Inter-correlation of parameters of sedimentary rocks conductivity dispersion

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Introduction

The low frequency dispersion of conductivity is one of the most important properties of sedimentary rocks affecting the data of studying the Earth crust by geo-electromagnetics, in particular by transient electromagnetic sounding (TEMS or TS). The phenomenon is mainly investigated with the help of mathematical modelling of the transient field in horizontally-layered media. The latter are assumed to be composed by sedimentary rocks.

The low-frequency dispersion (LFD) of rocks conductivity or, in other words, induced polarisation (IP) is mostly introduced by use of phenomenological Cole-Cole or similar formulae. IP parameters are independently varied in a very wide limits: the chargeability - from 0 to 1, and the time-constant - from µsec to tens of sec (e.g., Svetov, Ageev and Lebedeva, 1996). It is also seen from previous modelling results that strong IP effects (sign-reversals etc.) are possible in cases where the higher chargeability values take place like several tens of percent, which causes a certain doubts in correspondence of modelling data to natural conditions. There are although many authors who emphasise that the chargeability of sedimentary rocks can not exceed several percents (e.g., Sochelnikov, 1993).

These divergences can be explained by a certain lack of the laboratory experimental data relating to the so-called "fast IP" activated in sedimentary rocks by the electromagnetic field, especially by use of induction energising method. The already known experimental data of the sort are usually of a qualitative character (Kamenetsky and Novikov, 1997).

Moreover, as it comes from physical arguments, LFD or IP parameters can hardly be treated as mutually independent ones, since they are defined by one and the same electro-physical (or electro-chemical) properties of dispersive media. That is way it looks useful to investigate the inter-correlation of IP parameters and estimate possible values of them on the basis of a suitable model with the known physical nature of the dispersion. The Cole-Cole and

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other phenomenological formulae do not give such a possibility since they are independent of the LFD nature.

The model in question

The nature of LFD or IP is connected with divergent electro-physical and electro-chemical phenomena, which makes each type of dispersion a separate subject to research. The dispersion of sedimentary rocks is mainly of an electro-physical nature due to their heterogeneous and poly-phase character. The first from these two types of the dispersion is well investigated. It is typical for composite materials (including natural ones) and is known in electro-dynamics as Maxwell-Wagner effect. The alternation of particles, mineral grains, micro-layers, pores and fractures of different conductivity or/and permittivity causes the accumulation of electric charges at interfaces between different components due to the current projection normal to the interfaces. This can be treated as an equivalent "natural capacitor"[3] (or "quasi-capacitor") effect resulting in frequency dispersion of effective (averaged) conductivity and permittivuty of rock considered as macro-homogeneous one. The effect had been first analysed by Maxwell (1873) and Wagner (1913). Later on it has been investigated in details as applied to conductivity dispersion of rocks (Alvarez, 1973 and Gubatenko, 1991). The effect is more in case where low-resistive parts are separated by high-resistive intervals. These intervals may be filled up by gas (air) or high-resistive minerals like calcite, quartz, salt, bitumen, ice in permafrost, oil films etc. The alternation of the sort is also typical for the structure of clay, clay-sand, slates, shists and other sheet-type sedimentary rocks.

A suitable model of the sort is a two-component periodically-layered micro-structure (Gubatenko, 1991) with parameters: d_1, σ_1, ε_1, and d_2, σ_2, ε_2 (thickness, conductivity and permittivity of alternating layers), and $d = d_1 + d_2$ is the period of the structure. The effective admittivity of this structure has been found by the averaging of the electromagnetic field in the small volume exceeding the period of the structure several times. In case of the current parallel to the bedding both conductivity and permittivity are independent of frequency, and in case of the current normal to the bedding the effective admittivity is equal to:

$$\gamma_{ef} = \sigma_{ef} - i\omega\varepsilon_{ef} = (\sigma_\infty - \frac{\sigma_\infty - \sigma_0}{1+\omega^2\tau_E^2}) - i\omega(\varepsilon_\infty - \frac{\varepsilon_\infty - \varepsilon_0}{1+\omega^2\tau_E^2}), \quad (01)$$

where the effective conductivity at higher frequencies (or the true conductivity in the absence of polarisation) is

