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Key Points:

- Survey of the electromagnetic characteristics of fast magnetosonic (MS) waves has been made with observations of the Van Allen Probe A
- Satellite observations verify the validation of cold plasma approximation in deriving electromagnetic characteristics of MS waves
- Background magnetic field and normalized wave frequency become two key factors controlling the electromagnetic characteristics for MS waves

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Electromagnetic Characteristics of Fast Magnetosonic Waves in the Inner Magnetosphere

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Abstract In evaluating the effects of fast magnetosonic (MS) waves on magnetospheric particles, their magnetic spectra are often obtained from satellite observations, while electric field components are usually derived under the cold plasma approximation. However, such an approximation has not been verified with in situ observations yet. In this study, we report the electromagnetic characteristic for MS waves in various plasma environments with observations of the Van Allen Probe A. It is found that a considerable number of observed MS waves consist of dominated electrostatic components, which also suggest the importance of inspecting the estimation algorithm for the electric field components. Moreover, the comparison between results from statistical and theoretical analysis shows that electromagnetic characteristics of the observed MS waves can be well predicted by cold plasma theory. Our result indicates the validation of cold plasma approximation to estimate the electric field components of MS waves from their magnetic counterparts in the inner magnetosphere.

Plain Language Summary According to the quasilinear theory, fast magnetosonic (MS) waves can interact with magnetospheric particles via cyclotron resonances. To examine the resonance condition and calculate the diffusion rate, cold plasma approximation is often utilized, from which the wave vector and the relation between wave field components (magnetic field and electric field) can be derived. Thus, validating such an approximation becomes an essential topic, which still lacks corresponding satellite observational supports. Here, we use the observations of the Van Allen Probe A to show the electromagnetic characteristic of MS waves, and then verify the cold plasma approximation for deriving the electromagnetic characteristics of MS waves in the inner magnetosphere.

1. Introduction

Fast magnetosonic (MS) waves, known also as equatorial noise (Gurnett, 1976; Russell et al., 1970), are one of the most common emissions in the terrestrial magnetosphere, especially near the magnetic equatorial plane (Ma et al., 2013; Meredith et al., 2008; Yao et al., 2020). MS waves are observed to be propagating nearly normal to the background magnetic field, and have frequencies between the proton cyclotron frequency and lower hybrid resonance frequency (e.g., Santolik et al., 2002). These emissions are often found to have harmonic structures in the frequency domain, with frequencies locating closely to harmonics of the proton cyclotron frequency (Balikhin et al., 2015), but sometimes these emissions display continuous spectrums (Chen et al., 2016; Sun et al., 2016). Recent studies have demonstrated that MS waves can make contributions to the dynamic evolution of magnetospheric particles, including heating cold ions (Yuan, Yu, et al., 2018), scattering the ring current ions (Yu, Wang, et al., 2019; Yuan, et al., 2020), accelerating radiation belt electrons (Horne et al., 2007), leading to butterfly distributions of radiation belt electrons (Xiao et al., 2015). Moreover, it has been demonstrated that besides Landau resonances, MS waves can also interact with radiation belt electrons through the transit-time effect (Bortnik & Thorne, 2010; Li et al., 2014; Yu et al., 2020), and bounce resonances (Tao & Li, 2016).

MS waves are widely thought to be resulted from ion Bernstein instabilities, driven by ring current protons with a ring feature in velocity distributions (f_p), that is, $\partial f_p(v_{||} = 0, v_{\perp})/\partial v_{\perp} > 0$ (Balikhin et al., 2015; Chen et al., 2011; Min & Liu, 2016; Wu et al., 2021; Xiao et al., 2013; Yuan, Ouyang, et al., 2018; Zhou et al., 2014, 2021). Through controlling linear growth rates, background plasma densities have been found to modulate the intensity of MS waves (Yuan et al., 2017). With satellite observations and computer simulations, it has been found that MS waves largely have both electric and magnetic fluctuations, with

Figure 1. Van Allen Probe observations on February 2, 2013. (a) Observed plasma densities (red curve) and calculated cB_w/E_w of magnetosonic waves (blue curves). (b and c) Observed magnetic and electric field spectra. (d and e) Calculated wave ellipticities ϵ and normal angles θ .

magnetic fluctuations of MS waves predominately aligned along with the background magnetic field (Sun et al., 2016). However, there are some other cases of MS waves observed with ordinary electric fluctuations but rather weak magnetic fluctuations (e.g., Yuan et al., 2017), suggesting that these MS waves are more electrostatic-like.

