation of quasi-linear diffusion coefficients, we should extract two pre-treatments (Glauert & Horne, 2005; Summers, 2005; Shprits et al., 2013). One is transferring the wave power into wave RMS amplitude B_w ; the other is separating dependences of the other variants from the dependence of L , MLT , and λ . Then the MS wave regression model can be written in the following formula:

$$\sqrt{\langle B_w^2(L, MLT, |\lambda|, kp^*, norF, WNA) \rangle} = f(L, MLT, |\lambda|) \cdot g(kp^*) \cdot h(norF) \cdot y(WNA). \quad (2)$$

B_w represents the MS wave amplitude; $norF$ represents the wave frequency normalized by local proton gyrofrequency; WNA is the wave normal angle. Here we treat the $f(L, MLT, \lambda)$ as a basic level of MS wave amplitude. That means if we have confirmed the spatial location, that is, the value of L , MLT , and λ , we will get a basic MS wave amplitude. Then we take the

Table 1
The Values of Fitting Coefficients for Functions $g_0(kp)$

	Inside the PP	Outside the PP
p_0	20.81	24.33
p_1	−1.08	6.36
p_2	2.17	

Note. PP means the plasmopause.

Figure 3. The B_w distributions of MS waves on the wave normalized frequency. The same format as Figure 2.

different geomagnetic storm activity (the value of kp^*) and the detailed $norF$ and WNA of waves into consideration, to confirm the values of the scaling factors, that is, $g(kp^*)$, $h(norF)$, and $y(WNA)$. Finally, these scaling factors will adjust the basic level of B_w to acquire a more realistic value, that is, a value under a specific condition with kp^* , a specific wave location, and detailed wave $norF$ and WNA . In the following subsections, we discuss each fitting function with the kp dependence, $norF$ dependence, WNA dependence, and L , MLT , λ dependence.

3.1. kp Dependence

Kp index is known as a geomagnetic index describing the intensity of the geomagnetic disturbance for a certain period of time. The Space Weather Prediction Center classifies geomagnetic activity into four levels: quite time ($kp < 4$), active time ($kp = 4$), minor storm ($kp = 5$), and major to severe storm ($kp > 5$). Therefore, we adopt kp as one of input variants of this model. Based on this knowledge, we then treat the function $g(kp^*)$ as a scaling factor using the following form:

$$g(kp^*) = \frac{g_0(kp^*)}{\frac{1}{\Delta kp} \int_{Min_kp}^{Max_kp} g_0(kp^*) dkp^*}, \quad (3)$$

where $g_0(kp^*)$ is the wave amplitude function of kp^* , kp^* is the maximum value of kp index during the preceding 24 hr, $\Delta kp = Max_kp - Min_kp$ is the variant range of kp , and Max_kp (Min_kp) is maxima (minima) of the kp observed. The denominator on the right of Equation 3 represents the average wave amplitude. As Figure 2 shown, the value of horizontal axis is from 0+ to 8+, and the gray solid lines represent the statistical wave amplitude mean value distribution on the kp^* . Based on the observations, we adopt the following formula to fit this distribution:

$$g_0(kp^*) = p_2 \cdot kp^{*2} + p_1 \cdot kp^* + p_0 \quad (\text{Inside}), \quad (4)$$

$$g_0(kp^*) = p_1 \cdot kp^* + p_0 \quad (\text{Outside}). \quad (5)$$

Equation 4 is a second-order polynomial distribution for the wave amplitude inside the plasmopause. Outside the plasmopause, it is a first-order linear distribution, as Equation 5 shown. The values of the

Figure 4. The B_w distributions of MS waves on the wave normal angle. The same format as Figure 2.

fitting function coefficients are listed in Table 1. The red solid lines corresponding to the right-hand axis Y in Figure 2 represent the scaling factor $g(kp^*)$ distribution. Using scikit-learn (Pedregosa et al., 2011), we evaluate this regression model by calculating the coefficients of determination R^2 defined by

$$R^2(y, \hat{y}) = 1 - \frac{\sum_{i=1}^n W_i (y_i - \hat{y}_i)^2}{\sum_{i=1}^n W_i (y_i - \bar{y})^2}, \quad (6)$$

where n is the total number of bins, y_i is the observed value, $\hat{y}_i$ is predictive value from the regression model, and $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i W_i$ is the weighted average value of RMS wave amplitude. W_i is a weighting factor defined by $W_i = \frac{N_i}{N_{total}}$, where N_i is the number of measurements in the i th bin; N_{total} is the total number of measurements.

To be noted that the wave amplitude is monotonically increasing as the geomagnetic activity intensifies both inside and outside the plasmopause in general. However, when major or severe storms happen ($kp > 5$), the wave amplitude starts to undulate considerably outside the plasmopause. The undulation should be attributed to the competition of two mechanism when moderate or severe storms happen. One is there are more energies swarm into ring current with the energetic protons, which provide more free energies to excite the MS wave and the adiabatic acceleration of the ring current protons promotes the MS wave emerge (N. Liu et al., 2018); the other is the enhanced proton thermal pressure distorted the background magnetic field configuration and the cold plasma density distribution. The reduced phase velocities allow the weak growth or even damping of magnetosonic waves (Dai et al., 2019). On the other hand, the competition of these two mechanisms also accounts for the drastic fluctuation of observed distribution when $kp^* > 5$ in Figure 2b.

