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SYMMETRY PROPERTIES OF ELECTROMAGNETIC TOMOGRAPHY

INTRODUCTION

Interest in large-scale imaging of formation conductivity has generated many studies in recent years. The most suitable techniques for such imaging are electrical tomography and electromagnetic tomography. For practical reasons, increased emphasis is being laid on the development of electromagnetic tomography in the frequency domain [Spies, 1992; Lee and Xie, 1993; Gasnier et al, 1994]. These tomography techniques are attractive for a wide range of purposes: environmental and engineering applications, mineral exploration, and also reservoir characterization. The availability of modelling and inversion software is a crucial factor in the development and in the applicability of such new techniques, and many studies have been undertaken in order to develop such capabilities. Modelling is usually performed in 2-D model geometry with 3-D source geometry; various methods can be found in the literature: finite element [Unsworth et al., 1993], domain integration with nonlinear approximation [Torres-Verdin and Habashy, 1993], and use of the t-q or ω-q transform [Lee and Xie, 1993].

We have solved the modelling problem with a formulation based on boundary integral equations [Straub, 1994]. We assume a two-dimensional model geometry with piecewise homogeneous domains. From the reduced wave equation, we deduce a set of boundary integral equations for both electrical and magnetic fields. The model geometry is given by the contours or boundaries in the transverse plane. Their discretization leads to polygonal contours. In our particular version, the unknowns in the wavenumber domain are the boundary values of the axial fields and the normal derivatives of these fields. After computation of these boundary values, it is possible to obtain the axial and transverse fields at any point of the model.

The edges of the polygonal contours introduce angular singularities. These singularities are treated in an accurate manner. Validation of the numerical results is an important and sometimes neglected step in such software development. We explain in this paper how the symmetry properties of the synthetic tomogram can be exploited in order to easily test the numerical accuracy of the code.

DATA ACQUISITION AND DATA REPRESENTATION

Tomography consists in making cross-well measurements. One borehole contains an electromagnetic source, the other borehole contains the receiver. A commonly-used source is a magnetic dipole, whose axis is coincident with the borehole axis. The receiver consists of a three-axis magnetic sensor. Such a configuration enables the acquisition of bivariate data. The location of the source may vary within a given depth interval in the source borehole; in the same way, the receiver may vary within the same depth inteval in the receiver borehole. The two intervals approximately delimit a study region. Coverage of this study region is made possible by collecting the data corresponding to all pairs of source and receiver points. The first requirement for two-dimensional imaging is the availability of bivariate data. This dimensionality constraint is met in such a tomography. The second requirement for the uniqueness of the solution is the necessity for a complete coverage over the boundary surrounding the study region, as partial coverage will limit the spatial resolution of the image. Unfortunately this requirement is not met in geophysical tomography; this limitation is inherent in this specific geophysical application.

For control purposes, it seems important to be able to depict the entire set of data acquired on a given pair of boreholes. A convenient way of doing this is to introduce the tomogram concept. Each measured parameter (e.g. a component of the magnetic field in the receiver borehole) is plotted on a plane as a function of the source position Z_S and the receiver position Z_R. This type of representation in the ZZ plane is called a tomogram and enables visual control of the entire of experimental data. It can be applied with synthetic data as well. In the general case, there is no obvious relationship between the geoelectrical model and the tomogram in the ZZ plane. However it is interesting to observe how a given intrinsic symmetry of the geoelectrical model is transformed in the ZZ plane.

SYMMETRY PROPERTIES

We will establish here a correspondence between the symmetry of the geoelectrical model and the symmetry of the tomogram. Our reasoning is based on the reciprocity theorem [Harrington, 1961]; we first recall the general form of this theorem.

