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Comparison of 2D modelling methods: rapid inversion vs. polynomial fitting

Introduction

Some years ago, a paper describing a new 2D MT modelling scheme drew the attention of the induction community (Smith and Booker 1991). Known as the « Rapid Relaxation Inverse » (RRI) method, its corresponding FORTRAN programme has been made publicly available a short time after. Since then, it has probably become the most used 2D MT modelling method, owing its success to the short computing time required, even for large models.
This scheme has its drawbacks, however. The main one is that the resulting model lacks focus. This behaviour can be particularly well observed in the Fig. 3 of the article by the RRI programme authors. The most striking effect is the downward distortion of the model prism.
In this article we compare the results of the RRI method with those of the polynomial fitting method (Schnegg 1993.) published in the framework of the MT-DIW1 interpretation workshop (Jones 1993).

The Rapid Relaxation Inverse method

The reader will find the full scheme description in the paper by Smith and Booker. But let us recall how RRI works: Starting from a guessed 2D conductivity distribution σ_0 (y,z) (which can also be a simple homogeneous half-space), the field E_0 (y,z) (for TM mode: H_0 (y,z)) is calculated. A perturbation $\delta\sigma$ is applied to σ_0 and the produced variation δV of the field at the surface is computed. This computation is made easier by neglecting the horizontal field gradients with respect to the vertical ones, and results from an upward integration of an exact differential at each site and each frequency. The misfit between the measured and the predicted data is used to update σ_0 and the process is repeated iteratively until convergence is reached.

The polynomial fitting method

A 2D conductivity distribution can always be approximated by a patchwork of rectangular pieces of constant conductivity. When P.E. Wannamaker published his famous 2D forward modelling programme (Wannamaker 1985), most of the modellers were varying manually the size, the shape and the conductivity of each individual block until a reasonable model was found. The finite element mesh had to be rebuilt after every alteration of the model. This method was quite demanding in human resources and is probably no more used.

An attempt to make this task more automatic comes up against the problem of the large amount of free parameters that must be fed to the minimisation routine. However, numerous fast conductivity changes in the vertical and horizontal directions are rarely expected. A simplified model parametrisation could appropriately represent the conductivity distribution. The one which is used here is based on the representation by low-order polynomials of functions of the distance on the profile (Schnegg 1993). The half-space is divided into n_s contiguous vertical 1D models, one per measuring site, extending laterally between the points situated at half way to the right and left neighbouring sites (Fig. 1).

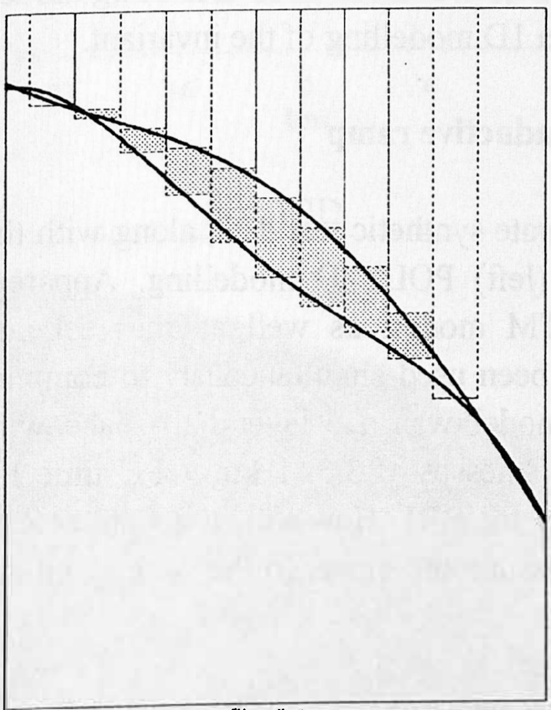
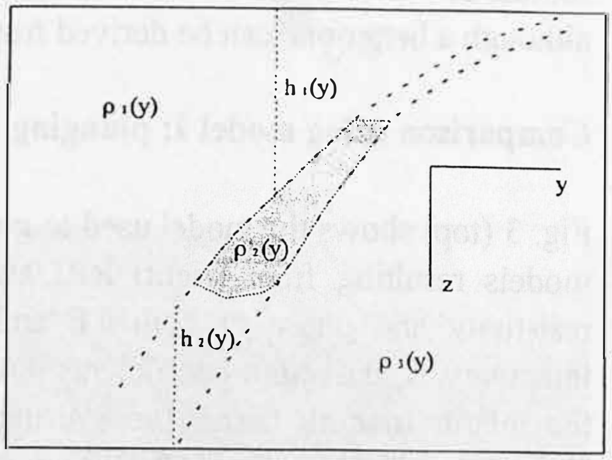


Fig. 1. 2D model showing contiguous 1D models with layer thicknesses described by polynomials.



$$h_j > 0$$

$$\rho_j > 0$$

$$\log h_j = \sum_i c_{hji} y^i$$

$$\log \rho_j = \sum_i c_{\rho ji} y^i$$

Fig. 2. Resistivities and thicknesses as polynomial functions of the profile distance y.