[3] To the contrary to electro-chemical LFD which can be treated as „natural accumulator" effect.

$$\sigma_\infty = \frac{\alpha\sigma_1\varepsilon_2^2 + \beta\sigma_2\varepsilon_1^2}{(\alpha\varepsilon_2 + \beta\varepsilon_1)^2}, \tag{02}$$

the effective conductivity at lower frequencies is

$$\sigma_0 = \frac{\sigma_1\sigma_2}{\alpha\sigma_2 + \beta\sigma_1}, \tag{03}$$

the effective permittivity at higher frequencies is

$$\varepsilon_\infty = \frac{\varepsilon_1\varepsilon_2}{\alpha\varepsilon_2 + \beta\varepsilon_1}, \tag{04}$$

the effective permittivity at lower frequencies is

$$\varepsilon_0 = \frac{\alpha\varepsilon_1\sigma_2^2 + \beta\varepsilon_2\sigma_1^2}{(\alpha\sigma_2 + \beta\sigma_1)^2}, \tag{05}$$

the relaxation time is

$$\tau_E = \frac{\alpha\varepsilon_2 + \beta\varepsilon_1}{\alpha\sigma_2 + \beta\sigma_1}, \tag{06}$$

$\alpha = d_1 / d,\ \beta = d_2 / d = 1 - \alpha.$

It is useful to mention here that the formulae above can be also obtained (King and Smith, 1981) by using a simple "electro-technical" approach. To do that one has to cut from the periodically-layered medium above the cylinder of any section S normal to the current direction. It is sufficient to examine one element of the medium consisting of two parts: one with the admittivity γ_1 and length d_1, and the other one with γ_2 and d_2 . The complex admittance of this element can be found as the admittance of the circuit composed by two complex resistors, so that

$$\frac{\gamma_{ef}}{d} = \frac{1}{\dfrac{d_1}{\sigma_1 - i\omega\varepsilon_1} + \dfrac{d_2}{\sigma_2 - i\omega\varepsilon_2}}. \tag{07}$$

After necessary transformations of (07) the formulae (01 - 06) are obtained.

From the phenomenological point of view, it makes no difference, whether the dispersion of complex admittivity is described as the dispersion of conductivity, or permittivity, or both parameters. In particular, it is shown (Kamenetsky and Timofeev, 1992) that the complex admittivity above (formula 01) can be presented in the form of Cole-Cole formula, provided the c factor equals 1 and the term $-i\omega\varepsilon_\infty$ is added describing displacement currents at higher frequencies, so that:

$$\gamma_{ef} = \sigma(p) + p\varepsilon_\infty = \sigma_\infty\left(1 - \frac{m}{1 + p\tau_E}\right) + p\varepsilon_\infty, \tag{08}$$

where m and τ_E are the chargeability (0<m<1) and IP time-constant in case of the step-shaped voltage is applied to a rock sample, and $p = -i\omega$.

In case of normal displacement currents are neglected one has the complete coincidence with the Cole-Cole formula at c=1:

$$\sigma(p) = \sigma_\infty \left(1 - \frac{m}{1 + p\tau_E}\right), \qquad (09)$$

or the complex resistivity

$$\rho(p) = \rho_\infty \left(1 - \frac{m_j}{1 + p\tau}\right), \qquad (10)$$

where $m_j = \dfrac{m}{1-m}$ and $\tau = \dfrac{\tau_E}{1-m}$ are the chargeability and IP time-constant in case of the step-shaped current is applied to a rock sample.

