Testing Vasseur and Weidelt's Thin-Sheet Algorithm

Paul Bruton
Applied Geophysics Unit, University College Galway, Ireland.

July 9, 1994

This paper deals with restrictions in the use of the Vasseur and Weidelt (1977) thin-sheet algorithm. For a more theoretical discussion on thin-sheet methods, the reader is referred to Vasseur and Weidelt (1977) and Dawson and Weaver (1979). For a comparison of these two algorithms the paper by Mareschal and Vasseur (1984) is instructive.

1 The Thin-Sheet Algorithm

Vasseur and Weidelt's (1977) algorithm uses the Gauss-Seidel iterative method to compute the surface electric field within a thin-sheet overlying a stratified Earth.

In initial tests of this algorithm I found that the Gauss-Seidel process did not converge for models with large cells or large conductances. Following discussions with Prof. P. Weidelt in July 1991 about the limitations of the thin-sheet method, he recommended the following should be true for the response of the thin-sheet to be accurate:

$$a\omega\mu_0\tau \ll 1 \qquad (1)$$

where a is the length of the side of each cell in metres, τ is the maximum anomalous conductance in Siemens, μ_0 is the permeability of free space and $\omega = 2\pi/T$ where T is the minimum period of investigation in seconds.

The expression in Equation 1 effectively limits the permissible cell size for a given anomalous conductance and period. I now examine the effect of changing the cell size for both the Pyrenean model of Vasseur and Weidelt (1977) and Mareschal and Vasseur's (1984) model of Scotland.

2 The Pyrenean Model of Vasseur and Weidelt (1977)

Vasseur and Weidelt's (1977) thin-sheet model of the Pyrenees was constructed to investigate the possibility of a conducting channel inside the continental crust linking the Atlantic ocean and Mediterranean Sea. The anomalous domain is 1260km north to south and 1440km east to west and contains $14 \times 16 = 224$ cells each 90km on a side (Figure 1).

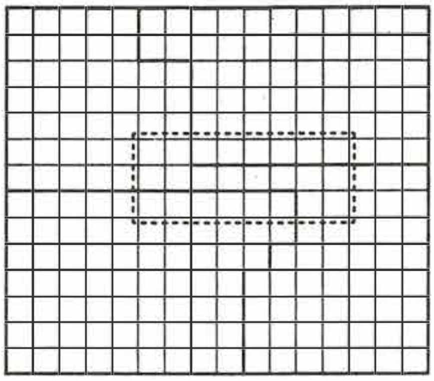

Figure 1: The thin-sheet used by Vasseur and Weidelt (1977). The cell size is 90km. The outlined regions at the top right and bottom left represent France and Spain respectively. The data domain is marked with a dashed box.

The thin-sheet is intended to represent the top 2km of the crust, so for the conductance of the oceanic region, a value of 8000S was used and for the resistive continental region a value of 500S. The underlying 1D structure consists of a 100km thick resistive layer of 2000Ωm and a conducting substratum of 10Ωm beneath.

The responses were computed at periods of 2700s, 5400s and 9000s for which Equation 1 suggests a maximum cell size ≪42.7km, 85.4km and 142.5km respectively. In order to demonstrate the effect of reducing the cell size, I have constructed a new model with smaller cells by replacing each 90km cell with four 45km cells. Since the response is computed at the centre of each cell, the locations of the responses of the two models do not coincide. I computed a bicubic spline (Press *et al.*, 1986, p100) of each field component (E_x, E_y, B_x, etc.) over the surface of the 45km model and used interpolated values to find the responses at points corresponding to those of the 90km model. The responses of the models are shown in Figure 2. It is obvious that both the real and imaginary components are affected by the increase in resolution, but it is interesting to note that though the magnitude of the real component is quite different, the direction is almost unchanged and that the opposite is true for the imaginary component.