Consider two states corresponding to a given geoelectrical model and two source models. The Maxwell equations in the frequency domain are:

$$\vec{\nabla} \times \vec{E}_I = -z\vec{H}_I - z\vec{M}^\bullet_I \qquad \vec{\nabla} \times \vec{E}_{II} = -z\vec{H}_{II} - z\vec{M}^\bullet_{II}$$

$$\vec{\nabla} \times \vec{H}_I = y\vec{E}_I + \vec{J}^\bullet_I \qquad \vec{\nabla} \times \vec{H}_{II} = y\vec{E}_{II} + \vec{J}^\bullet_{II}$$

The model is specified by the spatial distribution of the impedivity $z = i\omega\mu$ and the admittivity $y = \sigma + i\varepsilon\omega$. Each state is characterized by the distribution of magnetic sources $\vec{M}^\bullet$ and electrical sources $\vec{J}^\bullet$. These vectors $\vec{M}^\bullet$ and $\vec{J}^\bullet$ represent the volume density of the dipolar moment of these sources. In our application, the sources are point sources represented by Dirac distributions at points S_I and S_{II}. In a classical development, by forming well-chosen bilinear products, we obtain the equality:

$$\left\langle \vec{J}^\bullet_I . \vec{E}_{II} \right\rangle - \left\langle z\vec{M}^\bullet_I . \vec{H}_{II} \right\rangle = \left\langle \vec{J}^\bullet_{II} . \vec{E}_I \right\rangle - \left\langle z\vec{M}^\bullet_{II} . \vec{H}_I \right\rangle$$

This expresses the reciprocity theorem. Each term corresponds to an integration over the whole space i.e. IR^3. Each term can be understood as the result of a measurement. The quantities

$$\left\langle \vec{J}^\bullet .... \right\rangle \text{ or } \left\langle z\vec{M}^\bullet .... \right\rangle$$

are measurement operators, which are applied to the electrical field $\vec{E}$ or the magnetic field $\vec{H}$. So the source distributions play the role of sources in one state and of measurement operators in the other state. In our application to electromagnetic tomography, we will usually restrict this reciproctity theorem to magnetic dipoles and to media with constant permeability. In this case, we obtain the equality:

$$\left\langle \vec{M}_I^{*}.\vec{H}_{II} \right\rangle = \left\langle \vec{M}_{II}^{*}.\vec{H}_I \right\rangle \quad \text{or} \quad \vec{M}_I.\vec{H}_{II}\,(S_I) = \vec{M}_{II}.\vec{H}_I\,(S_{II})$$

In the same way we could write for the electrical dipoles:

$$\left\langle \vec{J}_I^{*}.\vec{E}_{II} \right\rangle = \left\langle \vec{J}_{II}^{*}.\vec{E}_I \right\rangle \quad \text{or} \quad \vec{J}_I.\vec{E}_{II}\,(S_I) = \vec{J}_{II}.\vec{E}_I\,(S_{II})$$

In these expressions, the vectors $\vec{M}$ and $\vec{J}$ are the dipolar moments of the sources. Due to the Dirac distributions, the volume integrals are transformed into quantities defined at point S_I and S_{II}.

We consider now a two-dimensional model geometry with a horizontal strike axis and a vertical dipolar source, for example a magnetic dipole. In the transverse plane of symmetry, which is of interest in our problem, the only non-zero fields are the axial electrical field and the transverse magnetic field. We will concentrate on the vertical magnetic field in the receiver borehole, because it represents the observed field. The reciprocity theorem leads to:

$$M_{Iz}.H_{IIz}(S_I) = M_{IIz}.H_{Iz}(S_{II})$$

This property can be combined with specific symmetries of the model. There are three basic symmetries: a symmetry with respect to a horizontal plane, a symmetry with respect to a vertical median plane, and a symmetry with respect to a central point. The important point is that in each symmetry transformation, the set of vertical boreholes remains invariant. As shown in Figures 1, 2, and 3, it is possible in each case to state an identity between one experiment given by a pair (S_1, M_1) and another experiment given by a pair (S_2,M_2).

a) Symmetry with respect to a horizontal plane (Figure 1)