Phenomenological IP parameters in (08) are of the form:

$$m = 1 - \frac{\sigma_0}{\sigma_\infty}, \qquad (11)$$

$$\tau_E = \frac{\varepsilon_0 - \varepsilon_\infty}{\sigma_\infty - \sigma_0}, \qquad (12)$$

and it can be shown as well that (12) coincides with (06).

Correlation of dispersion to medium parameters

Following to our previous works (Gubatenko 1991, Kamenetsky and Timofeev 1992) we shall take $\varepsilon_1 = \varepsilon_2 = \varepsilon$. This is quite reasonable, because of the relative permittivity $\bar{\varepsilon} = \varepsilon / \varepsilon_v$ (ε_v=$10^{-9}/36\pi$ F/m) of natural materials does not vary very wide (from 1 for the air to 81 for the water), whereas the conductivity can vary in limits of ten and more orders. Then the formulae (02, 04 - o6) are of a simpler form:

$$\sigma_\infty = \alpha\sigma_1 + \beta\sigma_2, \qquad (02')$$

$$\varepsilon_\infty = \varepsilon, \qquad (04')$$

$$\varepsilon_0 = \varepsilon \frac{\alpha\sigma_1^2 + \beta\sigma_2^2}{(\alpha\sigma_2 + \beta\sigma_1)^2}, \qquad (05')$$

$$\tau_E = \frac{\varepsilon}{\alpha\sigma_2 + \delta\sigma_1}, \qquad (06')$$

and the formula (03) is not changed.

Suppose also that $\sigma_1 < \sigma_2$. Then according to (06) $\tau_2 < \tau_E < \tau_1$, where $\tau_1 = \varepsilon / \sigma_1$, $\tau_2 = \varepsilon / \sigma_2$ are relaxation times of displacement currents in layers with indexes 1 and 2 correspondingly. Therfore the time-constant τ_E of composite medium does not excceed the time constant τ_1 of the resistive layer.

After substituting (02' – 05') into (11, 12) one obtains the chargeability

$$m = 1 - \frac{1}{\alpha^2 + \alpha(1-\alpha)(1/\bar{\rho} + \bar{\rho}) + (1+\alpha)^2}, \qquad (13)$$

and the relative time-constant

$$\bar{\tau}_E = \tau_E / \tau_1 = \frac{1}{1 + \alpha/\bar{\rho} - \alpha}, \qquad (14)$$

where $\bar{\rho} = \rho_2 / \rho_1 = \sigma_1 / \sigma_2$.

Let us consider further the dependence of IP parameters on geometrical α and electrical $\bar{\rho}$ parameters of the medium. It is clear that α and $\bar{\rho}$ are varying in limits 0 to 1. Graphs of the chargeability and time-constant are shown in Fig. 1 a, b.