Two additional 1D models at both ends extend laterally to infinity (i.e., to the mesh limits). All these models have the same number of layers. The logarithm of the layer thicknesses and resistivities are described by low-order polynomials of the abscissa y (Fig. 2). The coefficients of these polynomials are the parameters of the model. Their total number is independent of the number of sites:

$$2N - 1 + R_N + \sum_{k=1}^{N-1} (R_k + T_k)$$

where N is the number of layers, R_k and T_k the order of the polynomials for the resistivity and the thickness of layer k.

To prevent the minimising routine from assigning negative values to the resistivity and/or to the layer thickness, the logarithm of these values is used instead. The minimising routine varies the parameters until a minimum of the objective function is found. The objective function is the misfit between the measured data and the response of the model to the finite element forward routine of P.E. Wannamaker. A layered half-space can be used as starting model, although a better one can be derived from a 1D modelling of the invariant.

Comparison using model 1: plunging conductive ramp

Fig. 3 (top) shows the model used to generate synthetic test data, along with the models resulting from (right) RRI and (left) POLY2D modelling. Apparent resistivity and phase of both TE and TM modes, as well as the real and imaginary H_z induction coefficients have been used simultaneously to compute the misfit. In both cases, the starting model was a 3-layer half-space with resistivities of 5000, 10, 500 Ωm and thicknesses of 3.5, 1 km. CPU time for POLY2D was about 30 times longer than for RRI. However, the final results show that the POLY2D model is much sharper and closer to the starting model than the RRI model.

Comparison using model 2: conducting square box

Fig. 4 shows the situation for a conductive rectangular prism of 10 Ωm in a 300 Ωm matrix. Here the starting model is an homogeneous half-space of 50 Ωm. The same conclusions as for the ramp can be drawn for this model.

Although it looks simpler than the ramp model, it is more demanding due to its abrupt, discontinuous lateral character. If the value of the resistivity is not allowed lateral variations (zero degree polynomial), then the excess conductance at both sides of the box must be handled by the polynomials which control the layer thicknesses.