3 The Scottish Model of Mareschal and Vasseur (1984)

Mareschal and Vasseur (1984) compare the different thin-sheet methods of

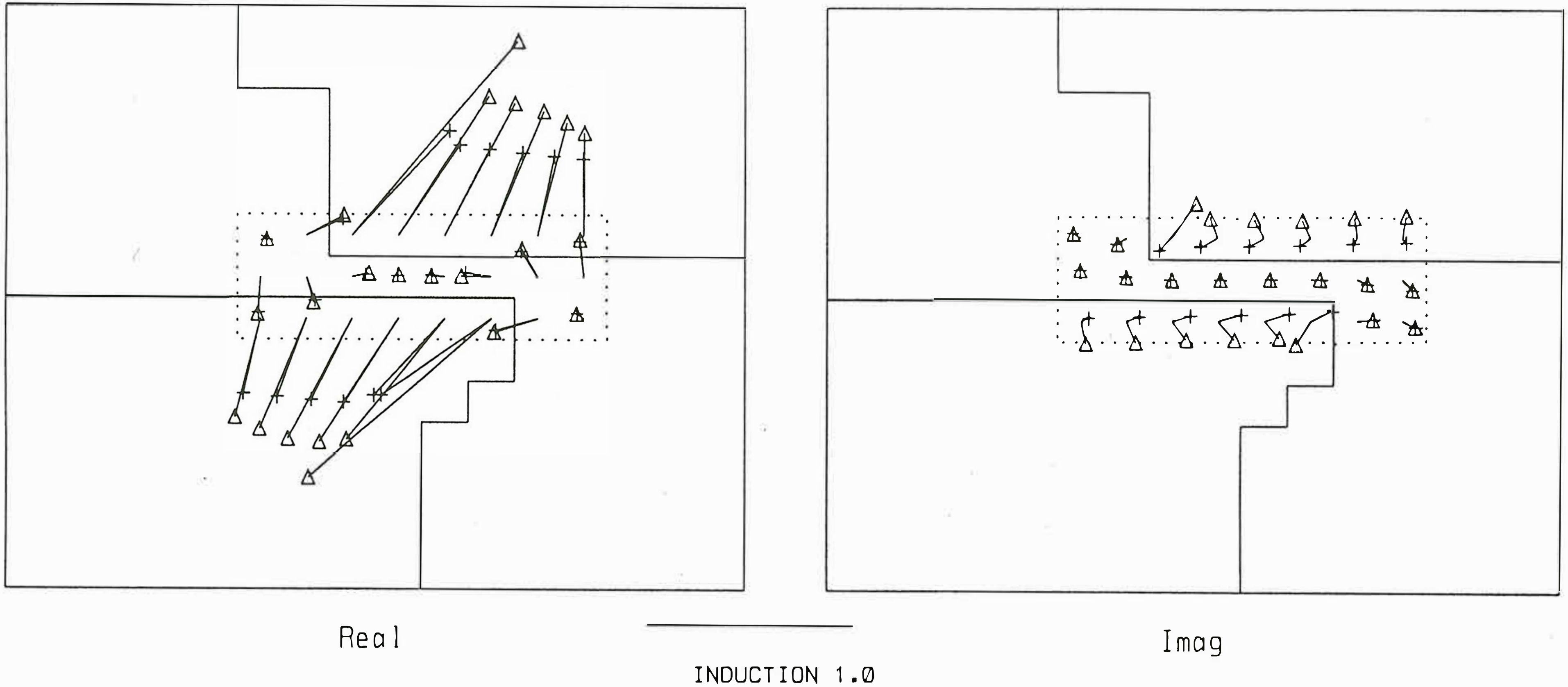

Figure 2: The response of the Pyrenean model of Vasseur and Weidelt (1977) (+) compared to the response of the same model with cell size reduced by half (△). The period is 2700s and the data presented are Wiese induction arrows. An induction arrow of length 1.0 is shown for comparison.

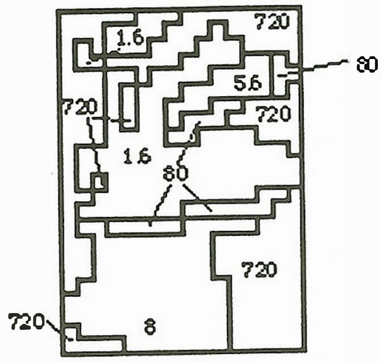

Figure 3: The thin-sheet model used by Mareschal and Vasseur (1984) The cell size is 20km. The conductances are given in Siemens.

