P2-1

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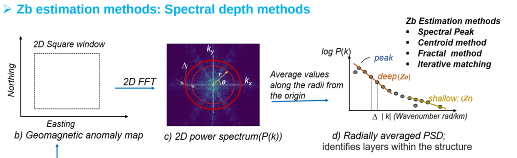
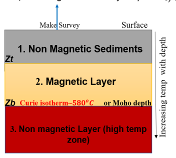
Testing the use of elastic-net constrained inversion for magnetic bottom depth(*Zb*) estimation.

Introduction and motivation:

Depth to bottom of magnetization (*Zb*) as controversial as it is has been a very important parameter that aids in geothermal system characterisation, heat flux & heat loss evaluation, and geothermal exploration. There are broadly two different schools of thought regarding *Zb.* The first has it that maximum depth of magnetization coincides with Curie temperature of magnetite (580^oC) or moho depth whichever is shallower, and is called Curie point depth(Wasilewski et al., 1979,Tenaka,*et.al,*1999); While the second suggests that this depth goes far beyond curie depth and into the mantle(McEnroe *et.al,*2018). The predominant method of estimation of Zb has been by the analysis of power spectral density of magnetic data(Fig1.0), though with notable assumptions and limitations. Here, we test with synthetic data an estimation of *Zb* through sparse 3D magnetic susceptibility (χ) inversion, with elastic-net (L1+L2 Norm combined) constraints(Fig2.0). The deepest non-zero susceptibility layer within a region is assumed a direct indicator of the magnetization depth. We showed that the sparsest solution of the ill-posed problem by L1-norm is the deepest possible distribution of magnetic anomalies, but there exists an optimal compromise between the sparsest and smoothest solution(L2) as the elastic-net solution which yield a better Zb estimate. We also remark that this inversion method is good in obtaining the 3D anomaly of structures as well.

➢ Zb estimation methods: Spectral depth methods



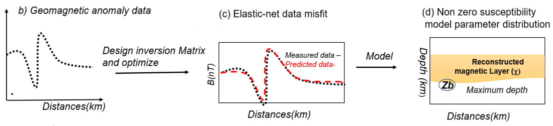


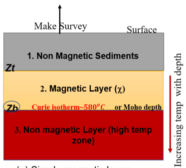
(a) Simple magnetic layers.

Fig1(b-d): schematics of spectral method of magnetic Zb estimate**;**

- peak relates to the thickness of a randomly magnetized "magnetic layer" (Spector & Grant, 1970).
- slope at small wavenumbers relates to the centroid depth (*Zo*) of parallelepiped sources (Bhattacharyya & Leu, 1975).
- slope at large wavenumbers relate to the top depth (*Zt*) of sources (Bhattacharyya & Leu, 1975).

➢ New Idea: Use elastic-net sparsity constrained inversion.





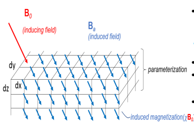
(a) Simple magnetic layers.

Claims

- Claim 1: The sparsest distribution of χ that fits the data is the deepest possible distribution of anomalies → Lasso(L1).
- Claim 2: There exists an optimal compromise between the sparsest(L1) and smoothest distribution(L2) → Elastic net .
- ❖ To infer *Zb,* use the maximum depth of non-zero χ layer as Zb indicator.

Fig2(b-d): Schematics of the new idea by elastic-net sparsity constrained inversion.

Inversion methodology.



- The magnetic field of a rectangular prisms was approximated by the field of a magnetic dipole in Eqn 1.0 to aid inversion speed.
- $\boldsymbol{B_a(r)} = m\frac{C_m}{r^3}\left[3(\hat{\boldsymbol{m}}.\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \hat{\boldsymbol{m}}\right]$ ***Eqn* 1.0** (Blakely,1995)
- **Forward operator:** superposition of point dipoles.
- dipole moment magnitude m_i =Induced magnetization (χ**B**$_0$) $*$ dipole volume $\nabla V_j(dx*dy*dz)$:
- TMI data: **d** = **B**$_a$· **B**$_0$/B_0 (projection onto main field)

→**Linear Inverse problem:** $\boldsymbol{m^{est}} = \boldsymbol{G^I d}$

- $\boldsymbol{G^I} \in R^{M\,x\,N}$ is the inversion matrix, where M>>N, underdetermined and ill posed .
- $\boldsymbol{d} \;=\; [B_1, B_2 \dots B_N]^{\boldsymbol{T}} ; \in R^N$ is the anomalous magnetic vector field
- $\boldsymbol{m} \;=\; [\chi_1, \chi_2 \dots \dots \chi_M]^{\boldsymbol{T}}; \chi \in R^M$ is the magnetic susceptibility as the model parameter
- Diagonal weighting Matrix of the form => $w_j = {}^1\!/\!\sqrt{\|g_j\|}$ was used.