Recall that in modeling the effects of MS waves on relativistic electrons, the observed magnetic fluctuations belonging to electromagnetic components are generally the main subject, from which electrostatic components (which are the components along the wave vector) are usually derived under the cold plasma approximation (Horne et al., 2007; Xiao et al., 2015; Yu, Li, et al., 2019). Whether such an approximation works well would determine the accuracy of estimating the contribution of MS waves into the evolution of magnetospheric particles. However, such an approximation has not been verified in detail with the satellite observations yet. Here, we report the electromagnetic characteristics of MS waves in the Earth's inner magnetosphere observed by the Van Allen Probe A. MS waves in the inner magnetosphere are found to largely have dominated electrostatic components. Comparison between results from statistical and theoretical analysis shows that electromagnetic characteristics of the observed MS waves can be well predicted by cold plasma theory. Our result provides an observational basis supporting the validation of cold plasma approximation for deriving the electromagnetic characteristics of MS waves in the inner magnetosphere.

2. Observations

Launched in August 2012, the Van Allen Probe missions (Mauk et al., 2012) consisting of two equivalent spacecrafts have orbited the Earth near the magnetic equator, measuring the particles, electric and magnetic fields, and waves in the inner magnetosphere. Data from this mission gives scientists a new insight

Figure 2. Comparison between the observed and theoretical electromagnetic characteristics of magnetosonic (MS) waves. (a) Ratios of magnetic and electric amplitude, cB_w/E_w , of the observed MS waves by the Van Allen A during September 2012 and February 2016. (b) Theoretical calculations of $c\delta B/\delta E$ for MS waves with $\theta = 89^\circ$. The red and blue stars in panel (a) denote the calculated $c\delta B/\delta E$ of the observed MS waves at about 1640 UT and about 1740 UT in Figure 1.

an power spectra density of electric field for each frequency channel during one day (Yuan et al., 2019). Furthermore, we have removed those emissions which last less than 5 min. The averaged cB_w/E_w of MS waves are illustrated in Figure 2a as a function of the plasma condition parameter $\alpha^* = \omega_{pe}^2 / \Omega_e^2$ (ω_{pe} is the electron plasma frequency, and Ω_e is the electron cyclotron frequency) and wave normalized frequencies $X = \omega / \Omega_H$ (ω is the wave frequency, and Ω_H is the proton cyclotron frequency). Note that the wave events reported in Figure 1 are marked as stars in Figure 2a (the red one for those waves observed at about 1640 UT, while blue one for 1740 UT). To verify whether the electromagnetic characteristics of MS waves can be described by the cold plasma approximation, the theoretical values of cB_w/E_w for MS waves have been evaluated from wave equations and shown in the next section.

into properties and distributions of various plasma waves and their effects on the dynamics of the terrestrial magnetosphere. In this paper we use tri-axial magnetic and electric spectrum data in the frequency range of 2 Hz to 12 kHz, and the high-resolution magnetic field vector data, both provided by the Electric and Magnetic Field Instrument Suites (EMFISIS) onboard the Van Allen Probe A (Kletzing et al., 2013). The background plasma density is derived from the upper hybrid resonance frequencies (Kurth et al., 2015) or the satellite potential acquired from the Electric Field and Waves (EFW) Instruments (Wygant et al., 2013).