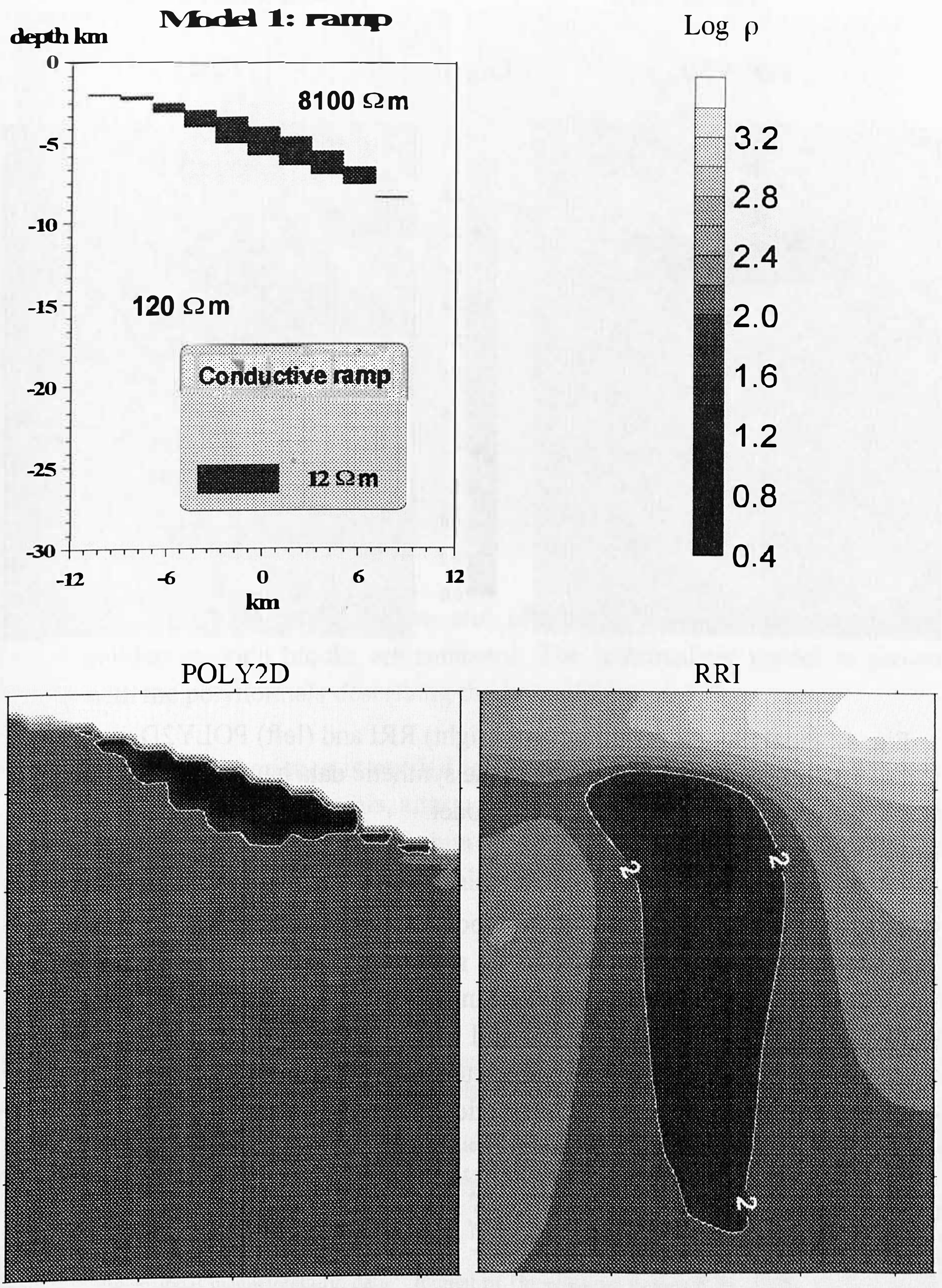


Fig. 3. Top: Model used to generate synthetic test data, and models resulting from (right) RRI and (left) POLY2D modelling.

