An Improved Phase-Dependent Regional Strike Angle for the Magnetotelluric Impedance Tensor

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This paper deals with the problem of determining regional strike from a magnetotelluric impedance tensor in the presence of galvanic distortion. For a more in-depth discussion on the sources of and proposed solutions to near surface and galvanic distortion the reader is referred to Jiracek (1990), Groom and Bailey (1989,1991) and Groom and Bahr (1992).

Bahr (1988,1991) discussed galvanic distortion and presented a classification of distortion types. He introduced the superimposition model in which the Earth is viewed as having a 2D regional structure but with local or near surface galvanic distortion. Using a real frequency-independent distortion matrix to represent the galvanic distortion, the measured impedance tensor takes the form

$$\tilde{Z} = \tilde{R}(\theta)\tilde{C}\tilde{Z}_2\tilde{R}^T(\theta) \tag{1}$$

$$\begin{aligned}
\tilde{R}(\theta) &= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \text{ is the rotation matrix} \\
\tilde{C} &= \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \text{ is the distortion matrix} \\
\tilde{Z}_2 &= \begin{bmatrix} 0 & Z_{xy_r} \\ Z_{yx_r} & 0 \end{bmatrix} \text{ is the impedance of the regional Earth}
\end{aligned}$$

In a coordinate system with x north, y east, z down, and rotation defined as above, the strike of $\tilde{Z}$ in Equation 1 is $-\theta$ east of north.

In the coordinate system of the regional structure (i.e. with $\theta = 0$) $\tilde{Z}$ has the form

$$\tilde{Z} = \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{bmatrix} 0 & Z_{xy_r} \\ -Z_{yx_r} & 0 \end{bmatrix} = \begin{bmatrix} -C_{xy}Z_{yx_r} & C_{xx}Z_{xy_r} \\ -C_{yy}Z_{yx_r} & C_{yx}Z_{xy_r} \end{bmatrix} \tag{2}$$

The key feature of $\tilde{Z}$ in Equation 2 is that the phases of the elements in the left-hand column are the same, as are the phases of the elements in the right-hand column.

To allow for cases where there is no strike angle at which the phases in each column are equal, Bahr (1991) extends the superimposition model to allow for a given phase difference δ_ϕ between the elements of each column. Here $\tilde{Z}$ takes the form

$$\tilde{Z} = \tilde{R}(\theta) \begin{bmatrix} -C_{xy} Z_{yx_r} \exp i\delta_\phi & C_{xx} Z_{xy_r} \\ -C_{yy} Z_{yx_r} & C_{yx} Z_{xy_r} \exp -i\delta_\phi \end{bmatrix} \tilde{R}^T(\theta) \tag{3}$$

Bahr (1991, Eqn. 30) presents an approximate expression (α_{30}) for this strike which I found to be inaccurate and occasionally undefined. A good example of this is shown in Figure 1.

I have extended Bahr's analysis to allow for the case where the magnitudes of the phase differences are the same, but their signs may differ and have determined a more accurate expression for the strike angle. In this analysis, $\tilde{Z}$ takes the form

$$\tilde{Z} = \tilde{R}(\theta)\tilde{C}\tilde{\Delta}\tilde{Z}_2\tilde{R}^T(\theta) \tag{4}$$

where

$$\tilde{\Delta} = \begin{bmatrix} e^{i\delta_\phi} & 0 \\ 0 & e^{\pm i\delta_\phi} \end{bmatrix}$$

We wish to find the angle α_E for which the following is true

$$\tilde{R}(\alpha_E)\tilde{Z}\tilde{R}^T(\alpha_E) = \tilde{C}\tilde{\Delta}\tilde{Z}_2$$

or explicitly

$$\begin{bmatrix} Z_{xx}^{\alpha_E} & Z_{xy}^{\alpha_E} \\ Z_{yx}^{\alpha_E} & Z_{yy}^{\alpha_E} \end{bmatrix} = \begin{bmatrix} -C_{xy} Z_{yx_r} \exp i\delta_\phi & C_{xx} Z_{xy_r} \\ -C_{yy} Z_{yx_r} & C_{yx} Z_{xy_r} \exp \pm i\delta_\phi \end{bmatrix}$$

There may be several such values of α_E in which case I choose that for which $|\delta_\phi|$ has the smallest value.

The phase of a complex number is given by

$$\varphi(c) = \tan^{-1}\left(\frac{\Im(c)}{\Re(c)}\right)$$

and the phase difference between two complex numbers is

$$\varphi(c_1) - \varphi(c_2) = \varphi(c_1/c_2) = \varphi(c_1 c_2^*)$$

We wish to find the angle α_E at which the magnitude of the phase difference between $Z_{xx}^{\alpha_E}$ and $Z_{yx}^{\alpha_E}$ is equal to the magnitude of the phase difference between $Z_{yy}^{\alpha_E}$ and $Z_{xy}^{\alpha_E}$.

$$\varphi(Z_{xx}^{\alpha_E} Z_{yx}^{\alpha_E *}) = \pm\varphi(Z_{yy}^{\alpha_E} Z_{xy}^{\alpha_E *})$$

Considering the two signs separately we have

$$\varphi(Z_{xx}^{\alpha_E} Z_{yx}^{\alpha_E *} Z_{xy}^{\alpha_E} Z_{yy}^{\alpha_E *}) = 0$$

or

$$\varphi(Z_{xx}^{\alpha_E} Z_{yx}^{\alpha_E *} Z_{xy}^{\alpha_E *} Z_{yy}^{\alpha_E}) = 0$$

Using the condition

$$\varphi(c) = 0 \iff \Im(c) = 0 \qquad (\text{assuming } \Re(c) > 0)$$

we have

$$\Im(Z_{xx}^{\alpha_E} Z_{yx}^{\alpha_E *} Z_{xy}^{\alpha_E} Z_{yy}^{\alpha_E *}) = 0 \tag{5}$$

or

$$\Im(Z_{xx}^{\alpha_E} Z_{yx}^{\alpha_E *} Z_{xy}^{\alpha_E *} Z_{yy}^{\alpha_E}) = 0 \tag{6}$$

These equations can be expressed as two polynomials in $t = \tan(\alpha_E)$.

