3D NLCG inversion algorithm for CSEM data - discussion about mesh design -

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Introduction

We want to discuss the mesh design for 3D non-linear conjugate gradient (NLCG) inversion of controlled-source electromagnetic (CSEM) data using our 3D finite-element modelling code [1] implemented in the inversion software *emilia* [2, 3].

Gradient Computation

For the NLCG algorithm, gradients of the objective function Φ wrt. the model parameters m_k have to be calculated. The gradient of the data functional $\frac{\partial \Phi_d}{\partial m_k}$ is the crucial part in this computation obtained after [4] as

$$\frac{\partial \Phi_d}{\partial m_k} = -2Re\Bigg[-\sum_{n=1}^{N} (\Delta Z_n)^{*1} \gamma_n^T A^{-1} \left(\frac{\partial A}{\partial m_k} E_1\right) - \sum_{n=1}^{N} (\Delta Z_n)^{*2} \gamma_n^T A^{-1} \left(\frac{\partial A}{\partial m_k} E_2\right) \Bigg], \tag{1}$$

where $\Delta Z_n = [(\mathbf{d}_n^{obs} - F_n(\mathbf{m}))/\epsilon_n^2]$, n is the number of data, A the system matrix of the forward problem and E_1 and E_2 the forward solutions of two source polarisations. The factor γ comprises the interpolator functions of the finite-element method and linear combinations of several electric and magnetic fields.

Regularisation

For the model regularisation term, we have to calculate the inverse model covariance matrix $C_m^{-1} = C^T C$ (C: smoothness matrix), which we construct with a first order difference operator as suggested in [5] for tetrahedral elements. Taking the neighbouring element relations into account, we know, that two neighbouring elements i, j share a face f, while only inner faces and only Earth cells are considered, so that

$$C_{f,i} = -1 \quad \text{and} \quad C_{f,j} = 1. \tag{2}$$

References

[1] Paula Rulff, Laura Maria Buntin, and Thomas Kalscheuer. Efficient goal-oriented mesh refinement in 3-D finite-element modelling adapted for controlled source electromagnetic surveys. *Geophysical Journal International*, 227(3):1624–16450, 2021.

[2] Thomas Kalscheuer, Laust B. Pedersen, and Weerachai Siripunvaraporn. Radiomagnetotelluric two-dimensional forward and inverse modelling accounting for displacement currents. *Geophysical Journal International*, 175(2):486–514, 2008.

[3] Dominik Zbinden. Inversion of 2d magnetotelluric and radiomagnetotelluric data with non-linear conjugate gradient techniques. Master's thesis, Uppsala University, 2015.

[4] Gregory A Newman and David L Alumbaugh. Three-dimensional magnetotelluric inversion using non-linear conjugate gradients. *Geophysical journal international*, 140(2):410–424, 2000.

[5] Thomas Günther, Christian Rücker, and Klaus Spitzer. Three-dimensional meodelling and inversion of dc resistivity data incorporating topography - ii. inversion. *GJI*, 166:506–517, 2006.

1. Mesh Design

The general **correctness of the gradient computations** can be verified by comparing $\frac{\partial \Phi_d}{\partial m_k}$ (setting $\Delta Z_n = 1.0$) with sensitivities obtained with the perturbation method (Table 1) using one active receiver.

Table 1: Comparison of element gradients obtained with eq. 1 with element sensitivities obtained using the perturbation method (model parameter perturbation $dm = 0.01$) of a mesh refined only in the central region (Fig. 1). For larger elements (e.g. element 4), the perturbation method is quite sensitive to the choice of dm, which shows, that the method only works for a gross verification of the gradient values.

element	$\frac{\partial \Phi_d}{\partial m_k}$ for Z_{xy}	perturbation for Z_{xy}	$\frac{\partial \Phi_d}{\partial m_k}$ for Z_{yx}	perturbation for Z_{yx}
1 (receiver 1)	21.1	20.9	-7.2	-6.3
2 (receiver 2)	13.5	13.5	-4.3	-4.3
3 (receiver 3)	9.4	9.4	-6.9	-6.9
4 (250 m|0 m|300 m)	-6.7	-7.5	1.7	1.5

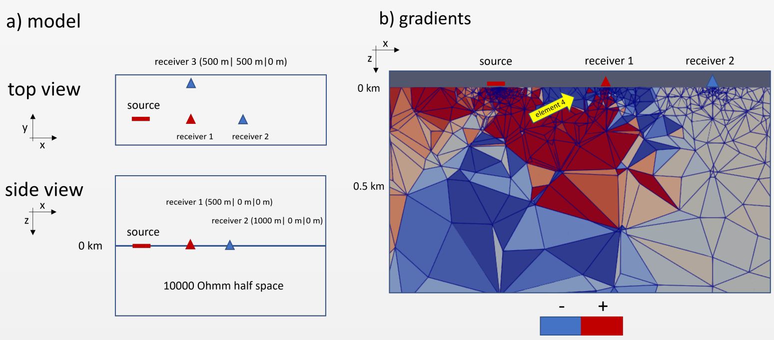

Figure 1: Slices through a 3D model (a) and the corresponding finite-element mesh with coloured gradients for the Z_{xy} data component of receiver 1 refined only in the central region around the source and receivers (b).

How to design the tetrahedral meshes, so that the gradient computations are accurate enough?

- **Receiver elements need to be small** to obtain accurate forward responses at the receivers.
- Do we have to design **more regular meshes** for gradient computation?
- How dense should a mesh be between receiver sites for expedient gradient computation?
- Is it meaningful to use **different meshes for forward computation and inversion (dual-mesh approach)**, although the forward solution vectors and system matrices (cf. eq. 1) can be re-used for gradient computation?

2. Refinement

Is automatic mesh refinement at every inversion step expedient for 3D inversion?

- only practicable, when using a dual-mesh approach
- idea: run the inversion on a coarse mesh without refinement, refine the best-fitting model, this refined model serves as the start model for a second inversion (cf. Fig. 2).

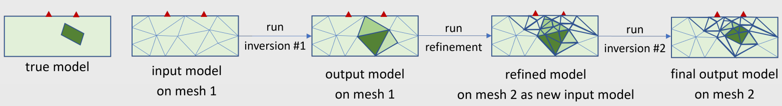

Figure 2: Sketch of a mesh refinement strategy for inversion.

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