A π rotation and a sign reversal lead to the following relations:

$$Z_{S1}+Z_{S2}= 2Z_0$$
$$Z_{M1}+Z_{M2}= 2Z_0$$

This expresses a symmetry with respect to the center 0 in the tomogram plane: the fields H_Z and E_y are symmetric, the field H_X is skew-symmetric.

b) Symmetry with respect to a vertical median line (Figure 2)

Combination of the reciprocity and a π rotation leads to the following relations:

$$Z_{S1}=Z_{M2} \quad \text{and} \quad Z_{M1}=Z_{S2}$$

This expresses a symmetry with respect to the first diagonal in the tomogram plane. This symmetry is valid only for the field H_Z.

c) Symmetry with respect to a central point (Figure 3)

Combination of the reciprocity and a π rotation leads to the following relations:

$$Z_{S1}+Z_{M2}= 2Z_0$$

$$Z_{S2}+Z_{M1}= 2Z_0$$

This expresses a symmetry with respect to the second diagonal in the tomogram plane. This symmetry is valid only for the field H_Z.
The correspondence between the symmetries of the model and the symmetries of the tomogram is summarized in Table 1.
It is clear that the symmetries which involve the reciprocity property are valid for a field whose nature and direction are identical to the source. It is important to mention that these symmetries are valid for the total fields and for the primary fields, and consequently also for the secondary fields. It can also be recognized that a combination of the two arbitrary symmetries implies the third symmetry for both the model and the tomogram. These properties can conveniently be exploited in order to check the accuracy of numerical modelling results.
We present in Figure 4 a model which possesses a symmetry with respect to a central point. It contains two square conductive bodies. The transmitter frequency is 10 kHz, so that the règime is situated in the resonance region. A control of the discretization of the boundaries (especially at the edges of the bodies) made it possible to obtain a symmetric tomogram for the field H_{ZS}, as shown in Figure 5. Less than 68 boundary elements were needed for each body in order to obtain the correct symmetry. This symmetry is in agreement with Table 1. It is important to observe that the symmetry and convergence criteria are simultaneously met. Thus visual validation of the tomogram becomes a sensitive and convenient tool for the numerical control of modelling results.

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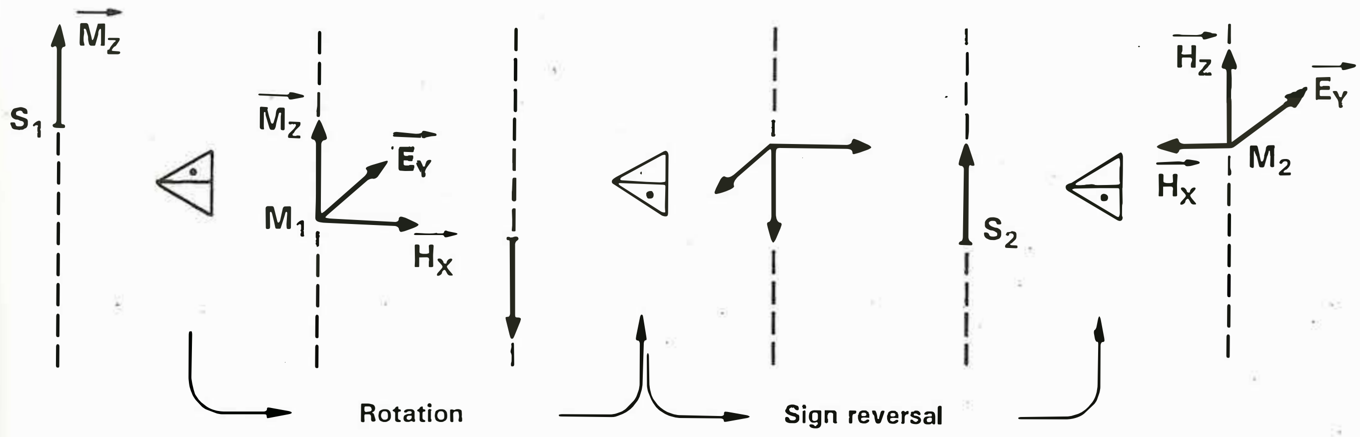