Figure 1 shows that on February 2, 2013, crossing from outside the plasmasphere through the plasmopause to inside (panel a), the Van Allen Probe A has measured several intense waves in both electric and magnetic spectrums (panel b and c). With the ellipticities (ϵ) and wave normal angles (θ) obtained from Singular Value Decomposition (SVD) analysis of magnetic fluctuations (Santolik et al., 2002), we can identify whistler-mode chorus waves ($\epsilon \approx 1$) observed outside the plasmopause with frequencies near $0.5f_{ce}$ (f_{ce} denotes the electron gyrofrequency), and hiss waves ($\epsilon \approx 1$, $\theta < 45^\circ$) observed inside the plasmopause with frequencies near 300 Hz (Yu et al., 2017). Meanwhile, MS waves ($\epsilon \sim 0$, $\theta \approx 90^\circ$) with frequencies between about the proton gyrofrequency (f_{ch}) and the lower hybrid resonance frequency (f_{LH}), occur both outside and inside the plasmopause. To study the compositions of electrostatic and electromagnetic components, analysis of cB_w/E_w (c is the speed of light, while B_w and E_w are magnitudes of magnetic field and electric field fluctuations, respectively), as well as the refractive index, is generally made. The calculated cB_w/E_w of MS waves, shown in Figure 1a (blue curves), indicates that in this case MS waves outside the plasmopause might have different electrostatic components from those inside the plasmopause, which will be discussed in more details in the following.

We have surveyed the observed MS waves by the Van Allen Probe A from September 2012 to February 2016, and obtained the statistical results of cB_w/E_w for MS waves in different plasma conditions. Since the orbits of the Van Allen Probe A during such interval have covered the whole equatorial plane, only the Van Allen Probe A observations will be used. In our survey of MS waves, we only select those fluctuations whose frequencies lie between 10 Hz and $0.1f_{ce}$, ellipticities are between -0.2 and 0.2 , wave normal angles are greater than 80° , and electric intensities are 1.5 times greater than background noise. The noise level is defined as the medi-

Figure 3. Theoretically calculated ratio of the electromagnetic component to the electrostatic component of magnetosonic waves with $\theta = 89^\circ$. (a) Absolute values of calculated $\delta E_\perp / \delta E_\parallel$ (in a co-k coordinate system) as a function of the plasma condition parameter α^* and wave normalized frequency X . (b) Schematic diagram for co-B and co-k coordinate systems, where $\hat{x}$, $\hat{y}$, and $\hat{z}$ are the three orthogonal axes of the co-B coordinate system, while $\hat{e}_{11}$, $\hat{e}_{12}$, and $\hat{e}_\parallel$ for the co-k coordinate system.

is generally difficult to be accurately determined in satellite observations so that $\delta E_\perp$ and $\delta E_\parallel$ are hardly separated (although several algorithms have been proposed in previous studies, see Chen et al., 2019; Liu et al., 2018; Walker & Moiseenko, 2013), the latter parameter is used for comparison with wave observations. On the other hand, when electrostatic electric fluctuations are included, the former parameter (which can be derived from the latter parameter after the cold plasma approximation is utilized) should be used to evaluate the comparative contributions of electromagnetic and electrostatic components of wave fluctuations made to particle evolutions.

3. Theoretical Modeling and Results

To describe wave equations, it is common to choose the wave vector $\vec{k}$ to lie in the x - z plane, the background magnetic field $\vec{B}_0$ to be aligned with the z axis, and the y axis to complete a right-handed orthogonal coordinate system (Swanson, 2003), which will be called as a co-B coordinate system here. However, it is quite difficult to investigate the electromagnetic characteristics of oblique waves in such a co-B coordinate system, since longitudinal electrostatic and transverse electromagnetic components are hardly separated. Instead, we will implement our study in a so-called co-k coordinate system, in which y axis is the same as that in a co-B coordinate system, but the z axis is aligned along with the wavevector $\vec{k}$, rather than the background magnetic field $\vec{B}_0$. Figure 3b shows the schematic diagram of these two coordinate systems. To avoid ambiguity, the three orthogonal axes of the co-k coordinate system are labeled as $\hat{e}_{11}$, $\hat{e}_{12}$, and $\hat{e}_\parallel$, respectively. As a result, wave equations in a co-k coordinate system can be given as:

$$\begin{pmatrix} S \cos^2 \theta + P \sin^2 \theta - n^2 & -iD \cos \theta & (S - P) \cos \theta \sin \theta \\ iD \cos \theta & S - n^2 & iD \sin \theta \\ (S - P) \cos \theta \sin \theta & -iD \sin \theta & S \sin^2 \theta + P \cos^2 \theta \end{pmatrix} \begin{pmatrix} \delta E_{11} \\ \delta E_{12} \\ \delta E_\parallel \end{pmatrix} = 0 \quad (1)$$

or equivalently,

$$\delta E_{11} = \frac{P \cos \theta}{(P - n^2) \sin \theta} \delta E_\parallel$$

$$\delta E_{12} = -i \frac{D(P - n^2 \sin^2 \theta)}{(S - n^2)(P - n^2) \sin \theta} \delta E_\parallel \quad (2)$$

$$\left(S \sin^2 \theta + P \cos^2 \theta \right) n^4 - \left[SP(1 + \cos^2 \theta) + (S^2 - D^2) \sin^2 \theta \right] n^2 + P(S^2 - D^2) = 0$$

where θ is the angle between the wave vector and background magnetic field, δE_{11} , δE_{12} , and $\delta E_\parallel$ are the three components of wave electric field fluctuations $\delta \vec{E}$ in the co-k coordinate system, while S , P , and D are defined Stix parameters (Stix, 1992). Thus, we can use two parameters to manifest the electromagnetic characteristics of plasma waves, defined as $\delta E_\perp / \delta E_\parallel = \sqrt{\delta E_{11}^2 + \delta E_{12}^2} / \delta E_\parallel$, and $c\delta B / \delta E = ck / \omega \cdot \delta E_\perp / \delta E$. Note a MS wave will be electrostatic-like (electromagnetic-like) when its $\delta E_\perp / \delta E_\parallel$ as well as $c\delta B / \delta E$ is small (large). Since the direction of wave vectors

Figure 4. Statistical analysis of plasma conditions where magnetosonic (MS) wave events occurred. (a) Sampling events for each set of plasma condition parameter α^* and normalized frequencies X . (b) Cumulative percentage of the proportion for MS waves as a function of X .

Figure 2b shows the theoretical values of $c\delta B / \delta E$ for MS waves calculated with the equations above, where the wave normal angle is set to be 89° ($\theta = 89^\circ$). It is worthy of noting that we have also calculated the cases where θ changes from 88° to 90° , and the results are not significantly different from that under $\theta = 89^\circ$. It is obviously shown that the electromagnetic characteristics of the observed MS waves can be well predicted by theoretical results under the cold plasma approximation. On the other hand, Figure 3a illustrates theoretical values of $|\delta E_\perp / \delta E_\parallel|$ for MS waves, which corresponds to the calculated cB_w/E_w in Figure 2b under the cold plasma approximated. Here the subscripts “ $\perp$ ” and “ $\parallel$ ” denote perpendicular and parallel components with respect to the wave vector $\mathbf{k}$, respectively (Figure 3b). Moreover, contour lines for $|\delta E_\perp / \delta E_\parallel| = 0, 0.05, 0.1$, and 0.5 are also illustrated as dark red curves. Both the observations and theoretical calculations demonstrate (exhibited in Figures 2 and 3) that near the so-called lower hybrid resonance frequency wave fluctuations are almost electrostatic in a cold plasma. Below the lower hybrid frequencies, wave fluctuations become more and more electromagnetic-like as their normalized frequencies X decrease.