Vasseur and Weidelt (1977) and Dawson and Weaver (1979) by computing the response of a model of Scotland originally created by Weaver (1982). I shall only deal with the Vasseur and Weidelt (1977) response here. The model has an anomalous domain 460km north to south and 320km east to west containing $23 \times 16 = 368$ cells of size 20km. The conductances vary from 1.6S to 720S and the underlying structure is a half-space of 2000Ωm(Figure 3).

The authors noted that at periods shorter than 30s the Gauss-Seidel iterations failed to converge and no response could be computed. However, Equation 1 suggests that the minimum period for an accurate response from this model with 20km cells must be $\gg$ 113s. If a response is to be computed at 30s, the maximum cell size must be $\ll$ 5.3km

I have constructed a new model in the same way as in Section 2 by replacing each 20km cell with four 10km cells of the same conductance. The response of the 20km model and the splined response of the 10km model are shown in Figure 4. As for the Pyrenean model, the magnitudes of the real induction arrows are longer for the model with the smaller cells. The directions of the real and imaginary arrows are also affected by the change in cell size.

These two tests illustrate the effect that a change in cell size has on the responses of a thin-sheet model. Even the cell sizes I have used are not small enough. If I take '$A \ll B$' to mean '$10A \leq B$' then I should use a cell size of <4.3km for the Pyrenean model and <530m for the Scottish model. This would entail thin-sheets with $(14 \times 21) \times (16 \times 21) = 98784$ and $(23 \times 38) \times (16 \times 38) = 531392$ cells respectively. Even taking a more liberal view that $3A$ should be $\leq B$ still requires thin-sheets with more than 10000 elements.

I have implemented Vasseur and Weidelt's (1977) algorithm on a 486

PC in order to model regional induction in Ireland (Bruton, 1994). Typical models covered an area around Ireland of $620 \times 620\text{km}^2$ and in order to accomodate large variations in conductance at the coast I required a cell size of 20km.

The period of interest was 1024s but with a maximum conductance of 5000S, Equation 1 implies a maximum cell size $a << 26km$. Taking the liberal view that $3A$ should be $\leq B$, a cell size of < 8.7km would be suitable, but this would be computationally time consuming. However in view of the effect of cell size on the induction arrows and in particular the imaginary arrow, I tested the algorithm with cell sizes of 10km and 20km. The responses were very similar in both cases indicating that, for some models, $a\omega\mu_0\tau < 1$ may be a sufficient condition in place of Equation 1. However, I recommend a discretisation test on any thin-sheet models before trusting the responses.

This work was funded jointly by GKSS (Germany) and EOLAS (Ireland). I would like to acknowledge the assistance given to me by the Applied Geophysics Unit UCG and the Institute for Geophysics at the University of Frankfurt, in particular to Colin Brown, Volker Haak, Karsten Bahr and Marcus Eisel. I would also like to acknowledge the hospitality and assistance of Peter Weidelt at the University of Braunschweig.

References

Bruton P (1994). *Analysis of broadband magnetotelluric data and an application to the Irish Variscides.* PhD thesis, University College Galway, Ireland.

Dawson T W and Weaver J T (1979). Three-dimensional induction in a non-uniform thin sheet at the surface of a uniformly conducting earth. *Geophys. J. of the Royal Astronomical Soc.*, 59:445–462.

Mareschal M and Vasseur G (1984). Bimodal inducton in non-uniform thin sheets, do the present algorithms work for regional studies? *Journal of Geophysics*, 55:203–213.

Press W H, Flannery B P, Teukolsky S A, and Vetterling W T (1986). *Numerical Recipes, The Art of Scientific Computing.* Cambridge University Press, Cambridge, UK.

Vasseur G and Weidelt P (1977). Bimodal electromagnetic induction in non-uniform thin sheets with an application to the northern Pyrenean induction anomaly. *Geophys. J. of the Royal Astronomical Soc.*, 51:669–690.

Weaver J T (1982). Regional induction in Scotland: an example of three dimensional numerical modelling using the thin sheet approximation. *Physics of the Earth and Planetary Interiors*, 28:161–180.