3.2. $norF$ and WNA Dependence

Besides the external effect of geomagnetic activities, the wave amplitude will also change in different wave normalized frequency and normal angle as denoted by gray solid lines in the Figure 3 (with the resolution of 0.1 for axis X) and Figure 4 (with the resolution of 0.05° for axis X). The red solid lines corresponding to the right-hand axis Y denote the scaling factors

Table 2
The Values of Fitting Coefficients for Functions $h_0(norF)$

	Inside the PP	Outside the PP
a_1	23.41	46.00
b_1	5.00	3.00
c_1	2.91	15.68
a_2	−0.98	482.85
b_2	46.29	6.00

Table 3
The Values of Fitting Coefficients for Functions $y_0(WNA)$

	Inside the PP	Outside the PP
p_0	1.91×10^{-3}	1.92×10^{-3}
p_1	-7.74×10^{-1}	-7.73×10^{-1}
p_2	1.25×10^2	1.24×10^2
p_3	-1.02×10^4	-9.99×10^3
p_4	4.10×10^5	-4.01×10^5
p_5	-6.63×10^6	-6.43×10^6

according to the fitting functions. The scaling factor functions $h(norF)$ and $y(WNA)$ have the same formula as the $g(kp^*)$, but the independent variant is changed from kp^* into $norF$ and WNA , and $g_0(kp^*)$ also is changed to $h_0(norF)$ and $y_0(WNA)$, respectively. We adopt the following formulas to fit the observed distribution:

$$h_0(norF) = a_1 e^{-\frac{(norF - b_1)^2}{c_1^2}} + a_2 \cdot norF + b_2 \quad (\text{Inside}), \quad (7)$$

$$h_0(norF) = a_1 e^{-\frac{(norF - b_1)^2}{c_1^2}} + a_2 \cdot (b_2 \cdot norF)^{-1} \quad (\text{Outside}), \quad (8)$$

$$y_0(WNA) = \sum_{i=0}^5 p_i \cdot WNA^i. \quad (9)$$

For the fitting function of normalized frequencies as shown in Equations 7 and 8, we adopt two different formulas to fit the B_w inside and outside the plasmapause. The coefficient values of these fitting functions are listed in Tables 2 and 3. From Figure 3 and Table 2, we can see the B_w becomes more intensive when the $norF$ approaches to lower harmonics both inside and outside the plasmapause. While both of MS wave amplitudes B_w inside and outside the PP decrease with the increase of the wave normalized frequency, there is a peak at $norF \sim 5$ inside the plasmapause. We use a Gaussian function to fit this peak part in Equation 7. This $norF$ (~ 5) is also the usual starting point of calculated wave growth value that becomes positive according to the previous works (Chen, 2015; Yuan et al., 2017). Additionally, B_w outside the plasmapause is larger than that inside the plasmapause, especially in the lower harmonics sector. Figure 4 shows the B_w distributes on the WNA. For waves inside and outside the plasmapause, the wave amplitudes both have the tendency toward rising as the WNA approaches to 89° . A five-order polynomial,

Figure 5. The observed global distributions of MS wave B_w and the fitting distributions with the dependence of L , MLT , and λ . (a–c) The observed distribution and (d–f) the fitting distribution inside the plasmapause; (g–i) the observed distribution and (j–l) the fitting distribution outside the plasmapause. Each row denotes different $|\lambda|$ ranges ($0^\circ\sim 20^\circ$, $0^\circ\sim 5^\circ$, and $5^\circ\sim 10^\circ$).

Table 4
The Values of Fitting Coefficients for Functions $f(L, MLT, MLAT)$

	Inside the PP	Outside the PP		Inside the PP	Outside the PP
a_0	-1.28×10^3	-2.60×10^4	a_{12}	-1.01	-0.09
a_1	9.99×10^2	1.28×10^4	a_{13}	46.78	-1.71×10^2
a_2	-6.99×10^2	-2.78×10^3	a_{14}	-9.76	8.42
a_3	-4.20×10^2	6.92×10^3	a_{15}	6.76	-6.43
a_4	13.59	-1.27×10^2	a_{16}	-1.68	-1.03
a_5	-39.86	-9.00	a_{17}	-5.84	-19.06
a_6	-0.07	-5.48	a_{18}	10.83	1.97
a_7	-1.50×10^2	-1.42×10^3	a_{19}	-0.54	-0.12
a_8	1.28×10^2	1.06×10^3	a_{20}	-1.49	2.67
a_9	1.75×10^2	-1.11×10^3	a_{21}	1.18	1.08
a_{10}	-8.84	23.57	a_{22}	1.13	0.75
a_{11}	6.22	-2.29	a_{23}	0.12	0.17

as Equation 9 shown, can reproduce their tendency well with high value of $R^2 = 0.91$. This observation agrees with the theoretical calculation; that is, MS wave growth rate will increase with the WNA approaching to 90° .