Figure 1: Symmetry with respect to a horizontal plane

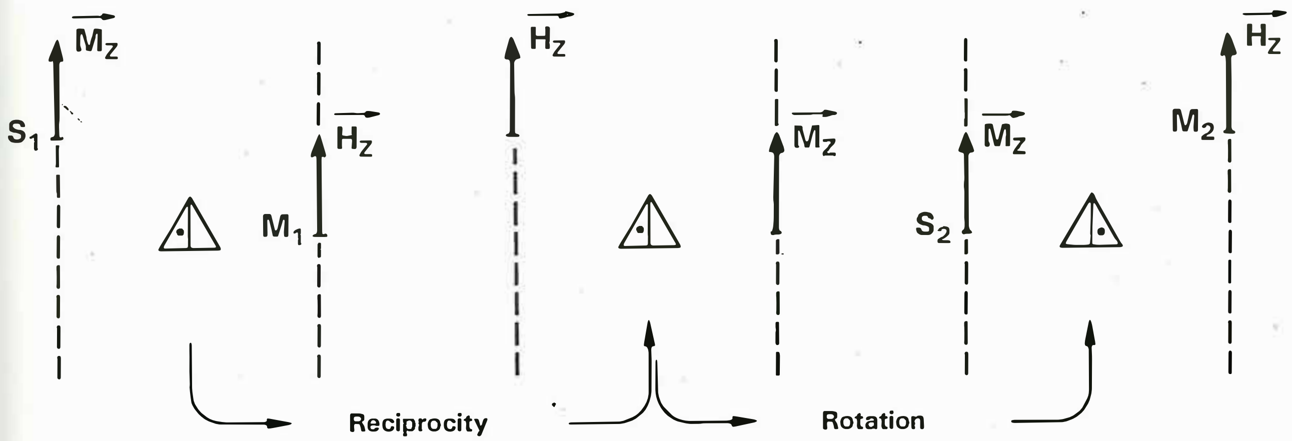

Figure 2: Symmetry with respect to a vertical plane

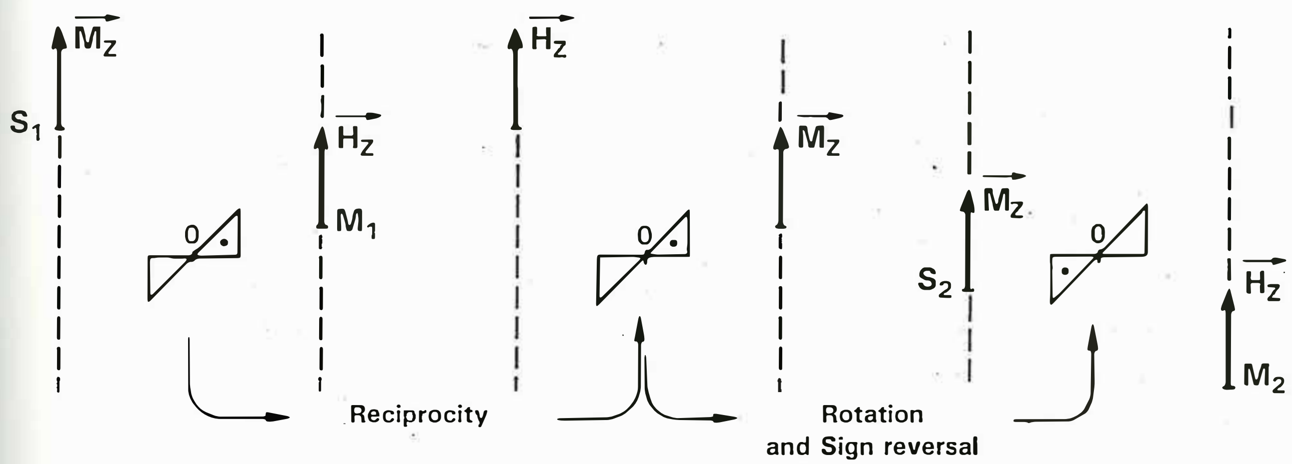

Figure 3: Symmetry with respect to a central point

SYMMETRY PROPERTIES
FOR THE ELECTROMAGNETIC TOMOGRAPHY
(Verical magnetic dipolar source M_z)