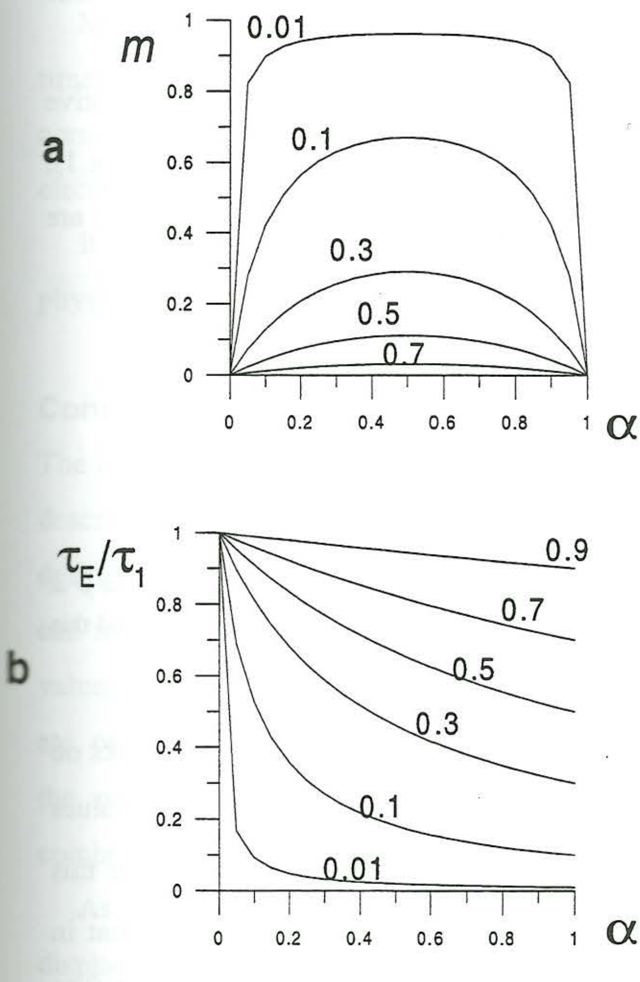

Fig. 1. Chargeability (**a**) and time constant (**b**) versus geometrical (α) and electrical ($\bar{\rho}$ as index of curves) parameters of the medium.

The chargeability $m \to 1$ at $\rho \to 0$ for any $0 < \alpha < 1$, has a maximum at $\alpha = 0.5$ (i.e. at $d_1 = d_2$) irrespective of the $\bar{\rho}$ value and increases with the decreasing of $\bar{\rho}$. In particular at $\alpha = 0.5$

$$m_{max}(0.5, \rho) = 1 - \frac{2}{1 + 0.5(\bar{\rho} + 1/\bar{\rho})}. \qquad (15)$$

The time-constant is maximal ($\bar{\tau} = 1$) at $\alpha = 0$ for any value of $\bar{\rho}$, decreases monotonously with the increasing of α and increases with the increasing of $\bar{\rho}$. In particular, at $\alpha = 0.5$ (i.e. for the maximal chargeability)

$$\bar{\tau}_E(0.5,\bar{\rho}) = \frac{2}{1+1/\bar{\rho}}. \qquad (16)$$

Corresponding graphs of m and $\bar{\tau}_E$ for α=0.5 are shown in Fig. 2.

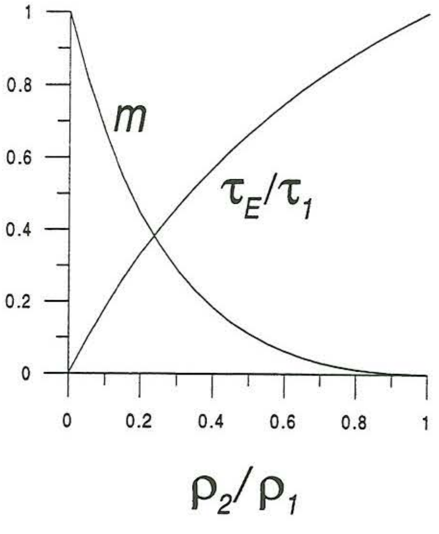

Fig. 2. Maximum chargeability and corresponding time constant versus $\bar{\rho}$ parameter (at α=0.5).

Results obtained show that the chargeability can reach a very high value, close to the limit value equal to one, for the quite reasonable values of medium parameters which explains the possibility of phenomena like sign-reversals etc. observed in practice. In this case, however, a small IP time-constant takes place, and v.v. the processes with a big time-constant posses a very low value of the chargeability.

As far as the combination is concerned of small values α and $\bar{\rho}$, where low-resistive thick layers (layers with index 2) are separated by high-resistive thin intervals (with index 1), we shall consider it separately. Since $\alpha << 1$ in this case and $\bar{\rho} << 1$ the formulae (13, 14) are of a simpler forms:

$$m = x/(1+x), \qquad (17)$$

$$\bar{\tau}_E = 1/(1+x), \qquad (18)$$

where. $x = \alpha/\bar{\rho} = d_1\sigma_2/d\sigma_1$.