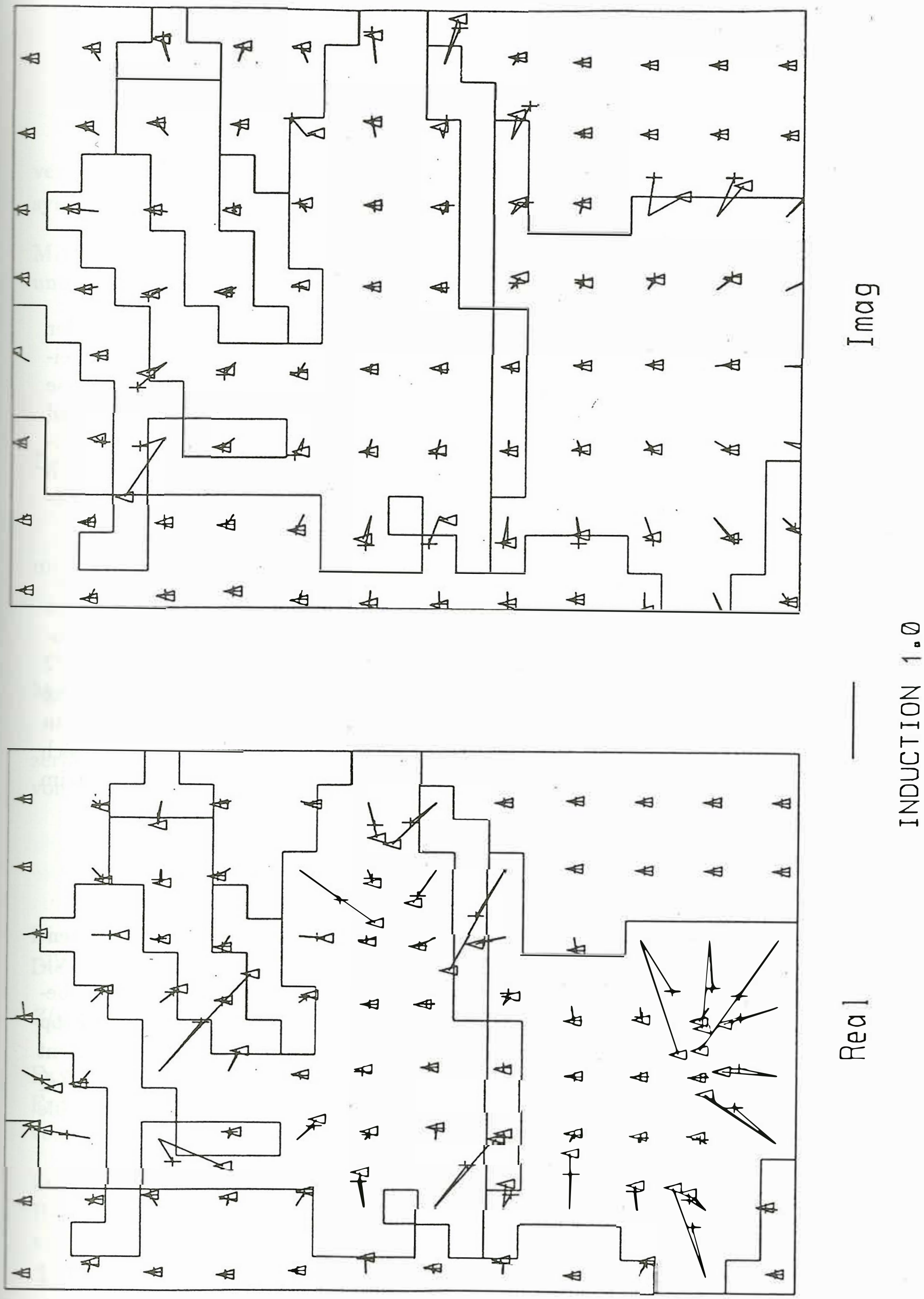

Figure 4: The response of the Scottish model of Mareschal and Vasseur (1984) (+) compared with the response of the same model with the cell size reduced by half (△). The period is 30s and the data presented are Wiese induction arrows. For clarity of presentation, only every second data point is shown. An induction arrow of length 1.0 is shown for comparison.