TECHNISCHE UNIVERSITÄT BERGAKADEMIE FREIBERG
Die Ressourcenuniversität. Seit 1765.

UNCERTAINTY QUANTIFICATION USING BAYESIAN INVERSION APPLIED TO THE DC RESISTIVITY PROBLEM

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INTRODUCTION

A geophysical investigation of a subsurface is an ambiguous endeavor. In the geoelectrical field inversion results are inconclusive because of the equivalence principle. Thus, results need to be quantified and reviewed.
One way to verify an inversion outcome is by quantifying uncertainties, that arise from imperfect input data. The Bayesian approach considers every parameter as a random number described by a probability distribution. Therefore, the outcome of the Bayesian inversion is not just one model, but a variety of possible models.
This work emphasizes the benefits of multiple model outcomes and strives to examine given data more extensively.

THEORY

- Bayes' theorem:

$$p(\boldsymbol{m}|\boldsymbol{d}) \propto p(\boldsymbol{d}|\boldsymbol{m}) \times p(\boldsymbol{m}) \;\rightarrow\; posterior \propto likelihood \times prior\ knowledge$$

d recorded data, **m** random model parameter

- Likelihood function $p(\boldsymbol{d}|\boldsymbol{m})$: normal Gaussian distribution.
- Prior knowledge $p(\boldsymbol{m})$ contains pre-information on investigated area
- Markov-Chain-Monte-Carlo (MCMC) method solves inversion problems by sampling from posterior distribution without its explicit knowledge
- Requirement of high number of sampling iterations
  → High cost of calculation time
- Metropolis-Hastings-Algorithm (MHA) realizes MCMC method with an acceptance ratio α. Each iteration, MHA proposes new values for set of model parameters and generates uniform random number u between 0 and 1. Acceptance ratio compares posterior probability of the proposed value with the previous one.

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Pseudo code:
Repeat for total number of iterations
Generate a uniform random number u
If u < α
    set x_proposed to x_new    (accepting)
else
    set x_proposed to x_old    (rejecting)
```

RESULT AND OUTLOOK

- Preliminary results for a 1D problem
- Improvement of 2D algorithm by adjusting prior distribution for geometrical parameters and execution with larger number of iterations.
- Result evaluation by MAP-estimator
- Introduction of new model class: complex system of layers represented by checkerboard pattern → analysis only for resistivities

REFERENCES

- Ray, A., Alumbaugh, D. L., Hoversten, G. M. and Kerry, K., 2013, Robust and accelerated Bayesian inversion of marine controlled-source electromagnetic data using parallel tempering: Geophysics, **78**, no. 6, E271–E280.

APPLICATION: 1D MODEL

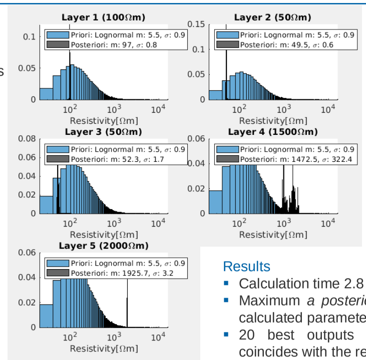
Problem

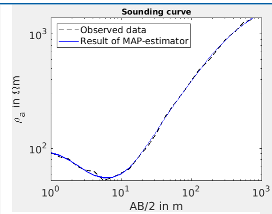
- DC resistivity Depth Sounding
- Model with five layers, layer thickness is fixed
- Gaussian noise is added

Resistivity [Ωm]	Thickness [m]
100	1
50	5
50	5
1500	10
2000	∞

MCMC Data

- Prior: Lognormal distribution with log mean m = 5.5 and log standard deviation σ = 0.9
- 39000 iterations

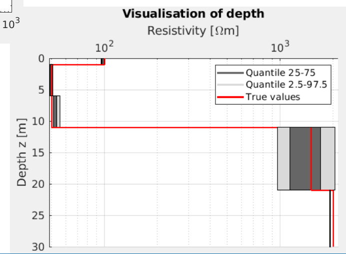




Results

- Calculation time 2.8 s.
- Maximum *a posteriori* probability (MAP) estimator sorts calculated parameter values by posteriori probability
- 20 best outputs according to the MAP estimator coincides with the recorded data significantly
- Tightening and shifting of the posterior distribution in comparison to the prior
→ information gain

- Large standard deviation in fourth layer, but true value located within 75 % quantile and resolved separately from substratum
- Standard deviation increase from second to third layer
  → conductive layer above channels electrical currents
- Large spread of 750 m results in clear resolution of last layer



APPLICATION: 2D MODEL

Problem

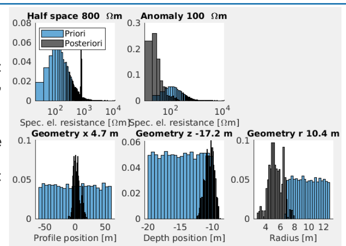
- Cylindrical anomaly in half space.
- Randomized geometrical parameters: its position regarding the profile x, depth z and radius r.

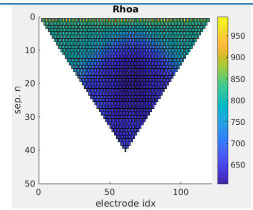
MCMC Data

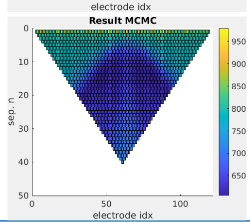
- Prior for resistivity parameters: same as before
- Prior for geometrical parameters: uniform distribution
- 6000 iterations

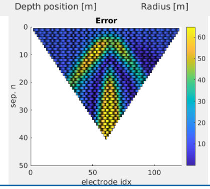
Results

- Calculation time 3.1 h.
- Result not generated by a MAP estimator, but by mean over chain minus burn-in amount
- Only result of half space reveals true value
- Better resolution of anomaly through increasing number of iterations, investigating prior distribution more carefully.
- Resistivity of anomaly and its radius not independent
  → Consideration of equivalence principle



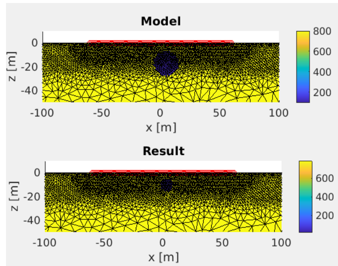






	Model	Result Mean	Result σ
Half space	800 Ωm	787.4 Ωm	23.5 Ωm
Anomaly	100 Ωm	38.7 Ωm	31.2 Ωm
x	4.7 m	3.0 m	5.6 m
z	-17.2 m	-10.5 m	0.7 m
r	10.4 m	5.3 m	0.9 m

Tab. 1



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