



A MATLAB FE Library for the Simulation and Inversion of EM Problems

Mathias Scheunert^{1*}, Jan Blechta², Ralph-Uwe Börner¹, Oliver Ernst² & Klaus Spitzer¹

MT 2D & 3D

Reconstruction of seven conductive bodies and two conductive plates embedded in a homogeneous half-space from the synthetic data set COPROD_251.

Preliminary inversion of a GNS MT data set acquired in the vicinity of mount Tarawera, New Zealand.

Discretization of the Tarawera region using Gmsh.

DC 2D, 2.5D & 3D

Reconstruction of a conductive zone 'Erzgang' beneath a meadow of the university campus.

2D & 3D MESHING BY Gmsh [6]

- Triangle & tetrahedra meshing
- Boolean & refinement operators
- Handy .geo format

2D & 3D FEM BUILDING BLOCKS [9]

Matrix assemblies: Div-Grad, Mass, Curl-Curl

Element types and orders: Lagrange 1. to n-th order, Raviart-Thomas 1. order, Nédélec 1. & 2. order

Boundary conditions: Robin, Dirichlet inh. & hom., Neumann inh. & hom.

Source types: Point, Line, Face, Volume

3D VISUALIZATION BY ParaView [2]

- Export of .vtk & .xdmf

SOLVER

- MUMPS [1] parallel direct
- HSL_M120 [5] AMG

INVERSION BUILDING BLOCKS

- Scalable iterative solver [3] for H¹-regularized normal equations [14]
- Gauss-Newton approach
- Explicit Jacobian assembly
- Line search (Armijo) [10]

CSEM

Sensitivity distribution (real part) for an exciting line source (~40Hz) on top of a layered half-space and a B_r receiver coil at a depth of about 220m.

- Line sources (wires / loops supported)
- Data types: E, H
- Inversion prototype available
- Nédélec elements, hom. Dirichlet BC, total field
- Adaptable for HEM applications!

IP

- Complex-valued conductivities
- 2D, 2.5D, 3D

READY TO USE (top arc)

UNDER DEVELOPMENT (right arc)

FUTURE WORK (bottom arc)

TEG

- Spatial discretization available (CSEM)
- To do:
 - Measurement operator
 - Time integration by rational best approximation [4]
 - Assembly of Jacobian

Code development

- Complete documentation
- Expansion of CI test environment
- Enhance set up of inversion parameters
- Data visualization
- GUI interface
- Adaptive mesh refinement, secondary field, total variation
- In short: Enough tasks for many rainy afternoons ... :-)

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TU Bergakademie Freiberg¹
Institut of Geophysics and Geoinformatics
Gustav-Zeuner-Str. 12, 09366 Freiberg
mathias.scheunert@mailserver.tu-freiberg.de*

Chemnitz University of Technology²
Faculty of Mathematics
Reichenhainer Str. 41, 09107 Chemnitz

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