Objective function: $$\min \rightarrow \phi(m) = \underbrace{\frac{1}{2}\|\mathbf{d} - \mathbf{G}m\|^2}_{Data\ misfit} + \underbrace{\lambda(1-\alpha)\frac{1}{2}\|m\|_{L2}^2 + \lambda\alpha\|m\|_{L1}}_{(L1+L2)Elastic-Net\ constraints}$$

- λ (lambda) is the regularization parameter (user defined : λ>0).
- The sequence of models implied by λ was fitted by coordinate descent method, using generalised linear model function, both of (Friedman, et.al,(2010)). The optimal λ was selected by cross validation.
- α (alpha) is the elastic net mixing parameter (0>α<1) : Controls L1 and L2 mix. (Note that $\phi(m)$ will reduce to ridge regression (L2) if α= 0 and to lasso (L1) if α =1).
- Sample all α from [0.1 to 0.9] and select α that gave the smallest model norm($\boldsymbol{m^{est}}$ - $\boldsymbol{m^{true}}$).

Test model: Why elastic-net sparsity is better than L1 and L2 for Zb estimate.

➢ **Make synthetic magnetic data(Fig3a)**

- Assumed induced magnetization only, [D,I]=[30,45], B0=42000nT.
- Used Born approximation. Added Gaussian noise (std of 1nT).

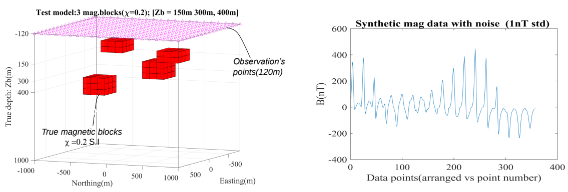


Fig3(a) Synthetic magnetic models with Zb of 0.15-0.4km; and synthetic magdata.

➢ **Inversion's results: Features of L2− (smooth model), L1−(sparse model), and elastic−net sparse model.**

The inverse problem is highly nonunique and regularization strongly determines solution. The Inversion result of Fig3a is shown below.

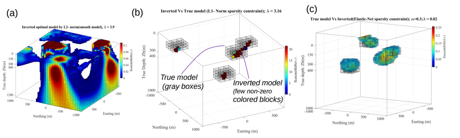


Fig4(a-c): optimal inverted 3D non-zero model parameters by L2,L1 & elastic-net Norm.

Fig4(a-c)
(a) L2: located the 3 blocks, but larger volumes of the magnetized regions and χ is smaller.
(b) L1: located the 3 blocks also but very sparse and the actual χ is larger.
(c) Elastic-net: The volume, shape and χ of the magnetized blocks are well reproduced, fairly comparable to the true model. (Suitable for Zb estimates)

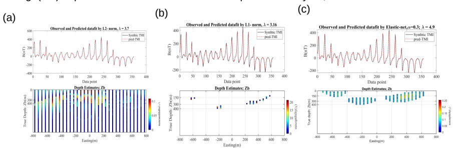


Fig5(a-c): Observed & predicted data misfit, and Zb estimates using deepest non−zero χ layers.

Fig5(a-c)
- (a) L2 Shows the smoothest solution that fits the data, but Zb cannot be discerned.
- (b) L1 Shows the sparsest solution that fits the data also, the deepest possible distribution of the magnetic anomaly but Unsuitable for Zb estimation.
- (c) Elastic net shows an ideal compromise of smoothness & sparsity and recovered true Zb, χ and dimensions approximately.(Useful for Zb)

Test model 2: Elastic-net inversion for deeper anomalies.

➢ **Make synthetic magnetic data(Fig6a)**

- Assumed induced magnetization only, [D,I]=[30,45], B0=42000nT.
- Used Born approximation. Added Gaussian noise (std of 5nT).

➢ **Results from the elastic-net inversion of Fig6a)**

- Successfully located and recovered all the 11 magnetic blocks and true *Zb* of between 18km -28km (Fig6b).
- Recovered χ of 0.2 and the blocks dimensions approximately(Fig6b.)

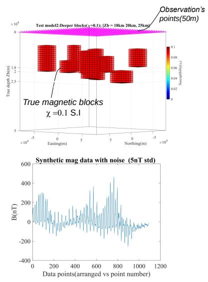


Fig6(a) Eleven synthetic magnetic model blocks with maximum Zb of 18km-25km; and synthetic magdata.

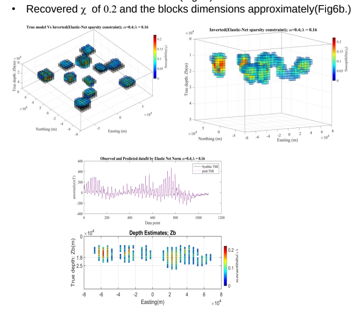


Fig6(b) Elastic-net inversion's results and data misfit.

Preliminary results and conclusions.

1. The sparsest solution corresponds with the **deepest** possible distribution of $\boldsymbol{\chi}$ → Lasso(L1) Fig4a;but not suitable for Zb estimate.
2. Elastic-net Fig4c and Fig6b clearly show an optimal compromise between the sparsest(L1) and smoothest solution(L2), and recovered the true ***Zb***, χ and dimensions. We view this as suitable for Zb estimation and will be used for field data.
3. The inversion method is good in obtaining 3D anomaly of structures as well.
4. Resolving the balance between L1 and L2 using alpha requires care and time.

Work in progress

We intend to apply this on real magnetic field data and compare the Zb with those obtained from the established spectral methods.

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