The coefficients of the polynomial derived from Equation 5 are

$$\begin{aligned}
t^8 &: \Im(P_4Q_4) \\
t^7 &: \Im(P_4Q_3 + P_3Q_4) \\
t^6 &: \Im(P_4Q_2 + P_3Q_3 + P_2Q_4) \\
t^5 &: \Im(P_4Q_1 + P_3Q_2 + P_2Q_3 + P_1Q_4) \\
t^4 &: \Im(P_4Q_0 + P_3Q_1 + P_2Q_2 + P_1Q_3 + P_0Q_4) \\
t^3 &: \Im(P_3Q_0 + P_2Q_1 + P_1Q_2 + P_0Q_3) \\
t^2 &: \Im(P_2Q_0 + P_1Q_1 + P_0Q_2) \\
t^1 &: \Im(P_1Q_0 + P_0Q_1) \\
t^0 &: \Im(P_0Q_0)
\end{aligned}$$

where

$$\begin{array}{lll}
P_4 = A + B & Q_4 = -A + B & A = S_2^* D_1 - D_2^* S_1 \\
P_3 = 2(C - D) & Q_3 = 2(-C - D) & B = D_2^* D_1 - S_2^* S_1 \\
P_2 = 2(A - 2E) & Q_2 = 2(-A + 2E) & C = D_1^* D_1 - S_2^* S_2 \\
P_1 = 2(-C - D) & Q_1 = 2(C - D) & D = D_1^* S_1 + D_2^* S_2 \\
P_0 = A - B & Q_0 = -A - B & E = D_1^* S_2 + S_2^* S_1
\end{array}$$

and

$$S_1 = Z_{xx} + Z_{yy} \quad D_1 = Z_{xx} - Z_{yy} \quad S_2 = Z_{xy} + Z_{yx} \quad D_2 = Z_{xy} - Z_{yx}$$

To obtain the coefficients of the polynomial derived from Equation 6 replace Q_i with Q_i^* above.

The solution to Equation 5 is the tan of that angle α_E at which the phase difference between the elements of the two columns of the impedance tensor is δ_ϕ. The solution to Equation 6 is that angle at which the phase differences are equal in magnitude but differ in sign.

I use Laguerre's numerical method (Press *et al.*, 1986, p263) for solving the two polynomials. Each polynomial has eight roots, some of which may be complex. The real roots are the tangents of several possible strike angles. The angle of interest is that for which $|\delta_\phi|$ in Equation 4 has the smallest value.

If $\alpha(\tilde{Z})$ denotes the strike of tensor $\tilde{Z}$ then, the following should be true for any value of θ

$$\alpha(\tilde{R}(\theta)\tilde{Z}\tilde{R}^T(\theta)) = \alpha(\tilde{Z}) - \theta \tag{7}$$

To determine if this is true for α_E or α_{30}, I constructed a rotated, distorted impedance tensor $\tilde{Z}$

$$\tilde{Z} = \tilde{R}(\alpha)\tilde{C}\tilde{\Delta}\tilde{Z}_2\tilde{R}^T(\alpha)$$

with

$$C_{xx} = 0.8 \quad C_{xy} = -0.05 \quad C_{yx} = 0.1 \quad C_{yy} = 0.9$$

$$Z_{xy_r} = 10400 + 4600i \qquad Z_{yx_r} = 6300 + 5800i$$

$$\delta_\phi = 5° \qquad \alpha = -45°$$

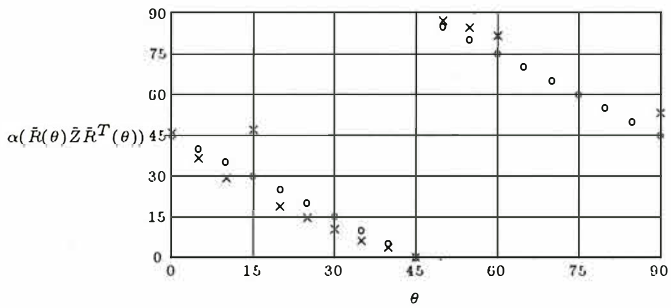

Figure 1: A test of the accuracy of two phase-dependent strike angles, α_E ($\circ$) and α_{30} ($\times$) for a synthetic impedance tensor. The strike angles were calculated for rotations of 0°, 5°, 10°... Wherever a $\times$ is absent this indicates that α_{30} is undefined.

Figure 1 shows the values of α_E and α_{30} for several values of θ for the rotated tensor $\tilde{R}(\theta)\tilde{Z}\tilde{R}^T(\theta)$. It is obvious that α_E fulfills the condition in Equation 7 and that α_{30} can be inaccurate by as much as 15°.

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