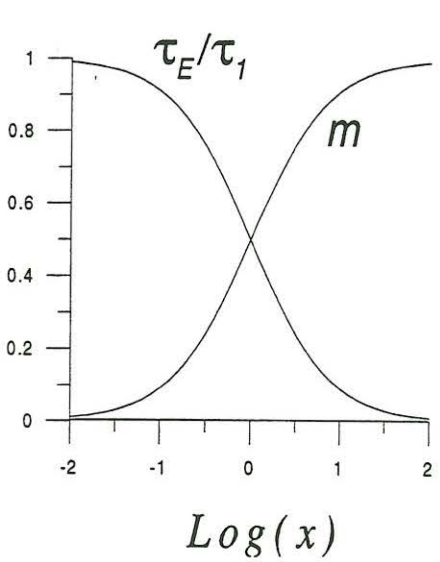

Fig. 3. Chargeability and time constant versus the ratio $x=\alpha/\bar{\rho}$ at small values of α and $\bar{\rho}$ (alternation of thick conductive and thin resistive layers).

It is seen that IP parameters of sedimentary rocks do not depend in this case on the α and $\bar{\rho}$ values separately, rather on their ratio $x = \alpha/\bar{\rho}$. As for this ratio of two small values, one have to suppose that in general case it can vary in a very wide range from 0 to

∞. It follows from (17, 18) that m changes in this case from 0 to 1, whereas $\overline{\tau}_E$ from 1 to 0. Changes of m and $\overline{\tau}_E$ values with change of the x value are shown in Fig. 3.

It should be emphasised that according to (17, 18) $\overline{\tau}_E = 1 - m$, i.e. the correlation of m and $\overline{\tau}_E$ values is vary clear and similar to previous cases: the less the time-constant $\overline{\tau}_E$, the more the chargeability, and v.v. It is quite probable that the chargeability m can also reach in this case very high values, like 0.5 and more. The matter is that the minimum α value is indeed limited by the physically possible minimum thickness d_1 . For example, α=10^{-1}, if d_1=10^{-4} and d=10^{-3} m, whereas the ratio $\overline{\rho} = \rho_2 / \rho_1$ may be equal to 10^{-1} and less. That corresponds to the chargeability m=0.5 and more.

As far as the time constant is concerned, remember that the $\overline{\tau}_E$ value does not exceed the $\overline{\tau}_1$ in any case. Let us estimate the value of $\tau_1 = \varepsilon\rho_1 = 10^{-9} \cdot \overline{\varepsilon}\rho_1 / 36\pi$. We shall take further the relative permittivity $\overline{\varepsilon} = 11.3$. Then $\tau_1 = 10^{-10}\rho_1$. Let the ρ_1 value is as high as 10^7 Ohm·m. Then τ_1=10^{-3} sec. Therefore, there is no practical sense to consider IP time-constants (τ_E or τ) of sedimentary rocks which are more than 1 msec.

Moreover, it is not possible to use the combination of big values of both chargeability and time constant. Modelling of processes of the sort can bring us to results which do not correspond to natural conditions, at least for sedimentary rocks containing no minerals with electronic type of conductivity (like dissiminated sulphides etc.).

It is expected also that the generalities above are common for other types of electro-physical dispersion. It is, however, the supposition only which takes a further research.

Conclusions

The chargeability and time-constant of the "fast IP" in heterogeneous sedimentary rocks, as described by Maxwell-Wagner effect, are not independent values. Their interconnection is defined by electrical and geometrical properties of heterogeneous medium. The chargeability can reach a very high value, close to the limit value equal to one, for the quite reasonable values of medium parameters which explains the possibility of phenomena like sign-reversals etc. observed in practice. In this case, however, a small IP time-constant takes place, and v.v. the processes with a big time-constant posses a very low value of chargeability. The combination of high values of both chargeability and time-constant does not take place.

As for the IP time constant, it does not exceed in any case the relaxation time of displacement currents in resistive layers. The estimation of this value shows that there is no practical sense to consider the IP time-constant of heterogeneous sedimentary rocks exceeding 1 msec.

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