On the other hand, effects of the plasma condition parameter α^* are more complicated. As shown in Figure 3a, for small plasma condition parameters ($\alpha^* \leq 1$), an increase of α^* could result in a significant increase of $|\delta E_\perp / \delta E_\parallel|$ and thus more electromagnetic components of MS waves. However, variations of large plasma condition parameters ($\alpha^* > 10$) are found to affect $|\delta E_\perp / \delta E_\parallel|$ less, indicating negligible effects on the electromagnetic characteristics of MS waves. This fact can also be obtained from the detailed shapes of contour lines for $|\delta E_\perp / \delta E_\parallel|$, as (quasi-)vertical contour lines (denoting by dark red curves in Figure 3a) for $\alpha^* > 10$ indicate crucial roles of normalized frequencies X , but inessential roles of plasma condition parameter α^* in controlling electromagnetic characteristics of MS waves. It is clearly shown in Figure 3a that for all the plasma conditions, MS waves with normalized frequencies greater than 8 ($X \geq 8$) are dominated by electrostatic fluctuations, whose magnitudes are 10 times larger than electromagnetic fluctuations ($|\delta E_\perp / \delta E_\parallel|$).

4. Discussions

In order to obtain the statistical distributions for the electromagnetic characteristics of MS waves, we have examined MS wave events in different plasma conditions. As illustrated in Figure 4a, MS waves are observed almost uniformly mainly during $10 \leq \alpha^* < 600$ and $X \leq 30$, with a uniform distribution for $8 \leq X < 30$ but an increasing number of events for $X < 8$. Since high plasma condition parameters ($\alpha^* > 10$) only slightly affects the electromagnetic characteristics of MS waves (cf. Figure 3), detailed histograms of these MS waves as a function of X are studied. As indicated by the cumulative percentage in Figure 4b, those MS waves with $X \leq 8$ (which have significant transverse electric fluctuations since $|\delta E_\perp / \delta E_\parallel| \geq 0.1$) make up less than 40% of the total amount, and a great number of MS waves have dominant electrostatic fluctuations.

It has been demonstrated that MS waves in the inner magnetosphere can propagate in an arbitrary direction near the equatorial plane (Xiao et al., 2012). During their propagations, MS waves would travel through different plasma environments with varying plasma densities and magnetic fields, so that electromagnetic characteristics of MS waves will change accordingly. An increase (a decrease) of plasma densities would directly result in an increase (a decrease) of plasma condition parameters α^* , but an increase (a decrease) of magnetic field would result in a decrease (an increase) of both plasma parameters and normalized frequencies (note that the frequencies of MS waves would be kept fixed during their propagations). That is to say, for MS waves with fixed frequencies, they would become more electromagnetic-like when observed in

regions with strong magnetic field but more electrostatic—like in regions with weak magnetic field. When MS waves propagate in the inner magnetosphere, the plasma density and magnetic field (and thus the plasma parameter and normalized frequency) vary in several orders of magnitude, which would lead to significant changes in their electromagnetic characteristics (Yu et al., 2021). As a result, some MS waves would be found with both intense electric and magnetic fluctuations while others are more electrostatic-like.

5. Conclusions

In this study, cold plasma dispersion relation and wave equations transformed into a $\text{co-}k$ coordinate system are given. In this coordinate system, electromagnetic and electrostatic components ($\delta E_{\perp}$ and $\delta E_{\parallel}$) of electric fluctuations are separated entirely and the ratios of these two components for MS waves under the cold plasma approximation are derived. The corresponding theoretical values of $c\delta B / \delta E$ for MS waves have been illustrated as a comparison to the statistical results of the MS waves observed by the Van Allen Probe A in the inner magnetosphere. It is shown that electromagnetic characteristics of the observed MS waves can be well predicted by cold plasma theory. Additionally, the roles of the two key factors (the plasma condition parameter α^* and the wave normalized frequency X) playing in controlling the electromagnetic characteristics have been analyzed. The background magnetic field is suggested to play a crucial role in determining the electromagnetic characteristics of MS waves in the inner magnetosphere.

However, our conclusions in this paper are drawn under cold plasma approximation, and thermal effects of the background plasma and the contribution of ring current particles have not included, which will be studied in the future work.

Data Availability Statement

The Van Allen Probes data are available at the web <http://emfisis.physics.uiowa.edu/data/index>.

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