3.3. L , MLT , and λ Dependence

We present the global distribution of observed MS wave amplitudes in the Figures 5a–5c and 5g–5i in different absolute value ranges of λ , that is, 0 – 20° , 0 – 5° , and 5 – 10° (10 – 15° and 15 – 20° are shown in Supporting Information S1). In order to obtain the regression model with the dependence of L , MLT , and λ , L is divided evenly into eight bins from 2 to 6. λ is divided evenly by its absolute value into four bins from 0 to 20, and MLT is divided evenly into eight bins from 0 to 24. Then we get 256 bins totally by crossing bins of these three variants. To eliminate the influence of stochastic cases, we abandon the bins in which the duration of MS wave detected is less than 1 hr. The L , MLT , and λ dependence function of the B_w is written as

$$f(L, MLT, |\lambda|) = f(L, A, B, C, D, |\lambda|) \quad (10)$$

where $A = \cos(MLT \times \pi/12)$, $B = \sin(MLT \times \pi/12)$, $C = \cos[(L-2) \times \pi/4]$, $D = \sin[(L-2) \times \pi/4]$, and

$$\begin{aligned} f(L, A, B, C, D, |\lambda|) = & a_0 + a_1L + a_2D + a_3C + a_4A + a_5B + a_6|\lambda| + a_7L^2 + a_8LD + a_9LC \\ & + a_{10}LA + a_{11}LB + a_{12}L|\lambda| + a_{13}CD + a_{14}AD + a_{15}BD + a_{16}B|\lambda| + a_{17}CA + a_{18}CB + a_{19}C|\lambda| \\ & + a_{20}AB + a_{21}A|\lambda| + a_{22}B|\lambda| + a_{23}|\lambda|^2. \end{aligned} \quad (11)$$

Equation 11 is a linear regression function, trained by the interaction features of independent variants with the highest order of 2. The coefficient values of this fitting function are listed in Table 4. According to the observed B_w global distribution, we find that B_w does not change monotonously along L and MLT dimension. Therefore, we use the normalized trigonometric functions of L and MLT to reproduce these alternate changes. The fitting results are shown in Figures 5d–5f and 5j–5l corresponding to different $|\lambda|$ sectors for each row, and the determination coefficients R^2 are 0.94 and 0.97 for wave model inside and outside the plasmopause, respectively. The high R^2 values show good performance of fittings. Additionally, the high resolution of wave location $L \times MLT \times \lambda$, that is, $0.5 \times 3 \text{ hr} \times 5^\circ$, gives a more detailed global distribution of MS waves.

4. Summary and Conclusions

In order to develop a comprehensive MS wave model, we have taken the geomagnetic activity, wave characteristic variants, and wave spatial locations into account based on observations of the Van Allen Probe-A. Our amplitude model of MS waves can be applied to the kp index of 0+ to 8+. It is valid in L -shell from 2 to 6 and all MLT s within $\pm 20^\circ \lambda$. Moreover, it has completed a quantitative description of the MS waves and could be used to research the effect of MS waves in the radiation belts accurately. The results of our MS wave model are summarized as follows:

1. According to the fitting function with kp dependence, we reveal the changes of MS wave B_w during different geomagnetic activities. The B_w will enhance as the geomagnetic activities become intensive generally. But we have found outside the plasmopause, when moderate or severe storm happens, the B_w starts to undulate considerably. That may because the excitation and depression of MS waves both exist and come into being competition. But our model shows that the overall trend is benefit for the growth of MS wave.
2. The lower harmonic part of MS waves accumulates most of wave energy. Based on the high-resolution B_w distribution on the $norF$, there is a peak of B_w at $norF \sim 5$ inside the plasmopause. Nevertheless, outside the plasmopause, lower harmonics is easier to be existed due to the low background electron density.

3. The largest wave amplitude is located at $WNA \sim 89^\circ$ from the B_w distribution on the WNA both inside and outside the plasmopause, due to the effects of MS wave excitation.

The developed MS wave B_w analytical model can give a new input for research of particle scattering in the radiation belts. For each fitting process, the high determine coefficient of R^2 shows the accuracy of our model. In the future, we will study the effect of MS wave in dynamic evolution of the radiation belts and ring current protons based on this model, including transit-time scattering on particles (Bortnik et al., 2015; Yu et al., 2020).

Data Availability Statement

We acknowledge the Van Allen Probe-A data from the EMFISIS instrument (obtained from <http://emfisis.physics.uiowa.edu/Flight/RBSP-A/>). We thank Craig A. Kletzing for providing the data of the EMFISIS and advice on their usage. We also acknowledge the plasmopause location data from Plasmopause Test Particle Simulation (obtained from <http://doi.org/10.5281/zenodo.3965048>).

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