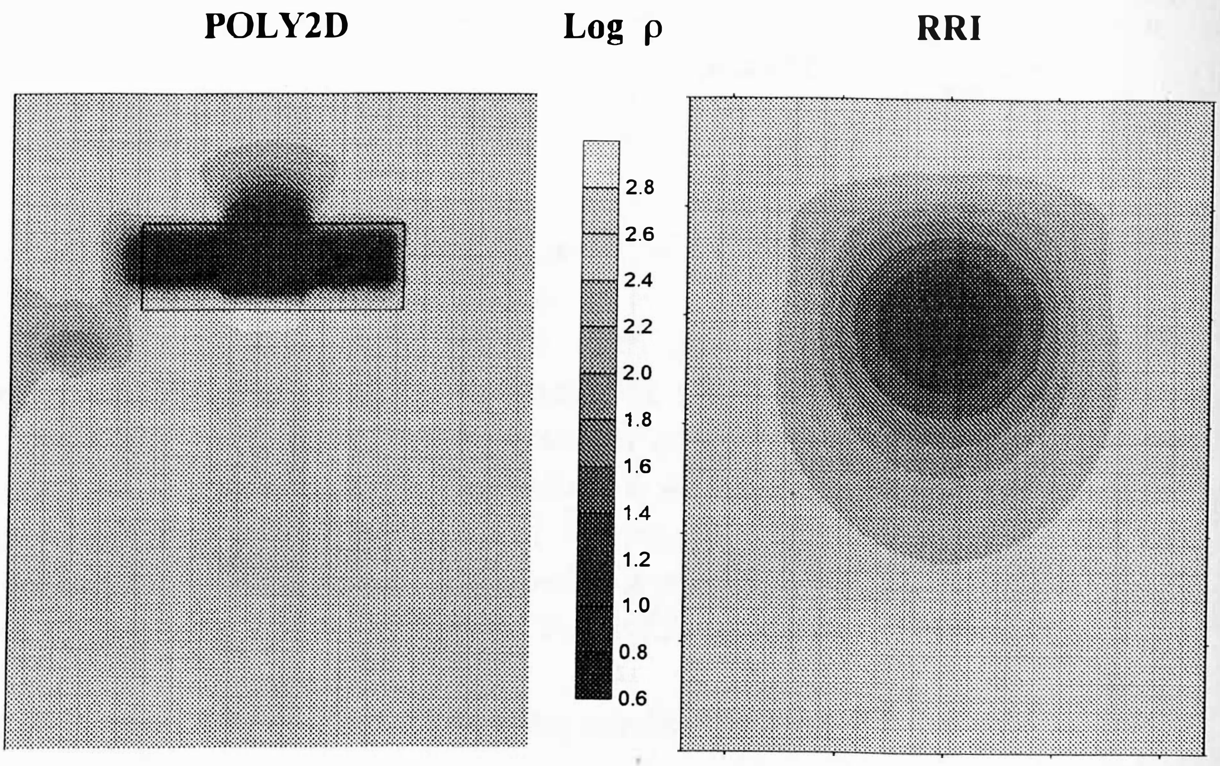


Fig. 4. Final models resulting from (right) RRI and (left) POLY2D modelling. The original model from which the synthetic data have been computed is superimposed on the POLY2D model.

This is obvious on the intermediate model of Fig. 5, where the 2 troublesome side blocks are pushed downwards. To remove any useless structure, a second modelling stage can be applied to this intermediate model. This time, the block thickness geometry is kept fixed, and the individual block resistivities (the logarithm of them) are used as parameters. The objective function is now the product of the data misfit and of the model resistivity roughness R:

$$R = \sum_{j=1}^{Nsites} \sum_{i=1}^{Nlayer-1} \left| \log \frac{\rho_{j,i+1}}{\rho_{j,i}} \right|$$

The final model shows that the side blocks have been successfully removed (Fig. 5.).

Intermediate model **Final model**

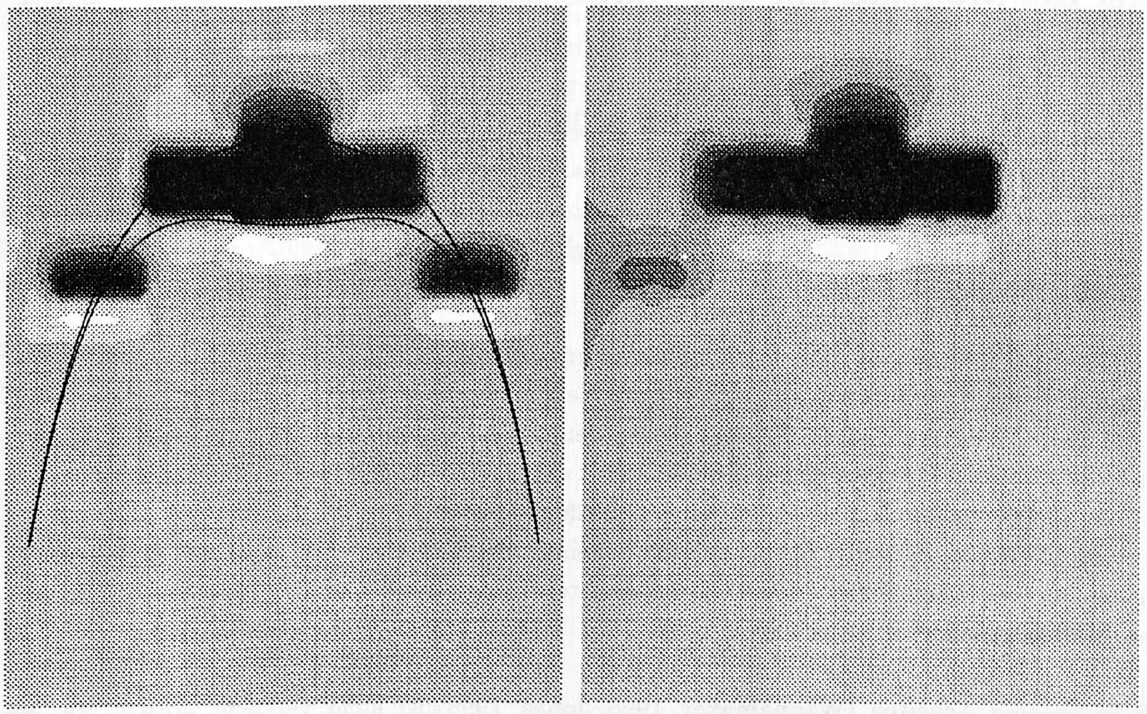


Fig. 5. POLY2D model before and after the 2nd modelling stage. The conductive side blocks are removed. The intermediate model is shown with the polynomials describing the layer thickness.

Some modelling experience is needed to choose a convenient number of layers and the order of the polynomials, although this choice is not critical. To avoid to be trapped into secondary local minima of the objective function, it proves safer to start with polynomials of order smaller than 3 and to increase the order on subsequent modelling runs.

References

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