Model geometry / Response function	Sym/ Horizontal plane	Sym/ Vertical plane	Sym/ Centre
Sym/centre	$+H_z$, E_y $-H_x$	0	0
Sym/First Diagonal	0	$+H_z$	0
Sym/Second Diagonal	0	0	$+H_z$

Table 1: Correspondence between model symmetry and tomogram symmetry

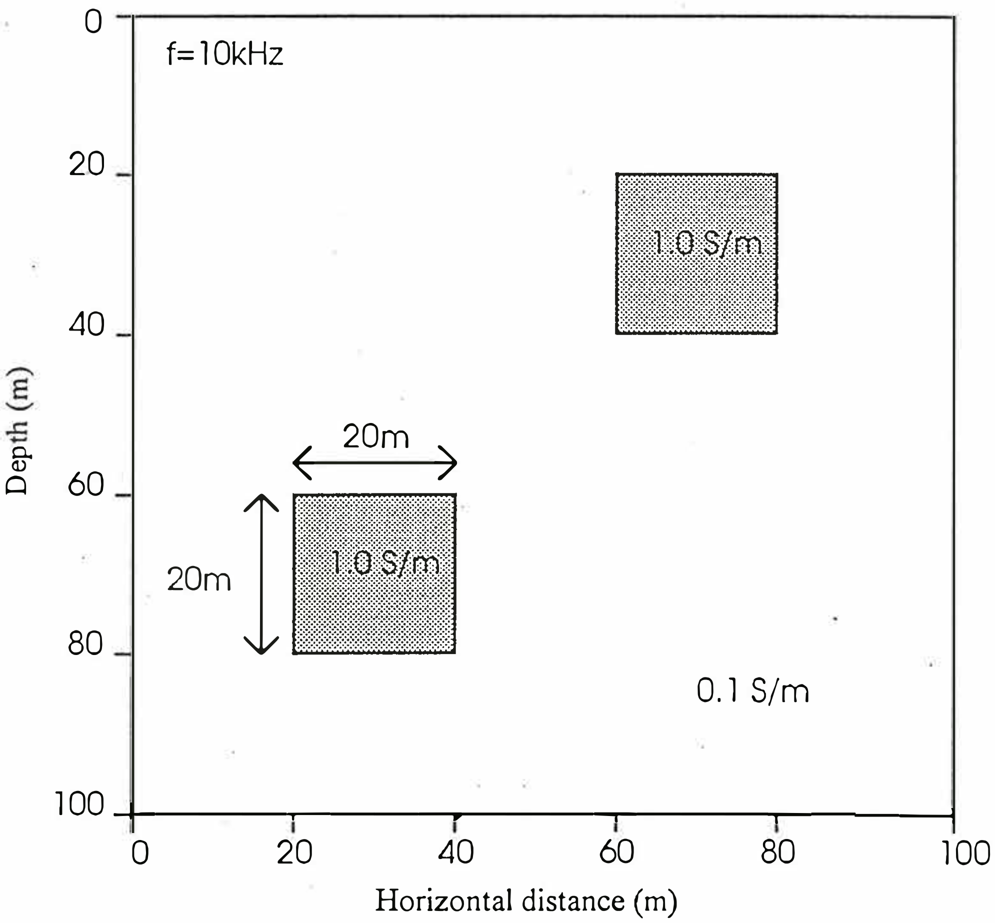

Figure 4: Model geometry

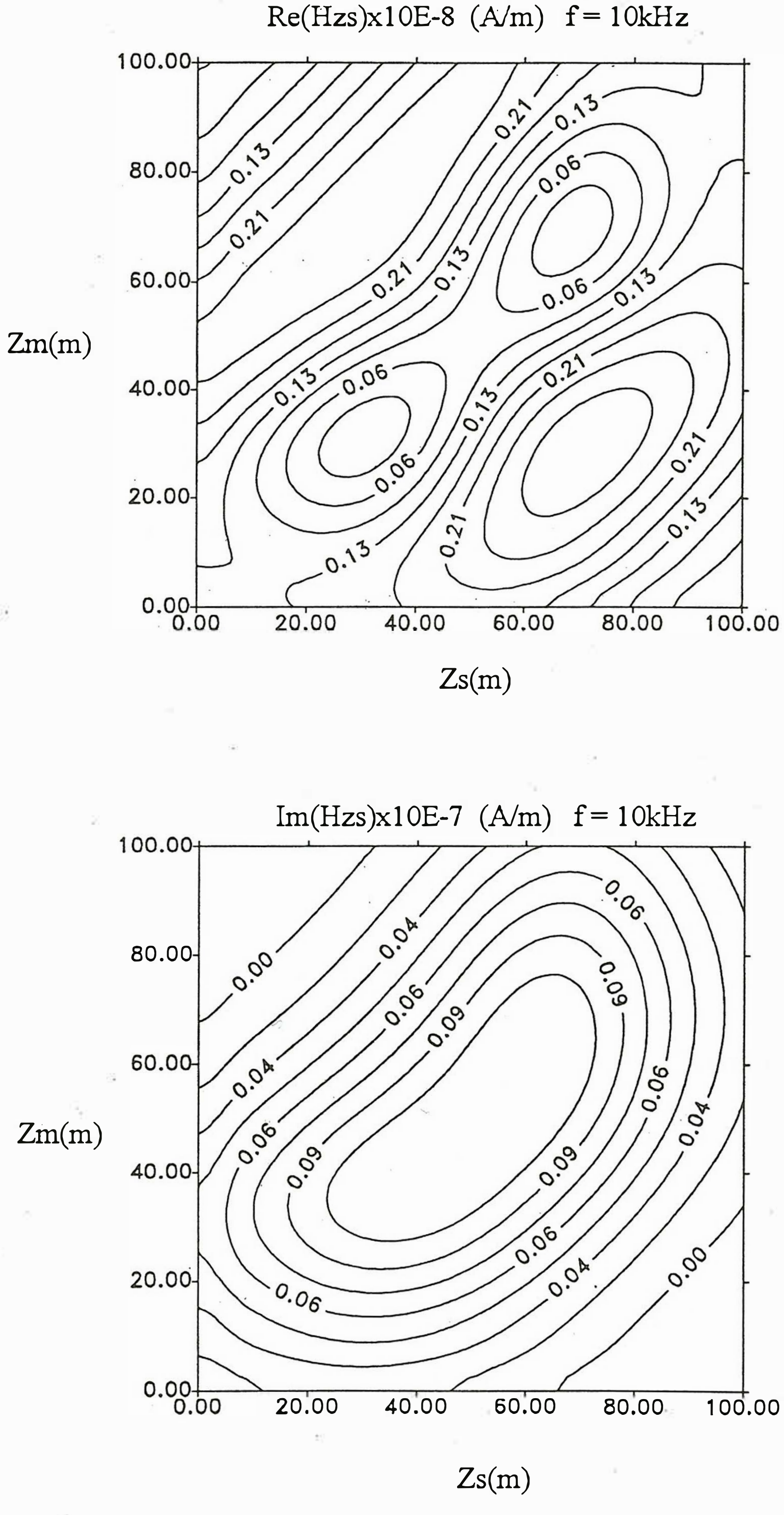

Figure 5: Tomograms for the vertical secondary magnetic field